A New Control Strategy for Optimum-Efficiency Operation of a Synchronous Reluctance Motor

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Abstract—In this paper, an optimum-efficiency control scheme of synchronous reluctance motors is presented. There exists a variety of combinations of $d$- and $q$-axis current which provides a specific motor torque. The objective of the optimum-efficiency controller is to seek a combination of $d$- and $q$-axis current components, which provides minimum input power, that is, minimum losses at a certain operating point in steady state. A small amount of perturbation is added to the $d$-axis current reference for the purpose of searching a minimum input power operating point.

Index Terms—Optimum-efficiency operation, synchronous reluctance motor.

I. INTRODUCTION

E NERGY SAVING is always an important issue for a motor drive system. A number of reports have been published on the subject of efficiency improvement for induction motor drives [4]–[6]. Induction motor efficiency can be improved by means of reduced voltage operation at light loads in the case of fixed frequency induction motor drives. It is also known that induction motor losses at partial loads can be reduced by operating an adjustable-frequency drive at the optimum slip which yields maximum efficiency. In an adjustable-frequency induction motor drive, the voltage and frequency applied to the motor are independently variable, and a specific torque-speed operating point can be achieved with a variety of different voltage frequency combinations. Each voltage frequency pairing defines a particular motor torque-speed characteristic, but motor efficiency may vary widely. If the voltage is high, then magnetizing current and core losses are large. If the voltage is reduced excessively, then motor currents and copper losses may rise. Consequently, there is an optimum current vector which gives a specified torque with maximum efficiency at every operating point.

For the practical realization of an efficiency-optimized synchronous reluctance motor drive, an optimum-efficiency controller may be accomplished with the aid of a loss model for the drive into which complete parameter values, including inductance saturation, coefficients of iron losses, temperature, and harmonic effects, must be programmed. At any operating point, the controller performs a computation on optimum-efficiency operating conditions and adjusts one, or more, variables in the model until the optimum values are found. These optimized values then become the commanded values for the drive regulator. The effectiveness of this approach obviously depends on the accuracy of the loss model.

Optimization can be accomplished by measuring the power input to the drive and perturbing one variable while seeking the minimum input power at the particular operating point. The minimization of total power input may not be a very sensitive procedure for minimization of losses, but accurate loss modeling and precise information regarding parameter values are not required. A block diagram for implementing the proposed optimum-efficiency controller of the synchronous reluctance motor is presented, and the overall control strategy for searching the minimum input power is discussed. An optimum-efficiency controller of the synchronous reluctance motor drive was implemented in the laboratory to verify the developed control scheme, and an experimental study was carried out with the implemented drive system.

II. $D$–$Q$ EQUATIONS OF A SYNCHRONOUS RELUCTANCE MOTOR

Since the stator winding of the synchronous reluctance machine is sinusoidally distributed, flux harmonics in the airgap contribute only an additional term to the stator leakage inductance. Hence, the equations which describe the behavior of the synchronous reluctance machine can be derived from the conventional equations depicting a conventional wound-field synchronous machine. In synchronous reluctance machines,
the excitation (field) winding is nonexistent. Also, in machines typically employing a modern axially laminated rotor structure, a rotor cage is normally omitted since the machine can be starting synchronously from rest by proper inverter control. Hence, eliminating both the field-winding and damper-winding equations from Park’s equations forms the basis for the \(d-q\) equations for a synchronous reluctance machine [3]. That is, in synchronous reference frame,

\[
\begin{align*}
\nu_{ds} &= r_s i_{ds} + p\lambda_{ds} - \omega_r \lambda_{qs} \\
\nu_{qs} &= r_s i_{qs} + p\lambda_{qs} + \omega_r \lambda_{ds} \\
\lambda_{ds} &= L_{ds} i_{ds} + L_{md} i_{ds} \\
\lambda_{qs} &= L_{qs} i_{qs} + L_{mq} i_{qs}
\end{align*}
\]

and where \(L_{ds}\), \(L_{md}\), and \(L_{mq}\) are, respectively, the stator leakage inductance, direct axis magnetizing inductance, and quadrature axis magnetizing inductance. The quantity \(r_s\) is the stator resistance per phase.

