SYNCHRONOUS RELUCTANCE MACHINES—A VIABLE ALTERNATIVE FOR AC DRIVES?

T. A. LIPO

University of Wisconsin—Madison
1415 Johnson Drive
Madison, Wisconsin 53706, USA

Abstract Although design of the variable reluctance (switched reluctance) type of synchronous machine has experienced intense activity in recent years, relatively little effort has been expended on improving the torque capability of the synchronous reluctance type of motor drive. Based on the analysis presented in this paper, it appears that substantial improvements can be made in the design of such motor drives resulting in performance characteristics which match or, indeed, perhaps even exceed that of the induction machine.

Introduction

In the past several decades remarkable progress has been made in the development of induction motor drives using both hard switched dc link and resonant-link schemes which utilize high speed switching devices such as fast recovery bi-polar junction transistors, insulated gate bi-polar transistors (IGBTs) and GTOs. These new resonant converters not only have high power density but also possess very low switching losses since switching of the devices are made at zero-voltage instants and thus enable the total system to operate at very high frequency. Switching losses of hard switched converters (dc voltage link converters) have also improved dramatically due to the substantially reduced turn off time of third generation IGBTs.

While the development of solid state power converters has proceeded at a rapid pace, the corresponding development of electric machines, specifically designed to take advantage of these new, high performance power converters has been disappointingly slow. For example, induction machines are presently designed for inverter supplies in a manner nearly identical to design for conventional sinusoidal supplies. That is, the machine is wound with sinusoidally distributed three phase stator windings and the rotor is constructed with a cast aluminum cage (squirrel cage). The only adjustment for operation from an inverter supply is to eliminate the double cage or deep bar rotor which is commonly used to improve the starting torque for fixed frequency operation. This feature is not required from modern inverter drives since the frequency of the converter can be smoothly varied to provide the necessary starting torque.

One of the few significant new development in electric machinery for variable speed operation is the resurgence of interest in the variable reluctance motor [1-4]. This machine, which was invented in the last century, traditionally found use in small positioning type actuators such as tape drives, disc drives, plotting heads and the like. A schematic plot of this type of machine is shown in Fig. 1. In these machines, both the stator and the rotor are equipped with saliencies. Because of this feature, the number of stator poles must be different from the number of rotor poles in order to prevent locking torques at zero speed. While any number of unequal stator/rotor pole combinations are possible requirements for minimizing the solid-state switching elements generally lead to an even number of stator poles which, in turn, leads to an even number of rotor poles in order to optimize torque.
production. The motor was operated open loop by simply applying voltage sequentially to
the windings concentrically wound around the projecting stator poles. If the load is
predictable and if the frequency of switching (slowing rate) is not too great, the rotor of the
machine follows the excited stator poles in a synchronous fashion without position
feedback.

It was recognized by Lawrenson, that if the current flow into the stator windings is
carefully controlled with respect to the instantaneous rotor position, a substantial increase in
torque production can be realized [1]. While such current regulation schemes were
common in induction motor drives at the time, the concept was considered as innovative
when applied to variable reluctance motors so that the resulting converter/motor
combination was christened the "switched reluctance" motor [1]. This motor created
immediate interest due to the fact that the construction of the rotor as well as the stator was
exceedingly simple and that unidirectional torque could be produced by

![Diagram of Variable Reluctance Motor]

**Fig. 1 Constructional Features of Variable Reluctance Motor.**

unidirectional current in the phases, thereby necessitating only a half bridge (chopper) for
each phase. Since the power density and efficiency of this machine approached or even
exceeded conventional induction motor drives, Ref. 1 initiated an intense period of research
attempting to alleviate the remaining disadvantages of this motor, namely high torque
ripple, acoustic noise and need for a position sensor. Unfortunately, after some 15 years
of development, significant progress has only been made on the position sensor issue [5-
7]. In particular, it now appears that the noise and torque pulsation issues can only be
solved by substantially increasing the air gap which, in turn, effectively eliminates the
power density and efficiency advantages of this machine.

