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L. Wei, Y. Matsushita, T.A. Lipo

Wisconsin Power Electronics Research Center
University of Wisconsin-Madison
Madison WI 53706-1691

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University of Wisconsin-Madison
College of Engineering
Wisconsin Power Electronics Research Center
2559D Engineering Hall
1415 Engineering Drive
Madison WI 53706-1691

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Lixiang Wei, Yoichi Matsushita and Thomas A. Lipo

Department of Electrical and Computer Engineering
The University of Wisconsin-Madison,
Madison, WI, 53705, USA

Abstract—The dual-bridge matrix converter concept has been gaining recognition as a promising circuit alternative in recent years. In this paper, a direct feed-forward unbalance control method with power factor adjustment ability is developed for a 9-switch dual-bridge matrix converter system. It firstly detects the line side source voltages. Then, after adjusting the switching functions of only the line side converter, the system can provide balanced output voltage with slightly distortion of line side current. Analysis demonstrates that the output voltage capacity and power factor adjustability is limited under unbalanced conditions. Theory analysis, simulation and experimental results are presented in the paper to verify the effectiveness of this control method.

I. INTRODUCTION

The dual-bridge matrix converter is a relatively new developed concept [1][2][3]. Compared to the conventional matrix converter [4], it possesses the same high quality performances, such as near sinusoidal input/output waveforms, adjustable input power factor, and compact system design due to no large energy storage components. Moreover, it has several other advantages over the conventional matrix converter, including easier and safer commutation, reduced number of switches, simple damp circuit, etc. However, since the concept is new, unbalanced operation of dual-bridge matrix converter, an important mode of operation, has not yet been discussed.

Since the matrix converter is a direct frequency conversion device, unbalances at the utility side can immediately be reflected to the load side and generate unwanted input/output harmonics to the system. Several techniques that reduce the influence of source voltage unbalance for the conventional matrix converter has been reported [5][6][7][8][9][10]. Among them, [5][6][7] synthesize the performance of two available unbalance control strategies when the source voltage contains only positive and negative sequence components. The first strategy provides the same sequence for both input current and input voltage vectors. The second optimizes the line side current distortion without degradation of load side voltage. Since the second strategy generates sinusoidal output voltages with slightly distorted input current waveforms, it has better features. In papers [8][9][10], the unbalance problem while line side voltage contains high order harmonics are analyzed. Several strategies were proposed and tested to eliminate the unbalances of load side voltage, but the distortions of line side current were not mentioned.

Since the dual-bridge matrix converter has the same input/output character under space vector PWM method as the conventional matrix converter, the same unbalanced control method discussed for conventional matrix converter can, in principle, also be applied. However, because the dual bridge matrix converter has an AC/DC/AC structure, some of these methods can be simplified. Specifically, a simplified unbalanced control method related to papers [5][6][7] can be applied to a 9-switch dual-bridge matrix converter (shown in fig.1) in this paper. Its main principle is to decompose unbalanced control switching function into line side and load side converters individually. For example, if line side voltage is unbalanced, only the switching function at line side converter is adjusted to remain the average DC side voltage $V_{dc}$ as a constant. On other hand, if the three-phase output load is unbalanced, only the load side switching function is adjusted to keep the average DC side current as a constant.

![Fig. 1. Configuration of matrix converter system](image-url)

This scheme has two advantages. Firstly, it eliminates the influence of unbalance from one side to the other side. For example, while the line side voltage is unbalanced, because the DC side voltage is adjusted to be constant, if sinusoidal switching function is applied to the load side converter, the output voltage/current does not have any low order harmonics. Secondly, it can simplify the unbalance control of the dual-bridge matrix converter to that of the current source rectifier discussed in paper [11].

The paper is organized through the following steps. After a brief discussion of the relationship between the unbalance source voltage and the switching functions, the unbalanced control method is derived. With this method, the dual-bridge matrix converter operates similar to the conventional matrix converter [5][6][7]. Implementation of this method and its limitations are also discussed. It is shown that the maximum
voltage transfer ratio is reduced under unbalanced conditions. Finally, simulation and experimental results are presented to prove the feasibility of this controller.