In terms of the \(d-q\) variables, the electromagnetic torque is identical to that of a synchronous machine, namely,

\[
T_e = \frac{3}{2} P \left( \lambda_{ds} i_{qs} - \lambda_{qs} i_{ds} \right)
\]

where \(P\) is the number of poles. The electromagnetic torque \(T_e\) can be described also as

\[
T_e = \frac{3}{2} P \left( L_{ds} - L_{qs} \right) i_{ds} i_{qs},
\]

III. CONSTANT-TORQUE OPERATION FOR VARIOUS CURRENT VECTORS

To produce certain constant torque \(T_e^*\), the required \(q\)-axis current \(I_{qs}\) can be expressed in terms of \(d\)-axis current \(I_{ds}\) as

\[
I_{qs} = \frac{3}{2} P \frac{T_e^*}{\frac{3}{2} (L_{ds} - L_{qs}) I_{ds}}.
\]

Using (7) for the expression of \(I_{qs}\), the stator phase current \(I_s\) and phase voltage are also expressed as a function of the \(d\)-axis current \(I_{ds}\):

\[
I_s = \sqrt{\frac{P_{ds}^2 + \left( \frac{3}{2} P \frac{T_e^*}{\frac{3}{2} (L_{ds} - L_{qs}) I_{ds}} \right)^2}{2}}.
\]

Figs. 1 and 2 show variation of stator current \(I_s\) and \(q\)-axis current \(I_{qs}\), and stator voltage \(V_s\) as a function of \(d\)-axis current \(I_{ds}\) for constant torque operation of 0.25 per unit. The parameters of the experimental motor are used for the calculation, which are as follows: the number of poles \(P = 4\), \(d\)- and \(q\)-axis inductances \(L_{ds} = 0.103\), \(L_{qs} = 0.016\) in henrys, the stator resistance \(r_s = 1.58\ \Omega\), the motor torque \(T_e^* = 2.2 \times 0.25\) (N-m). The calculations include the effects of the saturation of inductance \(L_{ds}\), which is a function of the \(d\)-axis current \(I_{ds}\). Larger value of \(L_{qs}\) is required to produce a specific torque when \(L_{ds}\) decreases due to saturation. Fig. 1 shows that there exists an operating point which gives a minimum current condition, where \(I_{ds} = 1.4\ A\) and \(I_{qs} = 1.6\ A\). Without the saturation effect, the minimum current condition occurs at the operating point where \(I_{ds} = I_{qs}\). A lookup table for \(L_{ds}\) as a function of \(I_{ds}\) was used to obtain Fig. 1, where the table was obtained from the measurement results of the experimental synchronous reluctance motor [7].

IV. IRON AND COPPER LOSSES

When evaluating the core loss, the different behavior of hysteresis and eddy-current losses with respect to frequency is taken into account. At fundamental frequency \(f\), the core losses are described as

\[
W_{core} = k_h f \phi^2 + k_e f^2 \phi^2
\]

where \(\phi\) is the magnetic flux and \(k_h\) and \(k_e\) are the hysteresis and eddy-current coefficients, respectively. Since the motor voltage (airgap voltage) is expressed as \(V_m = k_e f \phi\), the fundamental core loss is written as

\[
W_{core} = V_m^2 \left( \frac{k_h}{f} + k_e \right) = k V_m^2
\]

where

\[
k = \frac{\frac{k_h}{f} + k_e}{k_e^2}.
\]
The copper loss is expressed in terms of stator resistance \( r_s \) and stator current \( I_s \) as

\[
W_{\text{copper}} = 3r_s I_s^2.
\]  

(12)

The loss \( W_t \), which includes the fundamental core loss and the copper loss, is

\[
W_t = W_{\text{core}} + W_{\text{copper}} = k_1 I_m^2 + 3r_s I_s^2.
\]  

(13)

Using the following expressions for \( V_m^2 \) and \( I_s^2 \),

\[
V_m^2 = (\omega_p L_{md} I_{ds})^2 + (\omega_p L_{mq} I_{qs})^2
\]  

(14)

\[
I_s^2 = I_{qs}^2 + I_{ds}^2
\]  