Another recent development in variable speed electric machinery is the strides made
in permanent magnet technology which, in turn, has allowed an increase in the power
density and efficiency of permanent magnet machines [8]. These new machines
constructed of Neodymium-Iron-Boron magnets have the highest efficiency of any motor
yet developed. Major obstacles remain, however, including low Curie temperature and
high manufacturing cost both for the magnets and the machine itself. While the author
believes that such machines will eventually become dominant at nearly all power levels in
the future when oil supplies dwindle, they will remain suitable only for specialized
applications in which cost has a lower priority over the next decade.

Another class of machine, which appears to have been relatively overlooked, is the
synchronous reluctance machine. This machine differs from the variable reluctance motor
in that the stator is constructed from a cylindrical structure in identical fashion to an
induction motor and only the rotor has salient poles. Hence, the stator of both machines can be constructed on the same assembly line, a distinct advantage over the switched reluctance machine. The synchronous reluctance motor also has a long history [9]. Interest in this machine for variable speed applications peaked in the 1960s when fiber spinning plant for synthetic fibers were expanding rapidly. Since the synchronous reluctance motor operates in exact synchronism with the stator frequency, a higher grade of fiber could be produced compared to speed regulated dc motors or induction motors. In these applications, the synchronous reluctance motor was operated in open loop fashion in much the same manner as the early variable reluctance motor drives. In this case the inverter voltage and frequency was set independent of the rotor speed or position and the rotor of the machine followed along synchronously if the load torque and slew rate were not too large. When operated in such fashion, the motor exhibited poor damping so that speed response was very sluggish. Also, with the saliency \((X_d/X_q)\) ratios achievable at that time, the motor operated with a poor power factor and lower efficiency than a comparable induction machine.

With the emergence of induction motor drives with speed control accuracy to a fraction of a per cent, interest in these machines seems to have diminished. However, development of improved rotor structures and stator winding configurations have markedly improved the potential of this machine. In particular, in the sixties, the rotor configuration most often employed was a lamination with punchings or slits oriented so as to create a saliency effect as shown in Fig. 2 [10]. Because of the need for rotor bridges, this type of rotor, however, was only capable of a saliency ratio of two to three. Near the end of this decade, the saliency ratio had improved to values in the region of four to five by utilizing discrete poles mounted on a non magnetic shaft as shown in Fig. 3 [11]. Frame sizes comparable to the equivalent induction motor were claimed [11].

![Fig. 2. Illustrating Rotor Punching of Laminated Type Synchronous Reluctance Motor.](image)

While also maintaining a long history [12,13,14], the benefits of a rotor with axially aligned magnetic laminations have only recently been recognized [15,16]. In this case magnetic laminations are bent to produce paths of minimum reluctance in the direction of the laminations and maximum reluctance in the path normal to the laminations as shown in Fig. 4. Saliency ratios reaching 7-8 have been reported with such a construction [16]. With such high saliency ratios, difficulties associated with relatively low efficiency and power factor have essentially been eliminated. In addition, the benefits associated with careful current control of the armature currents, so effectively utilized on
switched reluctance motors, are only beginning to be quantified [15]. In particular, it can be shown that with the principle of current regulation, the synchronous reluctance motor can also achieve power densities and efficiencies approaching or, perhaps, exceeding the induction machine. Also, it has recently been demonstrated that many of the same advantages of the switched reluctance machine are common to the synchronous reluctance machine as well. For example, the stator of the switched reluctance motor need be fed only with unidirectional currents (two quadrant chopper) in much the same manner as a switched reluctance motor. Also, any number of phases can be considered including two phases, a number which can be achieved only with great difficulty in a switched reluctance motor. In this case the number of solid state switches (transistors) can be reduced to only two; a distinct advantage over a three phase induction motor.

Fig. 4 Axially Laminated Rotor Structure.

drive [15,17]. In addition, the torque pulsation and acoustic noise problems, so intractable in switched reluctance motors, can essentially be eliminated in a synchronous reluctance machine.

It should now be apparent that advances in the design and control of synchronous reluctance machines could open up new areas of research in this field. The purpose of this paper is to demonstrate how the advantages of current regulation of synchronous reluctance
Synchronous Reluctance Machines

Motors, a concept so far not exploited in such machines, result in a torque density very competitive with induction machines.