II. CONTROL PRINCIPLE

In order to simplify the analysis, the instantaneous space vector representation is defined as

$$\tilde{x} = x_n + j \cdot x_b = \frac{2}{\sqrt{3}} (x_a + x_b \cdot e^{\frac{j \cdot 2\pi}{3}} + x_c \cdot e^{\frac{j \cdot 4\pi}{3}}) \quad (1)$$

where, $x$ can be a voltage, current or switching function vector.

Supposing the line side voltage vector is unbalanced, it can be decomposed into positive and negative components

$$\tilde{V}_{in} = V_p \cdot e^{j(\omega_0 t + \theta_p)} + V_n \cdot e^{j(\omega_0 t + \theta_n)} \quad (2)$$

where, $V_p$ and $V_n$ are the amplitude of positive and negative sequence voltage vector; $\theta_p$ and $\theta_n$ are the angle of positive and negative vectors respectively.

Assuming that the expected output voltage vector is balanced, it can be illustrated as

$$\tilde{V}_{out} = V_o \cdot e^{j(\omega_0 t + \theta_o)} \quad (3)$$

If the vectors of line and load side switching functions are represented as $\tilde{S}_{rec}$ and $\tilde{S}_{inv}$ respectively, then

$$\tilde{V}_{dc} = \tilde{V}_{in} \cdot \tilde{S}_{rec}; \quad \tilde{V}_{out} = \tilde{V}_{dc} \cdot \tilde{S}_{inv} \quad (4)$$

Similarly, assuming that the three-phase output load is balanced and the output current vector is

$$\tilde{i}_{out} = I_{om} \cdot e^{j(\omega_0 t + \theta_o - \psi_o)} \quad (5)$$

where $\psi_o$ is the output power factor.

The following equations can be obtained

$$i_{dc} = \tilde{i}_{out} \cdot \tilde{S}_{inv}; \quad i_{in} = i_{dc} \cdot \tilde{S}_{rec} = \tilde{i}_{out} \cdot \tilde{S}_{inv} \cdot \tilde{S}_{rec} \quad (6)$$

Combining eqns. (3)–(6), the relationship of input and output power can be illustrated as

$$P_{in} = \tilde{V}_{in} \cdot \tilde{i}_{in} = \tilde{V}_{dc} \cdot i_{dc} = \tilde{V}_{in} \cdot \tilde{i}_{in} = P_{out} \quad (7)$$

This result proves that, in any given time, the input power must be same as the output power regardless of the input voltage unbalance. Thus, in order to get harmonic-free output power, the input power of the converter must also be harmonic-free. However, since the line side voltage is unbalanced, the line side current as well as the switching functions should also be distorted to obtain this harmonic-free condition \cite{5,6,7,12}. A compensation technique that was formerly applied to the PWM current source rectifier is employed. This technique yields line side switching function with low order harmonic components to generate a constant DC side voltage.

Supposing that the unbalanced line side switching function can also be expressed as a combination of positive and negative components:

$$\tilde{S}_{rec} = S_p e^{j(\omega_0 t + \theta_p - \psi_p)} + S_n e^{j(\omega_0 t + \theta_n - \psi_n)} \quad (8)$$

where, $S_p$ and $S_n$ are the amplitude of positive and negative sequence component; $\psi_p$ and $\psi_n$ are the angle differences between the switching function and input voltage vectors.

Combining eqs (4) and (8), the DC side voltage can be expressed as

$$\tilde{V}_{dc} = V_p S_p \cdot \cos \psi_p + V_n S_n \cdot \cos \psi_n + V_p S_n \cdot \cos(2\omega_0 t + \theta_p - \theta_n + \psi_p) + V_n S_p \cdot \cos(2\omega_0 t + \theta_n - \theta_p + \psi_n) \quad (9)$$

It shows that the DC side voltage consists of a DC term and a second order harmonics. Since the second order term brings low order harmonics to both sides of the converter, the switching function has to be carefully selected for purposes of elimination. The following equations can be obtained.

$$V_p S_n = V_n S_p; \quad \psi_p + \psi_n = \pi \quad (10)$$

or

$$\frac{S_p}{V_p} = ke^{-j\psi_p}; \quad \frac{S_n}{V_n} = ke^{-j\psi_n} = -ke^{j\psi_p} \quad (11)$$

in which case, $k = \left| \frac{S_p}{V_p} \right| = \left| \frac{S_n}{V_n} \right|$ is a constant.