(15)

the loss \( W_t \) can be expressed in terms of \( d \)-axis current as

\[
W_t = k_1 \left( \frac{\omega_p L_{md} I_{ds}}{2} \right)^2 + \left( \frac{\omega_p L_{mq} \frac{T_e}{2} \left( \frac{3P}{2} (I_{ds} - I_{qs}) I_{ds} \right)}{2} \right)^2 + 3r_s \left( \frac{\frac{T_e}{2} \left( \frac{3P}{2} (I_{ds} - I_{qs}) I_{ds} \right)}{2} + I_{ds}^2 \right).
\]  

(16)

Equation (16) can be written as

\[
W_t = a I_{ds}^2 + b I_{ds}^2 + c
\]  

(17)

and derivative of \( W_t \) in terms of \( I_{ds} \) is described as

\[
\frac{dW_t}{dI_{ds}} = 2a I_{ds} \left( I_{ds}^2 + \sqrt{\frac{b}{a}} \right) \left( I_{ds} + \left( \frac{b}{a} \right)^{1/4} \right)
\]  

(18)

where

\[
a = \frac{k_1}{k_e^2} \left( \omega_p L_{md} I_{ds} \right)^2 + 3r_s
\]  

(19)

\[
b = \left( \frac{k_1}{k_e^2} \left( \omega_p L_{mq} \right)^2 + 3r_s \right) \left( \frac{T_e}{2} \left( \frac{3P}{2} (I_{ds} - I_{qs}) I_{ds} \right) \right)^2.
\]  

(20)

Equation (18) indicates that \( \frac{dW_t}{dI_{ds}} < 0 \) when \( I_{ds} < (\frac{b}{a})^{1/4} \) and \( \frac{dW_t}{dI_{ds}} > 0 \) when \( I_{ds} > (\frac{b}{a})^{1/4} \), which means that there exists a minimum \( W_t \) at a specific value of \( I_{ds} \). The minimum loss \( W_{t_{\text{min}}} \) is given by

\[
W_{t_{\text{min}}} = a I_{ds_{\text{min}}}^2 + b I_{ds_{\text{min}}}^2 + c
\]  

(21)

in which

\[
I_{ds_{\text{min}}} = \left( \frac{b}{a} \right)^{1/4}.
\]  

(22)
V. EXPERIMENTAL STUDY OF INPUT POWER OF SYNCHRONOUS RELUCTANCE MOTOR DRIVE

An experimental study of the input power of the synchronous reluctance motor drive was carried out with the implemented experimental drive system. The speed of the synchronous reluctance motor is controlled at a certain speed and a partial load is applied to the motor by a dynamometer. The tested speeds were 600, 1200, 1800, and 2400 r/min, and the applied load is 0.55 N·m, that is, 25% of the rated torque of the experimental motor. The input power was measured at the dc bus of the inverter for various combinations of $d$- and $q$-axis current components in the steady-state condition.

Fig. 3(a) shows plots of input power $P_{in}$, stator current $i_s$, and $q$-axis current $i_{qs}$ for various $d$-axis current $i_{ds}$, where the rotor speed is controlled by the synchronous reluctance motor at 600 r/min, and the dc dynamometer provides the load torque of 0.55 N·m. The $q$-axis current $i_{qs}$ increases as $d$-axis current $i_{ds}$ decreases to maintain output torque constant. Fig. 3(a) indicates that there exists a minimum input power operating point at about $i_{ds} = 0.9$ A, while the stator current $i_s$ has minimum value at about $i_{ds} = 0.95$ A. It should be noted that there exists only one $i_{ds} - i_{qs}$ combination which provides minimum input power for this specific operating condition.

Fig. 3(b) illustrates the current vectors which produce the same torque of 0.55 N·m at 600 r/min. Fig. 3(c) shows experimental results of losses, where the total loss, the copper loss, and the difference between the total loss and the copper loss are plotted.

Fig. 4 shows the plots of the same items as Fig. 3 in the case of the rotor speed of 2400 r/min, where the load torque of 0.55 N·m is the same as in the previous case. Fig. 4(a) shows the plots of input power $P_{in}$, stator current $i_s$, and $q$-axis current $i_{qs}$ for various $d$-axis current $i_{ds}$. Again, the result indicates that there exists a minimum input power operating point between $i_{ds} = 0.6$ and 0.7 A, while the stator current $i_s$ has minimum value at about $i_{ds} = 0.9$ A. Fig. 4(b) illustrates the current vectors which produce the same torque of 0.55 N·m at 2400 r/min. Fig. 4(c) shows experimental results of losses, where the total loss, the copper loss, and the difference between the total loss and the copper loss are plotted.