Comparison of Torque Production in Synchronous Reluctance and Induction Machines

Induction Motor Torque

In order to demonstrate the inherent torque producing capability of a synchronous reluctance machine, it is instructive to compare this machine with that of a squirrel cage induction machine assuming that the stator winding is identical in both cases. Using d-q axis theory [18], the instantaneous torque produced by a symmetrical squirrel cage induction machine can be written:

\[ T_i = \frac{3}{2} P \left( \lambda_{ds} q_s - \lambda_{qs} d_s \right) \]  

where\n
\[ \lambda_{ds}, \lambda_{qs} = d, \text{ and } q-\text{axes stator flux linkage} \]
\[ i_{ds}, i_{qs} = d, \text{ and } q-\text{axes stator current}. \]

The stator fluxes are related to the machine currents by the equations,

\[ \lambda_{ds} = L_s i_{ds} + L_m i_{dr} \]  

\[ \lambda_{qs} = L_s i_{qs} + L_m i_{qr} \]  

In order to evaluate this torque equation it is useful to first express the flux linkages in terms of the rotor rather than the stator fluxes. The rotor flux linkages are similarly related to the machine currents by,

\[ \lambda_{dr} = L_r i_{dr} + L_m i_{ds} \]  

\[ \lambda_{qr} = L_r i_{qr} + L_m i_{qs} \]  

If Eqs. 4 and 5 are solved for \( i_{dr} \) and \( i_{qr} \) and the result substituted into Eqs. 2 and 3 we can obtain,

\[ \lambda_{ds} = L_s i_{ds} + L_m \frac{\lambda_{dr} - L_m i_{ds}}{L_r} \]  

\[ \lambda_{qs} = L_s i_{qs} + L_m \frac{\lambda_{qr} - L_m i_{qs}}{L_r} \]  

Substituting these expressions into Eq. 1 results in

\[ T_i = \frac{3}{2} P \frac{L_m}{L_r} \left( \lambda_{dr} q_s - \lambda_{qr} d_s \right) \]  

It is important to mention that Eq. 8 is valid in any reference frame, i.e. the d-q axes can be spinning or stationary. If the axes are made to rotate with the rotor flux vector, the two components project as constant values on the d-q axes (assuming that balanced conditions prevail). Furthermore, if we align one of the two axes with the flux vector itself (say the d-axis), then the corresponding q-axes component clearly becomes zero. Equation 8 then reduces to

\[ T_i = \frac{3}{2} P \frac{L_m}{L_r} (\lambda_{dr} q_s) = \frac{3}{2} \frac{L_m}{L_r} (\lambda_r i_{qs}) \]  

where \( \lambda_r \) denotes the amplitude of the rotor flux linkages. It is perhaps useful to point out that this equation is the form typically employed for “field oriented” control of induction machines. Since the time rate of change of rotor flux is impressed across the rotor resistance, the rotor current must be in phase with the time rate of change of rotor flux and
therefore in quadrature (at a right angle) with the rotor flux linkage itself. Since the rotor flux linkage is, from Eq. 9, entirely in the d-axis, it follows that the rotor current is entirely in the q-axis. That is,

\[ i_{q} = 0. \]

Equation 4 reduces to

\[ \lambda_{d} = L_{m} i_{ds} \]

and Eq. 9 becomes

\[ T_{i} = \frac{3}{2} \frac{P}{\pi} \frac{L_{m}}{L_{r}} (L_{m} i_{ds}) i_{qs} \]

(11)

**Synchronous Reluctance Motor Torque**

The torque produced by a synchronous reluctance motor is expressed by the identical expression, Eq. 1. That is,

\[ T_{r} = \frac{3}{2} \frac{P}{\pi} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \]

(12)

In this case the rotor currents are zero under normal steady state conditions and

\[ \lambda_{ds} = L_{ds} i_{ds} \]

(13)

\[ \lambda_{qs} = L_{qs} i_{qs} \]

(14)

where \( L_{ds} \) and \( L_{qs} \) are the d- and q-axes inductances and, in this case \( L_{ds} \neq L_{qs} \). Using Eqs. 13 and 14, Eq. 12 can also be written as

\[ T_{r} = \frac{3}{2} \frac{P}{\pi} (L_{ds} - L_{qs}) i_{qs} i_{ds} \]