Under this condition, the DC side voltage equals to

$$\tilde{V}_{dc} = \tilde{V}_{in} \cdot \tilde{S}_{rec} = V_p S_p \cdot \cos \psi_p \left(1 - \frac{V_n^2}{V_p^2}\right) \tilde{S}_{inv} \tilde{S}_{rec} \quad (12)$$

If the angle $\psi_p$ remains constant, the DC voltage also has a constant value without any low order harmonic components.

From eqs.(4) and (12), the expected output voltage is

$$\tilde{V}_{out} = V_p \cos \psi_p \left(1 - \frac{V_n^2}{V_p^2}\right) S_p \cdot \tilde{S}_{inv} \tilde{S}_{rec} \quad (13)$$

where, $S_{inv}$ is the amplitude of load side switching function

Eq. (13) shows that the amplitude of output voltage can be adjusted by simply changing the amplitude $S_{inv}$, and the angle of output voltage vector equals that of the load side switching function. By appropriately selecting these two
values, the output voltage can be the same as the expected output voltage without any low order harmonics.

Since the load side converter is balanced in the steady state, the average DC side current $i_{dc}$ is also a constant value. The line side current vector can then be illustrated as

$$i_{in} = i_{dc} S_{rec} = i_{dc} S_p \left( e^{j(\omega_t t + \phi)} \right) \frac{V_n}{V_p} e^{j(-\omega_t t + \phi_n)} \quad (14)$$

This result demonstrates that the input current vector is proportional to the switching function of line side converter. Because this switching function consists of both positive and negative sequence components, the input current also contains positive and negative sequence. Moreover, it shows that the line side power factor can be adjusted automatically by changing angle $\psi_n$.

**III. CONTROL IMPLEMENTATION**

The control block diagram of the dual-bridge matrix converter with unbalance compensation is shown in fig. 2. It consists of several functional blocks. Among them, the voltage decomposition block computes the positive and negative sequences of the line side voltage, the switching function block calculates the input current angle, output voltage vector angle and amplitude that are necessary for space vector PWM control, and the PWM control logic block calculates the duty cycles and the appropriate PWM sequences to the converter.

**A. Voltage decomposition block**

This block calculates the positive and negative sequence components from three phase input line voltages. Utilizing the same method as in paper [12], the positive sequence components are derived by

$$V_{ab}^p(t) = \frac{1}{3}(V_{ab}(t) + V_{bc}(t - \frac{2T}{3}) + V_{ca}(t - \frac{T}{3}))$$

$$V_{bc}^p(t) = \frac{1}{3}(V_{bc}(t) + V_{ca}(t - \frac{2T}{3}) + V_{ab}(t - \frac{T}{3}))$$

$$V_{ca}^p(t) = \frac{1}{3}(V_{ca}(t) + V_{ab}(t - \frac{2T}{3}) + V_{bc}(t - \frac{T}{3}))$$

The negative sequence components are calculated as

$$V_{ab}^n(t) = \frac{1}{3}(V_{ab}(t) + V_{bc}(t - \frac{T}{3}) + V_{ca}(t - \frac{2T}{3}))$$

$$V_{bc}^n(t) = \frac{1}{3}(V_{bc}(t) + V_{ca}(t - \frac{T}{3}) + V_{ab}(t - \frac{2T}{3}))$$

$$V_{ca}^n(t) = \frac{1}{3}(V_{ca}(t) + V_{ab}(t - \frac{T}{3}) + V_{bc}(t - \frac{2T}{3}))$$

From (15) and (16), this method can generate the positive and negative sequence components after 2/3 of the input time period under any kind of unbalanced inputs.

$$V_{n} = \frac{1}{3}(V_{ab}^n(t) - V_{ca}^n(t)) ; V_{p} = \frac{1}{\sqrt{3}} V_{bc}^n(t) \quad (16)$$

where, $T$ is the fundamental period of input voltage; $V_{n}^p, V_{p}^n$ and $V_{n}^p, V_{p}^n$ are the real and imaginary part of positive and negative sequence components of input voltage vectors respectively.