It should be noted that the minimum input power operating point for the rotor speed of 2400 r/min is shifted to the lower $i_{ds}$ value compared to the one for the rotor speed of 600 r/min. The difference of the total loss from the copper loss, of which the main part of the losses are the core losses, increases three times as the rotor speed increases from 600 to 2400 r/min, while the copper loss remains in almost the same range as the rotor speed increases.

Fig. 5 provides the additional experimental results of the measured input power for different rotor speeds. It can be found that each operating condition has a unique combination...
of \(d\)- and \(q\)-axis currents that provides minimum input power at the inverter dc bus.

Fig. 6 shows plots of the loss term, which is the difference between the total loss and the copper loss, that is, mainly iron losses versus \(d\)-axis current \(i_{\text{ds}}\) for different operating speed, while a dc dynamometer provides the load torque of 0.55 N·m. Fig. 7 shows the plots of the loss versus stator frequency for different \(d\)-axis current, while a dc dynamometer provides the load torque of 0.55 N·m.

The motor torque of synchronous reluctance motors is proportional to two components of the stator current vector, that is, \(d\)- and \(q\)-axis current, \(i_{\text{ds}}\) and \(i_{\text{qs}}\). Hence, there exist various combinations of \(d\)- and \(q\)-axis current which provide a certain amount of motor torque. The objective of the optimum-efficiency controller is to seek a combination of \(d\)- and \(q\)-axis current components, which provides minimum input power, that is, minimum losses at a certain operating point in steady state. A small amount of perturbation is added to the \(d\)-axis current reference for the purpose of searching a minimum input power operating point. The input power of the inverter is calculated from the measured dc-bus current and dc-bus voltage of the inverter. The dc-bus current was measured with a dc-current sensor and a 12-b A/D converter. The obtained input power includes motor output power, motor losses, and inverter losses. The motor losses include copper losses, iron losses, and stray load losses.

VI. OPTIMUM-EffICIENCY CONTROLLER

Fig. 8 shows a control configuration of the optimum-efficiency controller of a synchronous reluctance motor drive with a torque and speed control function.

By means of an absolute encoder or resolver, the sine and cosine of the angular position of the rotor is established. These sinusoidal components are used to refer those physical stator currents from the physical (stationary) reference frame to the rotating \((d-q)\) axes. The encoder is also used to measure speed and, based on the speed measurement, the desired (command) values of \(i_{\text{ds}}\) and \(i_{\text{qs}}\) are established. Current regulators guarantee that the desired and actual values of the \(d-q\) currents are obtained. The voltage command signals which are obtained in the synchronously rotating \(d-q\) frame are finally referred back to the stator frame before being used to switch the voltage source pulsewidth modulation (PWM) inverter [6].
expressed as

\[ T_e1 = \frac{3}{2} P \left( L_{ds1} - L_{qs1} \right) i_{ds1} i_{qs1} \]

\[ = \frac{3}{2} P \left( L_{ds2} - L_{qs2} \right) \left( i_{qs1} + \Delta i_{qs} \right) \left( i_{qs1} + \Delta i_{qs} \right) \]  

(23)

where the quantities with subscripts 1 or 2 denote quantities at each operating point. At the operating point 2, \( i_{ds2} = i_{ds1} - \Delta i_{ds} \) and \( i_{qs2} = i_{qs1} - \Delta i_{qs} \).

