Design and Optimization of Dual-Rotor, Radial-Flux, Toroidally-Wound, Permanent-Magnet Machines

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Abstract—A novel machine family – the dual-rotor, Radial-Flux, Toroidally-wound, Permanent-Magnet (RFTPM) machine – has been proven in a previous paper to be able to improve the machine efficiency and boost the torque density. This paper will present the key design equations and design procedure of the RFTPM machines, analyze parameter effects on machine performance, and propose an optimization method to achieve specific design objectives. In addition, Finite Element Analysis is employed to prove the effectiveness of the design equations and find the machine overload capability. Experimental measurements of a prototype, which match the design specifications well, verify the effectiveness of the design equations.

Keywords—RFTPM Machine, PM Machine Design, Optimization

I. INTRODUCTION

High torque density and high efficiency are two of the most desirable features for an electrical machine. Improvement of these features has been one of the main aspects of research on the electric machines in the last several decades. In order to provide a solution to this problem, a novel machine class – the Dual-Rotor, Radial-Flux, Toroidally-Wound, Permanent Magnet (RFTPM) machine was proposed in [1]. Its principle of operation, configurations, and features were discussed. Three low-cost techniques were proposed and proven to be valid to reduce the cogging torque. It is demonstrated that the dual-rotor RFTPM machine has the following features:

- Greatly shortened end windings
- High ratio of diameter to length
- High efficiency
- High torque density
- Low material costs
- Low-cost techniques available to reduce cogging torque

This paper will derive design equations and design procedure for such slotted, surface-mounted, RFTPM machines shown in Fig. 1. The analysis of parameter effects on machine performance, such as efficiency, torque density, and material costs, will be included as well. A proposed optimization method will be presented to achieve design objectives. In addition, Finite Element Analysis (FEA) is employed to prove the effectiveness of the design equations, find the machine overload capability, and verify if the machine can survive under the short-circuit fault. Finally, the proposed design and optimization method is used to design an actual prototype. The experimental measurements, which closely match the design specifications, verify the effectiveness of the design equations.

II. DESIGN EQUATIONS

This section will give the key design equations for a dual-rotor, slotted, surface-mounted RFTPM machine shown in Fig.1. The cross-section is depicted in Fig. 2. Since this topology has two air gaps, the design equations are separated into two portions: one is for the internal portion of rotors, permanent magnet, and stator, the other for the external.

A. Fixed Parameters

Many unknown parameters are involved in the design of the RFTPM machines. As a result, it is necessary to assign some fix-values, and then determine the remaining as part of the design. The fixed parameters will be further optimized in section V and VI based on a few different purposes. The fixed parameters may vary depending on the design purpose. Table I gives a list of the fixed parameters used in the proposed design approach. They include machine power or torque, speed, machine winding information, electrical and magnetic loadings, etc.

B. Inner Portion Design

Assume initially that the inner radius of the inner air gap, \( R_{PMI} \), shown in Fig. 2 is known, which will be adjusted later to meet the power or torque requirement when the overall torque or power output is available. The inner magnet thickness, \( H_{PMI} \), can be found from (1) [2], in which the air gap leakage flux is considered:

\[ \Phi_{g} = B_{g} \cdot A_{g} \]

Figure 1. Dual-rotor, toroidally-wound, slotted, RFTPM machine structure

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\[ B_{g1} = \left[ 1 + \frac{w_b}{w_{m1}} + \mu_r \frac{g_{e1}}{H_{\text{m1}}} w_{m1} + w_0 (1+2\eta_1+4\lambda_1) \right]^{-1} B_r \]  

where \( w_0 \) is the circumferential length between two inner magnets, \( w_{m1} \) is the inner magnet circumferential length, \( g_{e1} \) is the effective inner air gap, and

\[ \eta_1 = \frac{H_{\text{m1}}}{\mu_r w_{m1}} \ln \left( 1 + \frac{2g_{e1}}{H_{\text{m1}}} \right) \]

\[ \lambda_1 = \frac{H_{\text{m1}}}{\mu_r w_{m1}} \ln \left( 1 + \frac{2g_{e1}}{w_f} \right). \]

