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MULTIPLE REFERENCE FRAMES APPLIED TO IMPEDANCE UNBALANCES OF INDUCTION MACHINERY -- THE OPEN CIRCUITED STATOR PHASE

T. A. Lipo
General Electric Company
Schenectady, New York

Summary

In this paper, the method of multiple reference frames is applied to the analysis of a single phase open circuit of a three phase induction machine. This analysis demonstrates, for the first time, that multiple reference frames are equally suited to the analysis of impedance unbalances as to voltage unbalances on ac machinery. Hence, the method of multiple reference frames appears to be a tool that is equally powerful as symmetrical components in the analysis of unbalanced operation of ac machinery.

Introduction

The analysis of ac machines by means of symmetrical components has had a long and productive history.[1] Indeed, the need for a method of analyzing unbalanced ac machinery, in particular the single phase induction machine, was the primary motivating force in its development. The method has, over the years, been gradually extended and is today the accepted method of studying ac machine unbalances.

At a somewhat later date, the method also became widely used in the analysis of unbalanced static power networks. However, in this field, the need for symmetrical components has been diminishing. Other transformations, notably the Clarke Components,[2] have been developed which also decouple the impedance matrix for a static system. Since these newer transformations involve only the use of real elements, they are much more suitable for the analysis of power system transients as well as the steady-state.

An alternative to the use of symmetrical components in the analysis of ac machinery has also appeared.[3] This method, entitled the method of multiple reference frames, employs the repetitive use of balanced, synchronously rotating, two-phase d-q sets. It has been shown that any three periodic waveshapes applied to the three phases of a symmetrical induction machine can be resolved into a series of balanced, two-phase sets. Since the solution of machine currents for a single balanced d-q set is straightforward, stator and rotor currents for arbitrary periodic voltages, with constant speed operation, can be obtained by simple superposition.

The technique, which is, of course, related to symmetrical components since the method generates the same results, differs primarily in that the analysis is carried out and the solution is obtained in the time domain. It is not necessary to resort to complex numbers at any time. The d-q axis stator and rotor currents are expressed in terms of matrix equations involving only real numbers. Hence, this formulation is ideally suited to analyses involving a digital computer. Since the solution is in the time domain, the study of constant speed transients is a rigorous, straightforward extension of the method. Unfortunately, the analogous extension of symmetrical components using the instantaneous symmetrical components is not as straightforward.[4]

At the present time, a serious limitation to the application of the method is that the phase voltages must be defined uniquely as functions of time. Hence, ac machinery supplied by sources with unbalanced supply impedances presently cannot be studied by this technique. In this paper, the method of multiple reference frames is applied to the case of a single phase open circuit. It is shown that the solution of such problems are entirely within the framework set forth by Krause. Thus, it appears that multiple reference frames are a valid alternative to the use of symmetrical components over the entire class of problems which normally utilizes this technique.

Method of Multiple Reference Frames

It has been shown that the voltage equations of a three-wire symmetrical induction machine may be expressed in terms of a d-q axis which rotates at an arbitrary velocity ω (arbitrary reference frame).[5] In matrix form, these equations in per unit are:

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} x_s & \frac{\omega}{\omega_b} x_s & \frac{p}{\omega_b} x_m & \frac{\omega}{\omega_b} x_m \\ -\frac{\omega}{\omega_b} x_s & r_s + \frac{p}{\omega_b} x_s & -\frac{\omega}{\omega_b} x_m & \frac{p}{\omega_b} x_m \\ \frac{p}{\omega_b} x_s & \frac{\omega-\omega_r}{\omega_b} x_m & r_r' + \frac{p}{\omega_b} x_r' & \frac{\omega-\omega_r}{\omega_b} x_r' \\ -\frac{\omega-\omega_r}{\omega_b} x_m & \frac{p}{\omega_b} x_m & -\frac{\omega-\omega_r}{\omega_b} x_r' & r_r' + \frac{p}{\omega_b} x_r' \end{bmatrix} \times \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr}' \\ i_{dr}' \end{bmatrix} \quad (1)$$

$$T_e = x_m (i_{qs}\, i_{dr}' - i_{ds}\, i_{qr}') \quad (2)$$

where, p is the operator d/dt; r_s the stator resistance; r_r' the rotor resistance (referred to the stator winding); x_s, x_r' and x_m are the stator and stator-referred rotor self reactances and magnetizing reactance, respectively. The electrical angular velocity of the rotor is ω_r. The angular velocity ω_b is a reference or base quantity used to define the reactances x_s, x_r' and x_m.