Fig. 10 illustrates a pattern of \( d \)-axis current perturbation to find an optimum-efficiency operating point. It is assumed that the motor is generating a partial load torque at a certain speed, where \( d \)-axis current reference \( i_{ds}^* \) is decreased five steps, \( \Delta i_{ds} \) each step. Then, \( d \)-axis current reference \( i_{ds}^* \) is increased ten steps, again, \( \Delta i_{ds} \) each step. The input power is measured at each step while \( d \)-axis current reference \( i_{ds}^* \) is increased from \( i_{ds}^*(0) \) to \( i_{ds}^*(5) \) and, then, the optimum-efficiency controller determines at which step the input power has a minimum value. The controller decreases \( d \)-axis current reference to the current level where the input power has its minimum value. In this example, the input power has its minimum value at \( i_{ds}^*(5) \) and \( i_{ds}^*(10) = i_{ds}^*(5) - 5\Delta i_{ds} \). The amount of \( d \)-axis step change should be small enough not to create any significant torque disturbance to the drive system. The time period of one step change must be determined by the response time of the drive system against small \( i_{ds} \) disturbance.

VII. AN EXPERIMENTAL STUDY OF THE IMPLEMENTED OPTIMUM-EFFICIENCY CONTROLLER

The optimum-efficiency controller was implemented with digital signal processor (DSP) software, and an experimental study of the optimum-efficiency control of the synchronous reluctance motor drive was carried out with the implemented drive system.
The important issues to be addressed in the laboratory experimental study included the following:

1) behavior of the input power while \( d \)-axis current is continuously decreased from the rated operating condition;
2) response of the input power to a small \( d \)-axis current step;
3) optimum-efficiency control performance study.

Fig. 11 shows an experimental result of a behavior of input power \( P_{in} \) for continuous change of \( d \)-axis current \( i_{ds} \), while the rotor speed is controlled at 1800 r/min and dc dynamometer provides the load torque of 0.55 N\( \cdot \)m. The trace of input power \( P_{in} \) indicates that there exists an operating point where the input power has a minimum value, where the \( d \)-axis current was decreased with a constant reduction rate of the \( d \)-axis current simply to determine if there was an operating point with a minimum input power. The exact steady-state condition at each operating point was not established with this sweeping operation. In this case, the efficiency improvement of about 4 W is expected while the motor output power is 104 W. At this speed the motor full power is 415 W.

Fig. 12 shows an experimental result of a response of input power \( P_{in} \) for a step change of \( d \)-axis current \( i_{ds} \), while the rotor speed is controlled at 1800 r/min and dc dynamometer provides the load torque of 0.55 N\( \cdot \)m.

The input power decreased by about 0.5 W when the \( d \)-axis current was increased by 40 mA, which is about 2% of the rated \( d \)-axis current, as shown in Fig. 12(a). In the other case, the input power increased by about 0.7 W when the \( d \)-axis current was decreased by 40 mA, as shown in Fig. 12(b). The input power is not always strictly constant, as expected, and even in the experimental system, an input power fluctuation of about 1 W was observed while the output power is 104 W.

Fig. 13 shows a process of the optimum-efficiency control, where the input power is gradually minimized. Counter 1 counts up from zero to ten as the \( d \)-axis current reference is increased stepwise at every perturbation cycle, which is described in Fig. 10. The number of steps is ten in this case. Counter 2 counts down to the \( d \)-axis current reference where the input power is minimized among the perturbation at a specific perturbation cycle. The rotor speed is controlled at 1800 r/min and the dc dynamometer provides the load torque of 0.55 N\( \cdot \)m, while the optimum-efficiency controller seeks an operating point which provides the minimization of the input power. In this case, the improvement of the input power is about 6–7 W. The motor output power is 104 W at this speed, while the motor full power is 415 W.

VIII. CONCLUSION

The purpose of this paper was to present an optimum-efficiency control strategy for a synchronous reluctance motor drive, where the on-line controller seeks a combination of \( d \)- and \( q \)-axis current components, which provides minimum input power, that is, minimum losses at a certain operating point in steady state. A small amount of perturbation is added to the
$d$-axis current reference $i_{ds}^*$ for the purpose of searching a minimum input power operating point. The input power of the inverter is calculated from the measured dc-bus current and dc-bus voltage of the inverter. The obtained input power includes motor output power, motor losses, and inverter losses. A block diagram for implementing the proposed optimum-efficiency controller of the synchronous reluctance motor was presented, and the overall control strategy for searching the minimum input power was discussed. An optimum-efficiency controller of the synchronous reluctance motor drive was implemented in the laboratory to verify the developed control scheme.

REFERENCES


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