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Abstract — A novel machine family – the dual-rotor, Radial-Flux, Toroidally wound, Permanent-Magnet (RFTPM) machine – has been proven in a previous paper to be able to improve the machine efficiency and boost the torque density. This paper will develop the sizing equations for the RFTPM machines based on the machine overall sizes, material properties, and electrical and magnetic loadings to provide a quick method to evaluate the RFTPM machines. The accuracy of the developed sizing equations is proven by a prototype machine. A comparison among induction, axial-flux PM, and RFTPM machines are made based on the sizing equations. The RFTPM structure appears to be capable of substantially higher power density than equivalent induction machines, and more potential to achieve higher torque density than axial-flux PM machines.

Keywords — RFTPM Machine, Sizing Equations, PM Machine, Power Density

I. INTRODUCTION

In general, comparison of different machine types is a very formidable task since many variables exist for each machine and it is difficult to select those quantities which should be held constant and those that should be left free to vary. One traditional method of comparison is to use the $D_g L_e$ sizing equation, which compares the machine power on the basis of the airgap volume, where $D_g$ is the airgap diameter and $L_e$ is the effective stack length. However, the machine outer diameter $D_o$ is more directly related to the volume and thus to the cost and size of the machine. Meanwhile, given the great number of possible motor choices it has become important to compare power potential of machines with vastly different topologies, having a variety of different waveforms of back EMF and current. A systematic and easy-to-use method based on sizing equations is therefore very desirable to compare the capability of machines with different structures.

The general-purpose sizing and power density equations have recently been developed in terms of the machine outer diameter and power equations have been accomplished. The application to induction machines with small and medium power ratings (up to 100 hp) have been proven to be valid by comparing the actual motor data to the results estimated by the sizing equations.

Meanwhile, a novel machine family – the Dual-Rotor, Radial-Flux, Toroidally Wound, Permanent Magnet (RFTPM) machine – has been proposed and proven to be able to improve the machine efficiency and boost the torque density. It will be very interesting to develop sizing equations for the RFTPM machines based on the machine overall volume, material properties, and electrical and magnetic loadings, so that the RFTPM machine can be easily compared as other machines based on the same method – sizing equations.

In this paper, after the introduction of the general-purpose sizing and power density equations and the RFTPM machines, the sizing and power density equations based the machine outside diameter and length are developed for both the non-slotted and slotted RFTPM machines. Then, the power density of this machine type is compared with induction machines and axial-flux toroidally wound PM (AFTPM) machines.

The following SI units are used as default units for all the variables in this paper except those specially noted.

<table>
<thead>
<tr>
<th>Unit</th>
<th>SI Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Meter</td>
</tr>
<tr>
<td>Voltage</td>
<td>Volt</td>
</tr>
<tr>
<td>Power</td>
<td>Watt</td>
</tr>
<tr>
<td>Power density</td>
<td>W/m³</td>
</tr>
<tr>
<td>Current</td>
<td>Ampere</td>
</tr>
<tr>
<td>Current density</td>
<td>A/m²</td>
</tr>
<tr>
<td>Flux density</td>
<td>Tesla</td>
</tr>
<tr>
<td>Electrical loading</td>
<td>A/m</td>
</tr>
<tr>
<td>Frequency</td>
<td>Hertz</td>
</tr>
<tr>
<td>Flux linkage</td>
<td>Web</td>
</tr>
<tr>
<td>Time</td>
<td>Second</td>
</tr>
<tr>
<td>Torque</td>
<td>Nm</td>
</tr>
</tbody>
</table>

II. DUAL-ROTOR, RADIAL-FLUX, TOROIDALLY WOUND, PERMANENT-MAGNET MACHINES

Briefly, the dual-rotor, RFTPM machine is constructed so that two machines nested inside one another. The outer alternator has magnets at the outside surface of the outer airgap with the flux directed inward/outward, and the inner alternator has magnets at the inside surface of the inner airgap with the flux directed outward/inward. The two sets of stator coils are back-to-back toroidally wound, sharing common back iron. In this topology, the magnets drive flux across the two radial air gaps into the stator core; the flux then travels circumferentially along the core, back across the air gaps, and then through the rotor back iron of the rotor. Fig. 1 shows an example of the slotted RFTPM machine.