In the case of a three-wire system, the d-q stator voltages (ds-qs voltages) in the arbitrary reference frame are related to the stator phase voltages v_{as}, v_{bs}, v_{cs} by

$$v_{qs} = v_{qs}^{s} \cos \omega t - v_{ds}^{s} \sin \omega t \quad (3)$$

$$v_{ds} = v_{qs}^{s} \sin \omega t + v_{ds}^{s} \cos \omega t \quad (4)$$

where

$$v_{qs}^{s} = v_{as} \quad (5)$$

$$v_{ds}^{s} = \frac{1}{\sqrt{3}} (v_{cs} - v_{bs}) \quad (6)$$

The intermediate variables v_{qs}^{s} and v_{ds}^{s} are d-q voltages defined by fixing the arbitrary reference in the stator (d^s - q^s voltages). The superscript s is used to denote that these variables are expressed in the stationary reference frame.

The inverse relationships to (3)-(6) are

$$v_{qs}^{s} = v_{qs} \cos \omega t + v_{ds} \sin \omega t \quad (7)$$

$$v_{ds}^{s} = -v_{qs} \sin \omega t + v_{ds} \cos \omega t \quad (8)$$

$$v_{as} = v_{qs}^{s} \quad (9)$$

$$v_{bs} = -\frac{1}{2} v_{qs}^{s} - \frac{\sqrt{3}}{2} v_{ds}^{s} \quad (10)$$

$$v_{cs} = -\frac{1}{2} v_{qs}^{s} + \frac{\sqrt{3}}{2} v_{ds}^{s} \quad (11)$$

Similar definitions apply for the d-q stator currents and the d-q stator referred rotor currents. Space limitations prohibit listing these equations explicitly. However, replacing "v" with "i" in (3)-(11) will generate the analogous d-q equations for the stator currents. The proper expressions for stator referred rotor currents is also readily inferred.

Consider now the case where unbalanced voltages of an arbitrary periodic waveform is applied to the machine. In this case, the phase voltages can be expressed

$$v_{as} = V_{oa} + \sum_{k=1}^{\infty} V_{ka\alpha} \cos k\omega_e t + V_{ka\gamma} \sin k\omega_e t \quad (12)$$

$$v_{bs} = V_{ob} + \sum_{k=1}^{\infty} V_{kb\alpha} \cos k\omega_e t + V_{kb\gamma} \sin k\omega_e t \quad (13)$$

$$v_{cs} = V_{oc} + \sum_{k=1}^{\infty} V_{kc\alpha} \cos k\omega_e t + V_{kc\gamma} \sin k\omega_e t \quad (14)$$

The d^s-q^s voltages contain terms of the same frequency and are defined

$$v_{qs}^{s} = \sum_{k=o}^{\infty} V_{kq\alpha} \cos k\omega_e t + V_{kq\gamma} \sin k\omega_e t \quad (15)$$

$$v_{ds}^{s} = \sum_{k=o}^{\infty} V_{kd\alpha} \cos k\omega_e t + V_{kd\gamma} \sin k\omega_e t \quad (16)$$

where, it is clear, the definitions for $V_{kq\alpha}$, $V_{kq\gamma}$, $V_{kd\alpha}$, $V_{kd\gamma}$ are readily defined by (12)-(14) and (5)-(6).