In (1), the only unknown is \( H_{\text{m1}} \), which can be solved by iterative methods. The inner magnet working point \( B_{m1} \) can be calculated from the geometric parameters given above using (4) [2,4].

\[ B_{m1} = \frac{\left( 1 + \frac{2g_{e1}}{w_{m1}} \right) \frac{H_{\text{m1}}}{\mu_r g_{e1}} + 2\eta_1 + 4\lambda_1}{\left( 1 + \frac{2g_{e1}}{w_{m1}} \right) \frac{H_{\text{m1}}}{\mu_r g_{e1}} + 1 + 2\eta_1 + 4\lambda_1} B_r \]

Then the back iron width, \( d_{r1} \), and the inner radius of the inner rotor or the shaft radius, \( R_{r1} \), can now be written as

\[ d_{r1} = \frac{w_{m1} B_{m1}}{2B_{e1}} \]

and

\[ R_{r1} = R_{r01} - d_{r1} \]

where the outer radius of the inner rotor, \( R_{r01} \), is directly related to the inner radius of the inner air gap, \( R_{\text{m1}} \), by the magnet thickness.

The inner stator tooth width is given by

\[ A_{\text{slot1}} = \frac{2\pi R_{r01} K_{s1}}{K_{sca} N_s J_c} = \frac{B_s R_{r01} K_{s1}}{K_{sca} J_c}. \]

Figure 2. Cross-section of a dual-rotor, slotted, RFTPM machine

Figure 3. Geometry of the inner rotor and magnets

Figure 4. Geometry of the outer rotor and magnets

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( P_s ) or ( T )</td>
<td>Power, W, or rated torque, Nm</td>
</tr>
<tr>
<td>2</td>
<td>( N )</td>
<td>Rated speed, rpm</td>
</tr>
<tr>
<td>3</td>
<td>( N_s )</td>
<td>Number of conductors in each slot</td>
</tr>
<tr>
<td>4</td>
<td>( J_c )</td>
<td>Current density, A/m²</td>
</tr>
<tr>
<td>5</td>
<td>( K_{s1} )</td>
<td>Inner electrical loading, A/m</td>
</tr>
<tr>
<td>6</td>
<td>( m )</td>
<td>Number of phase</td>
</tr>
<tr>
<td>7</td>
<td>( p )</td>
<td>Number of poles for inner or outer magnets</td>
</tr>
<tr>
<td>8</td>
<td>( q )</td>
<td>Number of slots per phase per pole for inner or outer</td>
</tr>
<tr>
<td>9</td>
<td>( C )</td>
<td>Number of parallel circuits</td>
</tr>
<tr>
<td>10</td>
<td>( g_1 )</td>
<td>Inner air gap length, m</td>
</tr>
<tr>
<td>11</td>
<td>( g_2 )</td>
<td>Outer air gap length, m</td>
</tr>
<tr>
<td>12</td>
<td>( L )</td>
<td>Motor effective axial length, m</td>
</tr>
<tr>
<td>13</td>
<td>( \alpha_e, \alpha_m )</td>
<td>Half electrical/mechanical angular width of each magnet, radian, refer to Fig. 3 and 4</td>
</tr>
<tr>
<td>14</td>
<td>( B_e, \mu_r )</td>
<td>Magnet residual flux density, T, and recoil relative permeability</td>
</tr>
<tr>
<td>15</td>
<td>( B_{g1}, B_{g2} )</td>
<td>Average flux densities of inner / outer air gap, T</td>
</tr>
<tr>
<td>16</td>
<td>( B_{r01}, B_{e1} )</td>
<td>Flux densities of inner / outer rotor core, T</td>
</tr>
<tr>
<td>17</td>
<td>( B_{r1}, B_{e2} )</td>
<td>Flux densities of stator core / tooth, T</td>
</tr>
<tr>
<td>18</td>
<td>( \rho )</td>
<td>Conductor resistivity, ohm m²</td>
</tr>
<tr>
<td>19</td>
<td>( K_{st} )</td>
<td>Stator bare copper filling factor</td>
</tr>
<tr>
<td>20</td>
<td>( K_{fr} )</td>
<td>Loss factor of friction and windage</td>
</tr>
<tr>
<td>21</td>
<td>( K_{s1}, \rho_h )</td>
<td>Lamination stacking factor and lamination mass density, kg/m³</td>
</tr>
<tr>
<td>22</td>
<td>( \rho_{nc} )</td>
<td>Non-laminated steel mass density, kg/m³</td>
</tr>
<tr>
<td>23</td>
<td>( \rho_{cm}, \rho_{cm} )</td>
<td>Material mass densities of copper and magnet, kg/m³</td>
</tr>
<tr>
<td>24</td>
<td>( N_{slot} )</td>
<td>Number of layers per slot</td>
</tr>
<tr>
<td>25</td>
<td>( \tau_c )</td>
<td>Coil pitch in slot pitch, ( \tau_c \leq \tau_g )</td>
</tr>
</tbody>
</table>
where $K_{co}$ is the copper filling factor, $N_s$ is the slot number, and $\beta$ is the angular slot pitch in mechanical radians. The inner radius of the stator core is now easily found based on the slot shape.