The toroidal machine works like two conventional machines in series. One is inside, the other is outside. They have the same armature current. Their back EMFs are in series. The outer and inner portion of the rotor are connected together by one end disc, which can work as a cooling fan. The stator is fixed at the other end to a frame.
It has been demonstrated that the dual-rotor RFTPM machine has the following features [6-8]:

- Greatly shortened end windings
- High efficiency
- High torque density
- Low-cost techniques available to reduce cogging torque

### III. GENERAL PURPOSE SIZING EQUATIONS AND POWER DENSITY

The general-purpose sizing equations have been developed in [1-5] and take the form of

\[
P_R = \frac{1}{1+K_\phi} \frac{m \pi}{2} K_i K_p \eta_B \pi_2 D_o^2 L_e \tag{1}
\]

and

\[
P_R = \frac{1}{1+K_\phi} \frac{m \pi}{2} K_i K_p \eta_B \pi_2 D_o^3 \tag{2}
\]

The definitions of the variables in both equations are listed below:

- \( P_R \): rated output power
- \( K_\phi \): ratio of electric loading on rotor and stator.
- \( m \): number of phases of the machine
- \( m_1 \): number of phases of each stator (if there is more than one stator, each stator has the same \( m_1 \)).
- \( K_i \): current waveform factor in order to indicate the effect of the current waveform, \( K_i = \frac{I_{\text{phmax}}}{I_{\text{rms}}} \)
- \( I_{\text{phmax}} \): the phase current and the peak phase current, respectively, and \( I_{\text{rms}} \) is the rms current.
- \( K_p \): electrical power waveform factor, \( K_p = \frac{1}{T} \int_0^T \frac{E_{pk} I_{\text{phmax}}}{I_{\text{phmax}}} dt = \frac{1}{T} \int_0^T f_p(t) f_l(t) dt \) where \( f_p(t) = \frac{e(t)}{E_{pk}} \) and \( f_l(t) = \frac{i(t)}{I_{\text{phmax}}} \) are the expressions for the normalized EMF and current waveforms. \( e(t) \) and \( E_{pk} \) are the phase air gap EMF and its peak value. \( T \) is the period of one EMF cycle.

- \( K_L = \frac{L_e D_g}{D_o} \), defined as the aspect ratio coefficient
- \( \lambda_o = \frac{D_o}{D_e} \), the diameter ratio
- \( \eta \): machine efficiency,
- \( B_{gmax} \): flux density in the air gap, maximum value
- \( A \): total electric loading, including stator and rotor loading
- \( N_t \): number of turns per phase,
- \( f \): power supply frequency
- \( p \): number of machine pole pairs

Finally, the machine power density for the total volume can be defined as

\[
\xi = \frac{P_R}{4 \pi D_o L_e} \tag{3}
\]

where \( L_e \) is the total length of the machine including the stack length and the protrusion of the end winding from the iron stack in the axial direction.

By examining the back EMF and current waveform for a particular machine type, the factors \( K_i \) and \( K_p \) can be determined. \( K_i \) and \( K_p \) for several typical waveforms have been well summarized in [1].

### IV. SIZING EQUATIONS OF RFTPM MACHINES

In the last section, the general purpose sizing equation was introduced. For dual-rotor, radial-flux machines, the sizing equation is still applicable but may take different forms. As a further study, a detailed approach will be presented in this section for the application of the general-purpose sizing and power density equations to dual-rotor radial-flux machines, focusing specifically on the RFTPM machine.

Referring to the structure of the RFTPM machine shown in Fig. 2 and assuming the same air gap flux densities for inside and outside, the back EMF for the machines is given by

\[
e(t) = \frac{dA_e}{dt} = 2\pi K_c N_t B_{gmax} \frac{L_e}{p} I_{\text{rms}} \frac{1+\lambda_o}{2} D_o f(t) \tag{4}
\]

where \( A_e \) is the air-gap flux linkage per phase, \( N_t \) is the number of turns per phase. The ratio \( \lambda \) is defined as

\[
\lambda = \frac{D_o}{D_e} \tag{5}
\]