(15)

In general, \( L_{ds} \) and \( L_{qs} \) contain both leakage and a magnetizing component, i.e.

\[ L_{ds} = L_{ls} + L_{md} \]

(16)

\[ L_{qs} = L_{ls} + L_{mq} \]

(17)

so that Eq. 15 can also be written in terms of magnetizing components as

\[ T_{r} = \frac{3}{2} \frac{P}{\pi} (L_{md} - L_{mq}) i_{qs} i_{ds} \]

(18)

\[ = \frac{3}{2} \frac{P}{\pi} \left( 1 - \frac{L_{mq}}{L_{md}} \right) \left( L_{md} i_{ds} \right) i_{qs} \]

(19)

**Torque Ratio**

The performance of the synchronous reluctance motor relative to the induction motor can now be evaluated by taking the ratio of Eqs. 11 and 19. The result is

\[ \frac{T_{r}}{T_{i}} = \frac{(L_{md} - L_{mq})}{L_{m} L_{r}} \]

\[ = \frac{L_{md}}{L_{m}} \left( 1 - \frac{L_{mq}}{L_{md}} \right) \]

(20)

The ratio of torques is therefore dependant on three ratios, 1) the inverse of the saliency ratio \( \frac{L_{mq}}{L_{md}} \), 2) the induction motor magnetizing to total rotor inductance ratio \( \frac{L_{m}}{L_{r}} \), and 3) the ratio of the synchronous reluctance motor d-axis magnetizing inductance expressed as a per unit of the corresponding induction motor magnetizing reactance, \( \frac{L_{md}}{L_{m}} \). As noted previously, saliency ratios of up to 7-8 have been reported recently so that a representative (and slightly pessimistic) value for term 1 is 1/6. The induction motor magnetizing reactance is typically in the range 1.0 to 2.0 per unit while the rotor leakage reactance is
typically in the range 0.07 to 0.1 per unit. A representative value of term 2) is therefore 0.95. Finally, if the air gaps of the two machines are the same, the ratio of synchronous reluctance d-axis magnetizing reactance to induction motor magnetizing reactance is dependant primarily on the width of the rotor pole (pole arc) of the reluctance machine relative to the pole pitch of the stator poles of the two machines. The relevant equation, obtained by the method of winding functions [18] is

\[
\frac{L_{\text{md}}}{L_m} = \frac{\frac{\pi + \theta_p}{2}}{\frac{\pi - \theta_p}{2}} \int_0^{\pi} \sin^2 \theta \, d\theta
\]

where \( \theta_p \) is the pole arc in radians.

With modern axially aligned laminations [15,16], the pole arc of the reluctance machine can approach 180 degrees (i.e. pole arc = pole pitch). Using a very conservative value of 120 degrees (2\( \pi/3 \) radians) the ratio \( \frac{L_{\text{md}}}{L_m} \) becomes

\[
\frac{L_{\text{md}}}{L_m} = \frac{\frac{5\pi/6}{\pi}}{\frac{\pi/6}{\pi}} \int_0^{\pi/6} \sin^2 \theta \, d\theta
\]

\[
\frac{L_{\text{md}}}{L_m} = 0.94
\]

The torque ratio therefore becomes,

\[
\frac{T_r}{T_i} = \frac{0.94 \left(1 - \frac{1}{6}\right)}{0.95} = 0.82
\]

**Power Loss Ratio**

Equation 23 indicates that the synchronous reluctance motor is actually capable of less torque for the same amplitude of stator current. However, it must be remembered that the losses of the two machines are not the same since the induction motor must sustain rotor slip frequency losses to produce torque. In general, the issue of loss components in ac machines is very complicated. In general, however, the copper losses of any ac machine tends to dominate over the iron losses. If only copper losses are considered the power loss in the synchronous reluctance motor is simply

\[
P_r = \frac{3}{2} L_s^2 r_s
\]

while the losses in the induction machine are,

\[
P_i = \frac{3}{2} \left( L_s^2 r_s + L_r^2 r_r \right)
\]
The total copper loss of the induction machine depends upon losses in both stator and rotor circuits. If the machine is loaded to about rated current then the stator and rotor currents are nearly equal and thus (again pessimistically)

\[ I_d^2 \equiv I_s \]