C. Outer Portion Design

The outer portion shares some common parameters with internal portion: the slot number $N_s$, current density $J_c$, and the slot area ($A_{slot} = A_{slot}$). To begin the outer design work, it is convenient to assume the outer radius of the outer air gap, $R_{o2}$, Equation (1) through (3) can be used to find the outer magnet thickness, $H_{PM2}$, as well simply replacing the subscript 1 using 2, which stands for the outer rotor variables. Then the inner radius of the outer rotor $R_{o2}$ is the sum of $H_{PM2}$ and $R_{PM2}$.

Similar with (5), the outer rotor back iron width is found to be

$$d_{e2} = \frac{w_{e2} B_{e2}}{2B_{e2}}.$$  

Then the outer radius of the outer rotor $R_{o2}$ must be

$$R_{o2} = R_{o2} + d_{e2},$$

where $R_{o2}$ is the inner radius of the outer rotor.

The designed stator core flux density is now ready to be calculated as

$B_{co} = \frac{K_{L1} K_{L2} B_{e1} \tau_{e1} + K_{L2} B_{e2} \tau_{e2}}{2 K_{m} d_{m}}$  

(11)

where $d_{m}$ is the stator yoke thickness, $K_{L1}$ and $K_{L2}$ are the factors of the leakage flux traveling from shoe to shoe, not through the back iron, for inner and outer air gaps, respectively, and $\tau_{e1}, 2$ is the stator inner/outer pole pitch.

Equation (11) gives the calculated stator core flux density. The difference between the value of (11) and the desired one in Table I directs the designer to adjust the initial value of $R_{PM2}$, which forms the inner loop in the design flow chart shown in Fig. 5. Equations (1) through (11) describe the machine geometric sizes and flux distribution. Based on these parameters and kept in mind that the inner windings are in series with the outside one, the machine power output, losses, efficiency, material volumes, and weights are easily found using the permanent magnet machine theory [3,5,6]. Therefore, those equations are omitted in this paper.

III. DESIGN PROCEDURE AND FLOWCHART

The design procedure and flowchart is shown in Fig. 5. The first step to design a machine is to specify the input data needed in the process as per the design objective. After estimating the radius $R_{PM1}$, the equations in section II.B can be evaluated. Once the outer part design is completed based on the estimation of $R_{PM2}$, the stator yoke flux density $B_{co}$ found using (11) has to be compared with the desired value. If they do not match each other well, the outer part design should be repeated based on the refined $R_{PM2}$ until they match each other. According to the researcher’s experience, this loop needs run 2–4 times. Another loop is to make sure the power output matches the desired one by adjusting the radius $R_{PM1}$. The both loops can be automatically finished by a computer program.