From section II, the factor \( K_i \) is

\[
K_i = \frac{I_{\text{phmax}}}{I_{\text{rms}}} = \left[ \frac{1}{T} \int_0^T \left( \frac{e(t) I(t)}{I_{\text{phmax}}} \right)^2 dt \right]^{-1/2} \tag{7}
\]

where \( I_{\text{rms}} \) is the rms phase current which is related to the stator electric loading \( A_e \). For the RFTPM machine the electric loading \( A_e \) includes both the inner and outer parts. The inner electric loading \( A_{e1} \) has the form of

\[
A_{e1} = 2m_1 N_t \frac{I_{\text{rms}}}{\pi D_o} \tag{8}
\]
A. Sizing and Power Density Equations of Non-slotted RFTPM Machines

For the non-slotted RFTPM machines shown in Fig. 2A, the ratio \( \lambda \) can be derived using the relationship between \( D_i \) and \( D_o \),

\[
D_i = D_o - 2d_{ys}
\]

where \( d_{ys} \) is the thickness of the stator core.

If \( B_{cs} \) is defined as the flux density in the stator core, then \( d_{ys} \) can be found as

\[
d_{ys} = \frac{\pi \alpha \lambda_{max} (D_i + D_o) B_{max}}{4 p K_f e B_{cs} - \pi \alpha B_{max}}
\]

Combining above two equations, it can be determined that

\[
d_{ys} = \frac{D_i}{2 p K_f B_{cs} - \pi \alpha B_{max}}
\]

Substituting (19) and (21) into (5), the ratio \( \lambda \) turns out as

\[
\lambda = \frac{2 p K_f B_{cs} - \pi \alpha B_{max}}{2 p K_f B_{cs} + \pi \alpha B_{max}} \left[ 1 - \frac{2 \pi \alpha B_{max}}{2 p K_f B_{cs} + \pi \alpha B_{max}} \right]
\]

It is interesting to note that the ratio \( \lambda \) is not a function of \( D_i \) and \( D_o \). Table 1 shows \( \lambda \) values for the different pole pairs and magnet types at \( B_{cs} = 1.7 \text{ Tesla} \).

The machine total outer diameter \( D_i \) is given by

\[
D_i = D_o + 2 H_{cw} + 2 g + 2 H_{pm2} + 2 d_{ys}
\]

where

- \( H_{cw} \): outside part of the winding thickness in the radial direction.
- \( g \): air gap length.
- \( H_{pm2} \): thickness of the outer permanent magnets.
- \( d_{ys} \): outer rotor core thickness.

![Figure 2. Cross-section view of RFTPM machines](image)

Table 1. \( \lambda \) VALUES FOR THE DIFFERENT POLE PAIRS & MAGNET TYPES

<table>
<thead>
<tr>
<th>Pole pairs ( p )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{max} = 0.24 \text{ T} ) (Ferrite magnets)</td>
<td>0.669</td>
<td>0.820</td>
<td>0.876</td>
<td>0.906</td>
<td>0.924</td>
</tr>
<tr>
<td>( B_{max} = 0.6 \text{ T} ) (Rare earth magnets)</td>
<td>0.337</td>
<td>0.603</td>
<td>0.716</td>
<td>0.779</td>
<td>0.816</td>
</tr>
</tbody>
</table>

at \( B_{cs} = 1.7 \text{ Tesla}, \phi_p = 0.85, K_{fe} = 0.95 \)
The amplitudes of both the inner and outer winding thickness depend on the stator equivalent electric loading \(A_i\), the current density \(J_i\), and the copper filling factor \(K_{cu}\). For the RFTPM machine, there is no rotor winding, hence \(K_{slot} = 0\). Considering the typical trapezoidal waveforms in PM machines, \(K_{focus}\) is given as 0.881[2].

Thus, it is clear that \(H_{cu1}\) and \(H_{cu2}\) take the form of

\[
H_{cu1} = \frac{\lambda}{1 + \lambda} A_i \quad J_i K_{cu} \tag{26}
\]

and the power density of the RFTPM machine is

\[
\xi = \frac{P_{RFTPM}}{\pi D_t^2 L_t} \tag{39}
\]

Equation (22), (31), (33), (38), and (39) well define the RFTPM machine power density. In these equations, the only variable to be determined is efficiency \(\eta\). It closely depends upon the materials used, electric loading \(A\), and the selection of the number of pole pairs, frequency, machine structure and geometric sizes. Usually, the efficiency of PM machines may vary from 80% to 95% for small and medium power ratings.