Similar definitions apply for stationary d -q stator and rotor currents. For example

$$i_{qr}'^{s} = \sum_{k=o}^{\infty} (i_{kqr\alpha} \cos k\omega_e t + i_{kqr\gamma} \sin k\omega_e t) \quad (17)$$

$$i_{dr}'^{s} = \sum_{k=o}^{\infty} (i_{kdr\alpha} \cos k\omega_e t + i_{kdr\gamma} \sin k\omega_e t) \quad (18)$$

Eqs. (15) and (16) can be rearranged in the form

$$v_{qs}^{s} = \frac{1}{2} \sum_{k=o}^{\infty} (V_{kq\gamma} + V_{kd\alpha}) \sin k\omega_e t + \frac{1}{2} \sum_{k=o}^{\infty} (V_{kq\alpha} - V_{kd\gamma}) \cos k\omega_e t + \frac{1}{2} \sum_{k=o}^{\infty} (V_{kq\alpha} + V_{kd\gamma}) \cos k\omega_e t + \frac{1}{2} \sum_{k=o}^{\infty} (V_{kq\gamma} - V_{kd\alpha}) \sin k\omega_e t \quad (19)$$

$$v_{ds}^{s} = \frac{1}{2} \sum_{k=o}^{\infty} (V_{kd\gamma} + V_{kd\alpha}) \cos k\omega_e t - \frac{1}{2} \sum_{k=o}^{\infty} (V_{kq\alpha} - V_{kd\gamma}) \sin k\omega_e t + \frac{1}{2} \sum_{k=o}^{\infty} (V_{kq\alpha} + V_{kd\gamma}) \sin k\omega_e t - \frac{1}{2} \sum_{k=o}^{\infty} (V_{kq\gamma} - V_{kd\alpha}) \cos k\omega_e t \quad (20)$$

It can be noted that all terms correspond to balanced sets. The first summations as well as the second summations in (19) and (20) correspond to balanced two-phase sets of voltages which produce MMFs rotating in a positive direction at an angular velocity $k\omega_e$.

The corresponding third and fourth summations represent balanced voltage pairs which produce negatively rotating MMFs. Hence, voltages of arbitrary waveshape can be separated into pairs of balanced positively and negatively rotating d-q voltages. It can be noted that this symmetry was originally observed by Krause in a reference frame rotating at the angular velocity of the fundamental voltage component. Hence, his solution originally necessitated additional transformations in its application.

The method of multiple reference frames is simply a repetitive application of synchronously rotating reference frames. A particular k in (19) and (20), say n, and a direction of rotation, say positive, is selected. The speed of the arbitrary reference frame ω is set equal to $n\omega_e$ in (3) and (4) and the d^s-q^s voltages referred to this reference frame (+ ne frame). The currents which flow as a result of these voltage pairs are obtained from (1) by setting $\omega=n\omega_e$ and p=0.

The solution for the corresponding currents is obtained by inverting the resulting real, 4x4 matrix. An explicit expression for this inverse matrix is given by Krause.[3] The currents obtained are now referred back to the stationary reference frame by setting $\omega=n\omega_e$ in the current expressions analogous to (7) and (8). Similarly, the negatively rotating voltage sets in (19) and (20) are selected for k=n. These voltages are then referred to a negatively rotating reference frame where $\omega= -n\omega_e$. The corresponding currents are solved and then referred back to the stationary frame. The total stationary d-q axis stator and stator referred rotor currents are the sum of all such currents as k varies from 0 to ∞. For example, the rotor currents can be expressed compactly as

$$i'^{s}_{qr} = \sum_{k=o}^{\infty}\Big[(i'^{+ke}_{qr}+i'^{-ke}_{qr})\cos k\omega_e t + (i'^{+ke}_{dr} - i'^{-ke}_{dr})\sin k\omega_e t\Big] \tag{21}$$

$$i'^{s}_{dr} = \sum_{k=o}^{\infty}\Big[-(i'^{+ke}_{qr} - i'^{-ke}_{qr})\sin k\omega_e t + (i'^{+ke}_{dr} + i'^{-ke}_{dr})\cos k\omega_e t\Big] \tag{22}$$

where the superscript $\pm$ ke denotes the reference frame used to obtain the particular current component. Similar expressions apply for i^s_{qs} and i^s_{ds}. A comparison of (21) and (22) with (17) and (18) indicates for all $k \geq 0$

$$i_{kqr\alpha} = i^{+ke}_{qr} + i^{-ke}_{qr} \tag{23}$$

$$i_{kqr\gamma} = i^{+ke}_{dr} - i^{-ke}_{dr} \tag{24}$$

$$i_{kdr\gamma} = -(i^{+ke}_{qr} - i^{-ke}_{qr}) \tag{25}$$

$$i_{kdr\alpha} = i'^{+ke}_{dr} + i'^{-ke}_{dr} \tag{26}$$

Analogous results are readily obtained for the d-q stator currents.