Also, for most machines, the stator referred value of rotor resistance varies from 0.75 \( r_s \) to 1.25 \( r_s \). Hence, a nominal value of rotor resistance is simply,

\[ r_f \equiv r_s \]

The ratio of the power loss for the synchronous reluctance machine as a fraction of the copper losses of an induction machine thus becomes,

\[ \frac{P_T}{P_1} = 0.5 \quad (26) \]

**Torque Output for Equal Power Loss**

Equation 26 indicates that while the synchronous reluctance machine produces only 82% of the torque of the induction machine, it does so while producing only 50% of the losses. Clearly, the current can be increased in the synchronous reluctance machine to produce additional torque provided that the machine is not saturated in doing so. Since the power loss varies as the square of the current, it is apparent from Eq. 24 that if the power loss in the synchronous reluctance motor is doubled, making the losses in the two machines the same, then the amplitude of the current of this machine can be increased by \( \sqrt{2} \).

The precise manner in which the d and q components of current change as a function of the current amplitude depends upon the control algorithm used to establish the rated (nominal) condition. In general, however, the most effective control algorithm for operation up to the rated condition would produce the maximum torque per ampere. In general, in the steady state it can be shown that the vector diagram of Fig. 5 is produced whereupon it is evident that

\[ I_{qs} = I_s \cos \epsilon \quad (27) \]

and

\[ I_{ds} = I_s \sin \epsilon \quad (28) \]

where \( \epsilon \) is the angle of the current vector with respect to the q-axis.

Utilizing these expressions in Eq. 18, the torque equation of the synchronous reluctance machine becomes

\[ T_r = \frac{3}{2} \left( L_{md} - L_{mq} \right) I_s^2 \sin \epsilon \cos \epsilon \]

\[ = \frac{3}{4} \left( L_{md} - L_{mq} \right) I_s^2 \sin 2\epsilon \quad (29) \]

Equation 29 is clearly maximized when \( \epsilon = 45^\circ \) so that for loads less than and up to rated conditions the angle \( \epsilon \) is fixed at 45° and the current amplitude varied, realizing the minimum \( P_T \) loss as well as the smallest converter losses.

Figure 5 shows the condition for the case where the torque is assumed to have reached its rated value. The diagram is constructed for a machine in which the saliency ratio \( L_{ds}/L_{qs} = 7.0 \) and wherein the stator Ir drop is neglected. If the machine is optimally designed, the d-axis flux linkage is also at its rated value at this point. In this case the projection of the flux vector on the d-axis, (d-axis component of flux linkage) cannot increase further implying that the direct axis component of stator current also must become constant for torques greater than the rated value or,

\[ \Lambda_{ds} = L_{ds} I_{ds} = \text{Constant} \]
Fig. 5 Phasor Diagram of Synchronous Reluctance Motor.

Fig. 6 Phasor Diagram of the Machine of Fig. 5 in Which the Stator Current has Been Increased by $\sqrt{2}$, While Maintaining the Same Direct Axis Flux.

Referring back to Eq. 18 it is evident that torque in the synchronous reluctance machine can then only be increased only if the quadrature axis current $i_{qs}$ is increased. Figure 6 shows an increased torque condition in which the amplitude of the stator current $I_s$ has been
increased compared to Fig. 5 by $\sqrt{2}$ while the d-axis component has been maintained constant. In this case it is not difficult to show that the q-axis current then increases by $\sqrt{3} = 1.732$. Thus, if the copper loss in both machines is the same, and the flux in both machines are at their rated values, then from Eq. 23 the torque ratio becomes,

$$\frac{T_r}{T_i} = (0.82)(1.732) = 1.42$$

(30)

While the increased q-axis current will contribute to a cross magnetization effect, this is not expected to reduce the torque by more than 15-20%. Hence, the torque producing capability of the synchronous reluctance machine is inherently greater than the equivalent induction machine. Note, also that this ratio is conservative and that values greater than unity can be achieved if the saliency can be increased beyond 6.0 or if the pole arc of the machine can be increased beyond 120° (both of which appear to be achievable).