After that, back EMF, current, resistance and inductance of the phase can be obtained and the performance of losses, efficiency, material volumes, and weights can be calculated.

Usually this completes the design of a machine. However, if the designer is not satisfied with the performance, the third loop will be formed by backing to the beginning with the necessary adjustment of the input data. In addition, FEM analysis may be necessary to verify the analytical result, fine-tune it, and build confidence. Based upon the FEM analysis, the leakage flux factors may need to be updated to calculate the flux distribution more accurately.

Figure 5. Design flowchart

Objective data: $P_o$, $N$, $m$, $J_c$, $K_\phi$, $P$, $B_s$, $L$, $B'$, etc.

Estimation of $R_{PM2}$

Inner shoe, magnet, and slot design

Estimation of $R_{PM2}$

Outer shoe, magnet, slot and stator yoke design

$B_s$ match?

Refine $R_{PM2}$

Torque and power calculation

$P_s$ match?

Refine $R_{PM1}$

Loss, efficiency, volume, weight, cost, & EMF, I, R, L

Performance match?

Refine the associated inputs

FEM analysis: Flux densities, total and pulsating torque, and short circuit test

Flux densities & torque match?

Refine the leakage flux factors

Print the design results
The motor designed by this procedure should be quite causational, but may not be the optimal one since the input data have the significant effect on the motor and they have not been optimized. The input data to be optimized include parameters such as the magnetic loading, electrical loading, and the main aspect ratio. This work will be done in sections V and VI.

IV. FINITE ELEMENT ANALYSES

The Finite Element Analysis (FEA) method is employed in this section to verify the effectiveness of the design equations, refine the leakage flux factors, find the machine overload capability, and verify if the machine can survive under the short-circuit fault.

A. Flux Distribution

Fig. 6 shows the flux distributions at full load and no load for a 3 HP slotted dual-rotor RFTPM motor with 0.7 mm air gaps. The design data are: 1800 RPM, 8 poles, 120 Hz, efficiency 0.884, current density 551 A/cm², inner/outer electrical loading 200/159 A/cm. Comparison shown in Table 2 demonstrates that all the design values well match the measurements except the stator core flux density, which is lower than the designed value by about 15%. This is mainly caused by the leakage flux in the screw holes punched on the stator lamination to fix the stator. The slot leakage flux, which has not been considered in the equations in section II, also has contribution to the flux density discrepancy. The inner magnets were designed to have the “load” shape to reduce the magnet manufacture cost. The armature reaction can be observed comparing Fig. 3A and 3B or from Table 2, although it is as weak as expected.

B. Overload Capability

Overload capability is another important machine parameter. Short period overload capability is very desirable for some applications. The surface-mounted PM machines have high overload capability achieved by taking more current due to the low armature reaction. The torque associated with 2 times rated current is 23.87 Nm, which is 1.82 times the rated torque. When 3 times the rated current is fed, 2.31 times the rated torque will be produced. Fig. 7 demonstrates the torque variation with the armature current. It clearly shows the strong overload capability. The torque almost linearly increases as the current increases before it reaches the two times the rated current.

C. Short Circuit Protection

It is well known that permanent magnets must be protected against reverse fields exceeding some value $H_D$. The magnet flux density must not be reduced below a certain value of $B_D$. Typically, the value of $B_D$ is about $0.2$ Tesla at 100°C for currently available Nd-Fe-B materials [6].

One of the main reasons to cause the magnet demagnetizing is the short-circuited current, which is usually a few times the rated armature current. The effect of the stator field on the magnet is to increase its flux density on the leading edge and decreases it on the lagging edge. FEM analysis can simulate this situation by simply signing the short-circuited current to the windings.

Fig. 8 shows the $B$ vector (the flux density) distribution in the motor cross-sectional area during a short circuit (6 times the rated current). The four areas circled by black dash lines are the areas where the fluxes are opposite to that driven by the magnets. The amplitudes of the reversed fluxes are smaller in the inner magnets (0.05 Tesla) than those in the outer magnets (0.09 Tesla) since the magnet thickness outside is little smaller than the inside ones.