### B. Sizing and Power Density Equations of Slotted RFTPM Machines

The general sizing equations for the non-slotted RFTPM machines have been derived in the last section. For the slotted RFTPM machine shown in Fig. 2B, the expressions of the diameter ratio \(\lambda\) and the total length \(L_t\) should be updated due to the presence of slots. The new relationship among \(d_{ss}, D_t,\) and \(D_s\) is

\[
D_s = D_t - 2D_i - 2d_{ss} \tag{40}
\]

where \(d_{ss}\) is the sum of the depths of inner and outer slot slots, which is composed of the inner slot depth \(d_{in}\) and the outer slot depth \(d_{out}\) indicated in (41) and (42).

\[
d_{in} = \frac{A_{in}}{K_{ds} K_{slot} J_i K_{cu}} \tag{41}
\]

\[
d_{out} = \frac{A_{out}}{K_{ds} K_{slot} J_i K_{cu}} \tag{42}
\]

where \(K_{ds}\) is the ratio of the slot width to the total width occupied by a pair of slot and tooth. In two equations above, it is assumed that the factor \(K_{ds}\) and \(K_{slot}\) are same for
both inner and outer slots. Given \( A_s = A_{s1} + A_{s2} \), \( d_{s} \) can therefore expressed as

\[
d_{s} = d_{s1} + d_{s2} = \frac{A_s}{K_{s}K_{sd}K_{sc}K_{cu}} \tag{43}
\]

Substituting (20), which is still correct for the slotted case, into (40), and after some manipulation, it yields

\[
\frac{D_t}{D_o} = 1 - \frac{2\alpha\lambda B_{\text{max}}} {2pK_{s}B_{m}} - \frac{4\lambda B_m A_s}{2pK_{s}B_{m} + \alpha\lambda B_{\text{max}}} \frac{1}{D_o} \tag{44}
\]

Substituting (43), the ratio \( \lambda \) is finally found as

\[
\lambda = \frac{D_t}{D_o} = 1 - \frac{2\alpha\lambda B_{\text{max}}} {2pK_{s}B_{m} + \alpha\lambda B_{\text{max}}} - \frac{4\lambda B_m A_s}{2pK_{s}B_{m}} \frac{1}{D_o} \tag{45}
\]

Note that the first two terms are same as the non-slotted case in (22), the last one is caused by the slots and is inversely proportional to the stator out diameter \( D_o \).

Since the winding thickness is already included in \( D_o \) for this case, the machine total outer diameter \( D_t \) should be updated to

\[
D_t = D_o + 2g + 2H_{PM2} + 2d_{s2} \tag{46}
\]

Also, the magnet thickness, \( H_{PM2} \), will be different due to the change in the effective air gap.

\[
H_{PM2} = \frac{\mu K_{s2}gB_{\text{max}}}{B_r - B_m} \tag{47}
\]

where \( K_{s2} \) is the Carter coefficient for stator outside slotting. Substituting (47) and (30), which is kept unchanged for the slotted case, into (46), it yields

\[
D_t = D_o(1 + \frac{\pi K_{s2}B_m}{2pB_r} + 2g(1 + \frac{\mu K_{s2}B_{\text{max}}}{B_r - B_m})) \tag{48}
\]

The total machine length \( L_t \) can be expressed as

\[
L_t = L_e + 2L_{cue} + L_{rev} \tag{49}
\]

where \( L_{cue} \) is the protrusion of the end winding from the iron stack and should be updated to

\[
L_{cue} = \frac{A_s}{1 + \lambda K_{sd}K_{sc}K_{cu}} \tag{50}
\]

Equation (34) through (36) are still correct. Although the expressions of the overall sizing equation and power density indicated in (51), (52), and (53) have the same forms as those in the last section, the calculation formulas of \( \lambda, D_o \), and \( L_e \) are modified.

\[
P_{RFTPM} = 8.6951\eta K_{s}K_{FL}A_s \frac{L_e}{p} \lambda D_o^3L_e \tag{51}
\]

\[
P_{RFTPM0} = 11.07\eta K_{s}K_{FL} \sin \frac{K_{s}K_{FL} \pi}{2} B_m A_s \frac{L_e}{p} \lambda D_o^3L_e \tag{52}
\]

\[
\xi = \frac{P_{RFTPM}}{\pi 4D_o^2L_e} \tag{53}
\]

V. SIZING EQUATION EVALUATION AND POWER DENSITY COMPARISON

A three-phase 3 HP RFTPM prototype machine shown in Fig. 3 was designed based on the detailed design equations in [6], built, and tested. The main machine performance parameters as well as the machine sizes and power density are listed in Table II. The same prototype machine parameters are used in the sizing equations derived above to evaluate their accuracy. The resultant machine overall length and diameter are 10.24 cm and 17.47 cm, respectively. They are very close to the prototype data of 11.1 cm length and 17.7 cm diameter. The power density from the sizing equation is 0.91 W/cm³, which is only 11% higher than the prototype data of 0.82 W/cm³. For a sizing equation with simplifications, 11% error is reasonable, which proves the sizing equation’s accuracy.