From (15) and (16), this method can generate the positive and negative sequence components after 2/3 of the input time period under any kind of unbalanced inputs.

**B. Unbalanced control block**

The unbalanced control block calculates the necessary information for the space vector PWM block from the positive and negative sequence input voltage vectors. The line side current angle $\theta_a, \theta_b$, and $\theta_c$ are illustrated by

$$\theta_a = \angle S_{rec} ; \theta_b = \theta_a - \frac{2\pi}{3} ; \theta_c = \theta_a + \frac{4\pi}{3} \quad (17)$$

To simplify the analysis, it is assumed that the amplitude of the line side converter switching function equals to unity

$$S_{rec} = 1 \quad (18)$$

If one assumes that the expected output voltage $\bar{V}_o^e = k_u V_p S_{rec}$, the amplitude of the load side converter can be appropriately selected to generate the expected output voltage

$$S_{inv} = k_u \left| \frac{V_p^2 - V_n^2}{V_p} \right| = k_u \frac{V_p^2 - V_n^2}{V_p} \left[ V_p e^{-j\psi_n} - V_n e^{j\psi_n} \right]$$

$$\bar{V}_o^e = k_u V_p S_{inv}$$

(19)
If $V_n^2 \ll V_p^2$, this equation can be further simplified

$$S_{inv} = k_u \left| \frac{V_p e^{-j\psi_n} - \tilde{V}_n e^{j\psi_n}}{V_p} \right|$$

(20)

Fig. 3 demonstrates simulated amplitude of load side switching function $k$ and the angle of line side switching function while $k_u = 0.8$ under unbalance line voltage conditions. From this figure, the second order harmonics can be clearly identified.

### C. PWM control logic block

The PWM control logic block calculates the duty cycles of different vectors and finally provides the gate signals of each switch. The input of this block is line side current angle $\theta_{a}, \theta_{b}, \theta_{c}$, power factor angle $\psi_{in}$, amplitude of load side switching function $S_{inv}$, and the angle of output voltage vector $\theta_{o}$. Both the calculation and switching sequences are the same as that of the balanced space vector PWM illustrated in papers [1][2].

Supposing that the three phase line voltages is shown as

$$V_{sa} = V_{ma} \cos \theta_{av} = V_m \cos \theta_{o_a} t$$
$$V_{sb} = V_{mb} \cos \theta_{bv} = V_m \cos (\theta_{o_b} t - \frac{2\pi}{3})$$
$$V_{sc} = V_{mc} \cos \theta_{cv} = (1-a)V_m \cos (\theta_{o_c} t + \frac{2\pi}{3})$$

(21)

where, $0 < a < 1$ is the unbalance constant.

Then, from eqs.(15) and (16), the positive and negative voltage vectors can be derived as

$$\tilde{V}_p = (1 - \frac{a}{3})\tilde{V}_{in} \quad \tilde{V}_n = -\frac{a}{3}\tilde{V}_{in}$$

(22)

where $\tilde{V}_{in}$ is the line voltage vector at balanced condition.