Comparing with the $B_D$ value $0.2$ Tesla, the magnets are still can survive from the short circuit with little range of safety. If it is desired to increase the safety range, a longer air gap or a larger magnet can be a choice. It has also been found that the approach to increase the ratio of the leakage to magnetizing inductance are helpful [6].

Figure 6. Flux density distribution

Figure 7. Overload capability
V. ANALYSES OF EFFECTS OF PARAMETERS ON MACHINE PERFORMANCE

This section will analyze the parameter effect on machine main performances based on the equations derived in section II. Under an assumption that the leakage fluxes at both ends of the machine are negligible, some examples to show how to obtain an optimal design will be demonstrated in this section.

A. Aspect Ratio and Pole Number

Fig. 9 through Fig. 12 show the effect of the pole number and the main aspect ratio, $K_{L_b}$, of machine inner stator diameter, $D_m$, to length, $L_r$, on machine efficiency, torque density, material, and weight, respectively. Main machine parameters are 3 phases, 3 horsepower, 1800 rpm, $\alpha_e=75^\circ$, $B_{m1}=0.25$ T, $K_{L_b}=330$ A/cm, and $J_c=448$ A/cm$^2$. Ferrite magnets are employed.

Figs. 9 through 12 illustrate that the aspect ratio is one of the main parameters to be optimized. Efficiency, torque density, material cost and weight are all strongly a function of the aspect ratio. For the example shown in these plots, a aspect ratio of 0.5 – 1.5 can achieve the best torque density with the reasonable efficiency, material cost and weight.

Also note that the values of the aspect ratio for the maximum torque density slightly vary with the pole number. In addition, the curves may have the different shapes for the different speed, magnetic loading, and electrical loading.

For the constant speed (1800 rpm for this case), the power supply frequency increases as the pole number increases, therefore the iron losses increases as well and the efficiency decreases, which is shown in Fig. 9.

Fig. 10 shows that the torque density is enhanced by doubling the pole numbers, while the difference between two adjacent curves decrease as the pole number increases. This is the combined effects of the stator and rotor core thicknesses and the airgap leakage flux. When the pole number is small, both stator and rotor cores are large and decrease quickly as the pole number increases, so that the machine torque density increases. When the pole number is large, however, the leakage flux increases faster and faster. Thus, the increase of the torque density becomes slower as the pole number increases. It can be foreseen that the torque density will decrease when the pole number is so high that the leakage flux effect is dominant.

Figure 8. $B$ vector distribution in the motor cross-sectional area at the short circuited current

<table>
<thead>
<tr>
<th>TABLE II. COMPARISON OF THE DESIGNED AND FEA RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designed</td>
</tr>
<tr>
<td>$B_{m1}$ (T)</td>
</tr>
<tr>
<td>$B_{m2}$ (T)</td>
</tr>
<tr>
<td>$B_{m3}$ (T)</td>
</tr>
<tr>
<td>$B_{m4}$ (T)</td>
</tr>
<tr>
<td>$B_{m5}$ (T)</td>
</tr>
<tr>
<td>$B_{m6}$ (T)</td>
</tr>
<tr>
<td>$B_{m7}$ (T)</td>
</tr>
<tr>
<td>Torque $T$ (Nm)</td>
</tr>
</tbody>
</table>

Figure 9. Efficiency vs. the ratio of $D_m$ to $L_r$

Figure 10. Torque density vs. the ratio of $D_m$ to $L_r$

Figure 11. Material cost per horsepower vs. the ratio of $D_m$ to $L_r$
The curves of the material cost and overall machine weight vs the aspect ratio of $D_b$ to $L_r$ are shown in Fig. 11 and 12. The ripples in these curves and in those in following plots are caused by the discontinuity of the conductor number per slot.