It is now possible to compare the power densities of dual-rotor RFTPM machines and other machines based on the sizing equations. The power density comparison between IM, AFTPM and RFTPM machines is shown in Fig. 4, where the data of the RFTPM machines are calculated based on the sizing and power density equations derived before, while those of AFTPM and induction machines were summarized by Dr. Luo [1].

Every point on the curves represents a different machine design optimized for that particular rated mechanical speed. The curves show that the non-slotted rare earth RFTPM with 8 poles and 4 poles achieve the higher power density by a factor of 1.3–1.4 than the AFTPMs with 8 poles and 4 poles, respectively. Even the ferrite magnets are used in the slotted topologies, they still reveal as high power density capability as the AFTPM machines with rare earth magnets.

It should be mentioned that the sizing equations are derived to provide a simple comparison method of the capability of different machine topologies. They are not suitable and should not be used for the machine detailed design due to the simplification, which usually introduces some error. For example, the error caused by the simplification makes the power density lines, Curve 3 and 6, Curve 2 and 4 in Fig. 4, crossed, respectively.

<table>
<thead>
<tr>
<th>Torque</th>
<th>12.87 Nm</th>
<th>Speed</th>
<th>1800 r/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>0.87</td>
<td>Power factor</td>
<td>0.78</td>
</tr>
<tr>
<td>DC bus voltage</td>
<td>380 V</td>
<td>Phase current</td>
<td>7.15 A, rms</td>
</tr>
<tr>
<td>Inner elec. loading</td>
<td>245 A/cm</td>
<td>Current density</td>
<td>452 A/cm³</td>
</tr>
<tr>
<td>Outer elec. loading</td>
<td>186 A/cm</td>
<td>Air gap flux</td>
<td>0.217 T</td>
</tr>
<tr>
<td>Overall diameter</td>
<td>17.7 cm</td>
<td>Overall length</td>
<td>11.1 cm</td>
</tr>
<tr>
<td>Volume</td>
<td>2730 cm³</td>
<td>Power density</td>
<td>0.82 W/cm³</td>
</tr>
</tbody>
</table>

Figure 3. RFTPM prototype machine

Table II RFTPM PROTOTYPE PARAMETERS
Figure 4. Power density comparison among RFTPM, AFTPM and induction machines

For IM and AFTPMs, \( A = 60 \text{ kA/m}, J_c = 6.2 \times 10^6 \text{ A/m}^2, P_R = 75 \text{ kW} \)

For RFTPM, \( A = 50/45 \text{ kA/m}, J_c = 6.2 \times 10^6 \text{ A/m}^2, P_R = 65 \text{ kW (Rare)} / 75 \text{ kW (Ferrite)} \)

VI. CONCLUSION

After the review of the general purpose sizing and power density equations and novel RFTPM machine structures, the sizing equations for the radial-flux, toroidally wound, permanent magnet machines have been derived. Both slotted and non-slotted machine topologies are included. These equations are in terms of the machine overall sizes, material properties, electrical loading, magnetic loading, and some factors and constants. They are easy to be employed.

This new approach permits a comparison of the novel RFTPM machines with other machine topologies based upon the overall occupied volume, not the airgap volume. It has been proven by a prototype machine that the sizing equation, together with experience parameters, can provide quite accurate estimation for the machine sizes and performances.

A fair comparison among induction, AFTPM, and RFTPM machines was carried out based upon the sizing and power density equations. The RFTPM structure appears to be capable of substantially higher power density than equivalent induction machines, and more potential to achieve higher torque density than AFTPM machines.

The sizing equations were derived to provide a simple and quick comparison method among different machine topologies. They are simplified and lack of design details. To design a RFTPM machine, more detailed calculations may be needed.

REFERENCE