**Permanent Magnet Assisted Synchronous Reluctance Motor**

It is important to note at this point that while the torque of the synchronous reluctance motor has been increased to a value in excess of the induction motor (Eq. 30) this has been accomplished at the expense of increasing the current in the synchronous reluctance machine by a factor of $\sqrt{2}$ higher than the induction motor. While thiscreates the same losses in this machine as the induction motor, the increased current clearly imposes a penalty on the converter since the switch ratings must be increases by $\sqrt{2}$ compared to an induction motor. In effect, the synchronous reluctance motor runs at a poorer power factor than the induction motor.

This problem can alleviated by inserting permanent magnets between rotor laminations such that the permanent magnets “assist” the torque production. The principle can be illustrated by again referring to Eq. 1. That is,

$$T_{pmr} = \frac{3P}{2} \left( \lambda_{ds}i_{qs} - \lambda_{qs}i_{ds} \right)$$

(31)

When permanent magnet are included in the q-axis flux path the flux linkage expressions become

$$\lambda_{ds} = L_{ds}i_{ds}$$

(32)

$$\lambda_{qs} = L_{qs}i_{qs} + \lambda_{m}(pm)$$

(33)

where $L_{ds}$ and $L_{qs}$ are the d- and q-axes inductances and, again case $L_{ds} \neq L_{qs}$, also be written as

$$T_{pmr} = \frac{3P}{2} \left[ L_{ds}i_{qs} - \lambda_{m}(pm)i_{qs} - \lambda_{qs}i_{ds} \right]$$

(34)

If we examine the first term in Eq. 34, i.e., the saliency term we can note that the quantity $L_{ds}i_{qs}i_{ds}$ while the quantity $L_{qs}i_{qs}i_{ds}$ is negative. In effect, the q-axis saliency (q-axis flux) acts to lower the torque production which is primarily coming from the d-axis saliency (d-axis flux) interacting with the q-axis current. We could consider that a theoretical maximum torque for a synchronous reluctance machine can be reached if we were able to make $L_{qs} = 0$. This possibility can be reached by use of the second term of Eq. 34. In particular, assume that the polarity of the magnets are reversed relative the positive direction defined by the q-axis (direction of the stator q-axis MMF). Then, Eq. 34 can be written,

$$T_{pmr} = \frac{3P}{2} \left[ L_{ds}i_{qs}i_{ds} - L_{qs}i_{qs} - \lambda_{m}(pm)i_{ds} \right]$$

(35)

Hence, the second (negative torque producing term can be made as small as necessary by placing magnets in the q-axis of the rotor. In effect, the q-axis inductance can be made to approach zero.

Figure 7 shows a phasor diagram including the effects of a permanent magnet in which the quadrature axis current is assumed to be completely cancelled. Note the improved power factor compared to the normal synchronous reluctance motor. It is clear
that the power factor can be improved even more by reducing the d-axis component of stator current (which now not need be the same value since the torque has effectively been increased).

![Phasor Diagram]

Fig. 7 Phasor Diagram Corresponding to Fig. 5 of Permanent Magnet Assisted Synchronous Reluctance Motor.

A sketch of the basic rotor geometry for a permanent magnet assisted synchronous reluctance motor is shown in Fig. 8. Note in particular, that simple bar magnets are placed in the sections between lamination groups which make up the salient pole structure. These bar magnets are held in place by simple mechanical pressure produced by bolting the lamination groups. The magnets are magnetized so as to force the permanent magnet flux into the quadrature axis. It is important to mention that the amount of flux which is needed is not large since the machine is basically magnetized by the d-axis component of stator current as in a normal machine. Only a relatively small amount of quadrature axis flux is needed to counteract the q-axis armature reaction.

**Conclusion**

This paper has demonstrated, by means of analysis, that the conventional synchronous reluctance motor may be capable of better torque production than the squirrel cage induction motor. It becomes apparent that there appears to be rich research possibilities in the field of adjustable speed reluctance motor drives. It is hoped that the reader will join the author in exploring these exciting new possibilities.

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Fig. 8 Illustration of Rotor Configuration for Permanent Magnet Assisted Synchronous Reluctance Motor Drive.

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