Since the amplitude of line side switching function is bounded by

$$|S_{rec}| = \left| \frac{S_p(e^{-j\psi_n} - \tilde{V}_n e^{j\psi_n})}{V_p} \right| \leq 1$$

(23)

The amplitude $S_p$ is also bounded

$$S_p \leq \frac{1}{\max \left| e^{-j\psi_n} - \frac{\tilde{V}_n}{V_p} e^{j\psi_n} \right|} = \frac{1}{\left| \frac{\tilde{V}_n}{V_p} \right|} = \frac{3-a}{3}$$

(24)

From (20): $0.866 > k_u \frac{V_p^2}{V_p^2 - V_n^2} \frac{1}{|S_p|}$

(25)

The voltage transfer ratio under one phase voltage unbalance can be obtained

$$k_u < 0.866 \cdot \frac{(1-a^2)}{(3-a)} \cdot \left(1 - \frac{a}{3}\right)^2$$

(26)

Fig. 4 shows the maximum output voltage transfer ratio as a function of the unbalance ratio. From this figure, one can find that the maximum output voltage reduces quickly while the unbalance ratio increases. Finally, when this ratio equals to 1 and the phase C voltage becomes 0, the maximum output voltage reduces to only 28.9% of the input line voltage.

![Fig. 3. Amplitude of load side switching function and the angle of line side switching function under unbalanced condition](image)

**Fig. 3. Amplitude of load side switching function and the angle of line side switching function under unbalanced condition**

### III. LIMITATION OF UNBALANCE CONTROL

Although the unbalance control method can provide balanced output voltage with optimized input current waveform, it also reduces the maximum output voltage ratio and input power factor adjustability. These limitations under one phase unbalance are analyzed in this section.

Supposing that the three phase line voltages is shown as

$$V_{sa} = V_{ma} \cos \theta_{av} = V_m \cos \theta_{o_a} t$$
$$V_{sb} = V_{mb} \cos \theta_{bv} = V_m \cos (\theta_{o_b} t - \frac{2\pi}{3})$$
$$V_{sc} = V_{mc} \cos \theta_{cv} = (1-a)V_m \cos (\theta_{o_c} t + \frac{2\pi}{3})$$

where, $0 < a < 1$ is the unbalance constant.

The second limitation of dual bridge matrix converter under unbalance control is that the maximum available power factor angle is reduced when the dual-bridge matrix converter is configured in one of the single directional power flow topologies. Fig. 5 shows the maximum power factor angle versus one phase unbalance ratio. One can establish that
while phase C is zero, a sinusoidal output voltage can only be generated when line side power factor is unity.

![Graph showing maximum input power factor angle versus one phase unbalance ratio](image)

**Fig. 5. Maximum input power factor angle versus one phase unbalance ratio**

### IV. SIMULATION AND EXPERIMENTAL RESULTS

The unbalance control scheme has been studied thoroughly on a 9-switch dual-bridge matrix converter with three phase R-L load by both the simulation and experiment. Fig. 6 and Fig. 7 illustrate the simulation result of the converter without and with the unbalance control method. Fig. 8 and Fig. 9 demonstrate experimental results for the converter under same conditions. The waveforms shown in these figures are DC side voltage $V_{dc}$ and input/output currents. The following conclusions can be obtained.

- Both the simulation and experimental results of DC side voltage agree well with each other. This voltage remains positive under both conditions.

- If no unbalance control is applied, the load side current is obviously distorted; in contrast, if the unbalanced control method is applied, no large distortion can be found.

- If no unbalance control is applied, the line side current is distorted; on the other hand, if the unbalanced control method is applied, the distortions are slightly reduced.

### V. CONCLUSIONS

In this paper, an unbalanced control scheme is developed on a 9-switch dual-bridge matrix converter to eliminate low order harmonics under unbalanced operation. Theoretical analysis, simulation and experimental results are provided to verify its effectiveness. Following conclusions are derived.

- If no unbalanced control scheme is applied for the converter, the unbalanced source voltage can generate considerable low order harmonics on both sides of the converter;

- The proposed control method can generate balanced output voltage under unbalanced source voltage with optimized line side current waveforms and power factor adjustability.

- Under unbalance control, the highest output voltage ratio and input power factor adjustability with high quality input/output waveforms are reduced.

![Graph showing simulation results without unbalanced control](image)

**Fig. 6. Simulation result without unbalanced control**

![Graph showing simulation results with unbalanced control](image)

**Fig. 7. Simulation result with unbalanced control**

![Graph showing experimental results without unbalanced control](image)

**Fig. 8. Experimental result without unbalanced control**
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VII. REFERENCES