**B. Electrical Loading and Current Density**

The electrical loading and current densities are important elements to be optimized in machine design. It is common knowledge that the torque density will increase as either electrical loading or current density increases as shown in Fig. 14 for a constant power machines. In addition, the material cost and the overall weight decrease as well. However, the efficiency depicted in Fig. 13 decreases as the penalty of electrical loading and current density increasing. This implies the more cooling capability is required due to the higher power losses per unit air gap shown in Fig. 17. Thus, the design goal may become to achieve the highest torque density for a given efficiency $\eta_{\text{loss}}$ or a pound number of the weight, etc. Or the contrary, the goal may be to optimize the efficiency, weight, or material cost for a given torque density. In the curves in Figs. 13 through 17, 8-pole and an aspect ratio of 1.3 were used based on Figs. 9 through 12 in the last section.

The highest torque density can be obtained by selecting different points that have the same efficiency but on the different curves of electrical loading. For example, point A, B, C, D, and E, shown in Fig. 13, on the five different curves of $K_{r1} = 264, 297, 330, 363,$ and 396 A/cm, respectively, have the efficiency $\eta_{\text{loss}} = 0.87$. However, point B has the highest torque density of all shown in Fig. 14. Point E can achieve the lowest material cost shown in Fig. 15 and the lowest power losses per unit air gap shown in Fig. 17, while point A possesses the lightest weight shown in Fig. 16. Which point to be selected is dependent upon the optimizing purpose.

**C. Magnetic Loading**

The magnetic loading again plays a major role in PM machine designs. The optimization of the magnetic loading still remains a task. The air gap flux density and the angular width of each magnet ($2\theta_m$) may vary from motor to motor, and the resultant torque density, efficiency, and material cost also differ. Several plots in this section will demonstrate this influence. They are obtained using ceramic magnet, one of the ferrites. The magnet residual flux density $B_r$ is reduced to 0.322 from the original 0.4 Tesla to represent the temperature effect.
Although the efficiency shown in Fig. 18 monotonously increases as the more magnet material is used, the torque density is not in the same situation. Fig. 19 shows that the highest torque density is achieved by the curve with $B_{m1}$ of 0.25 Tesla, not by those with the flux densities having a scaling factor of 0.9, 0.95, 1.05, and 1.1. Meanwhile, the torque density increases with the magnet angle $\alpha_e$. When $\alpha_e$ is close to 90°, the magnet-to-magnet leakage flux increases quickly so that the increasing torque density slows. For $B_{m1} = 0.25$ T, the lowest material cost per horsepower shown in Fig. 20 and the highest weight in Fig. 21 are achieved when $\alpha_e$ is approximately 70° - 75°. It also has been noted that to obtain the maximum torque density, the magnet working point (flux density) are slightly different for the inner and outer magnets.

VI. OPTIMIZING PROCESS

An optimizing process is one of the necessary steps to achieve an optimal design. To emphasize this process, the optimization of the six important variables discussed in the last section is demonstrated in Fig. 22. It is helpful to note that the different optimization processes may be needed to achieve the different design purposes.

In each optimization step, the design process has to be run several times to reach the optimal point. The results from the previous step should be used as the next step initial conditions. The results of the last step usually do not match the initially assumed conditions used in the first step. Thus this optimization process has to be repeated with the updated initials until all the optimal results from each step well match the initial conditions of the others.
VII. PROTOTYPE AND EXPERIMENTAL RESULTS

A 3-horsepower, 1800 RPM prototype machine shown in Fig. 23 was designed using the design and optimization procedure presented in this paper with the designed output torque of 11.87 Newton-meters and efficiency of 87%. The experimental results of the prototype show that the output torque at the steady state is 11.78 Newton-meters and efficiency is 87.1%. Both values well match the design ones. The results prove the validity of the derived design equations and the proposed design procedure.

VIII. CONCLUSIONS

The key design equations and procedure of the RFTPM machines have been presented. Analyses of parameter effects on machine performances have been made. In addition, an optimization method to achieve the design objectives was proposed. The Finite Element Analysis results and the prototype measurements verify the effectiveness of the design equations and procedure.

REFERENCES


Figure 22. Optimization flowchart of the aspect ratio, pole number, electric, and magnetic loadings

Figure 23. Dual-Rotor RFTPM prototype