The electromagnetic torque can be computed by substituting (21), (22) and the related expressions for i^s_{qs} and i^s_{ds} into (2). The result is given by Krause.[3]

Open Circuited Windings

Krause has shown that multiple reference frames can be readily applied to any application where the phase voltages are known functions of time. In situations where the source impedances are unbalanced or an open circuit voltage appears across one of the phases, it is not apparent how the method can be applied. With suitable modifications, it will be shown that the method can be extended to include single phase operations as well as other types of phase impedance unbalances.

In the following analysis, it is assumed that a three-phase, three-wire induction motor, previously connected to a three-phase source, develops an open circuit in phase a. The source voltages e_{ag}, e_{bg}, e_{cg}, although possibly not balanced, are of a single frequency. Since currents flow in the shorted rotor windings, the voltage which appears across phase as is an induced emf owing to mutual coupling.

Since the stationary d^s_s-q^s_s axes are aligned so that $i^s_{qs}=i^s_{as}$ and $i'^s_{qr}=i'^s_{ar}$, the effect of open circuit may be expressed in the d^s_s-q^s_s axes as[5]

$$v^s_{qs} = \frac{p}{\omega_b} x_m i'^s_{qr} \tag{27}$$

The corresponding direct axis voltage remains defined as a function of time and is given by

$$v^s_{ds} = \frac{1}{\sqrt{3}}(e_{cg} - e_{bg}) \tag{28}$$

For purposes of analysis, it is assumed that

$$v^s_{ds} = V_{ds\alpha}\cos\omega_e t + V_{ds\gamma}\sin\omega_e t \tag{29}$$

where it is clear that constants $V_{ds\alpha}$ and $V_{ds\gamma}$ can be found from (28) for a specific application. Since a single frequency is applied to the system, i'^s_{qs} can be assumed sinusoidal

$$i'^s_{qr} = i_{qr\alpha}\cos\omega_e t + i_{qr\gamma}\sin\omega_e t \tag{30}$$

Hence from (27)

$$v^s_{qs} = -\frac{\omega_e}{\omega_b} x_m i_{qr\alpha}\sin\omega_e t + \frac{p}{\omega_b}(x_m i_{qr\alpha})\cos\omega_e t + \frac{\omega_e}{\omega_b} x_m i_{qr\gamma}\cos\omega_e t + \frac{p}{\omega_b}(x_m i_{qr\gamma})\sin\omega_e t \tag{31}$$

Employing (19) and (20), Eqs. (29) and (31) can be arranged in the form

$$\begin{aligned} v^s_{qs} = {} & \frac{1}{2}\Big[-\frac{\omega_e}{\omega_b} x_m i_{qr\alpha} + \frac{p}{\omega_b}(x_m i_{qr\gamma}) + V_{ds\alpha}\Big]\sin\omega_e t \\ & + \frac{1}{2}\Big[\frac{\omega_e}{\omega_b} x_m i_{qr\gamma} + \frac{p}{\omega_b}(x_m i_{qr\alpha}) - V_{ds\gamma}\Big]\cos\omega_e t \\ & + \frac{1}{2}\Big[\frac{\omega_e}{\omega_b} x_m i_{qr\gamma} + \frac{p}{\omega_b}(x_m i_{qr\alpha}) + V_{ds\gamma}\Big]\cos\omega_e t \\ & + \frac{1}{2}\Big[-\frac{\omega_e}{\omega_b} x_m i_{qr\alpha} + \frac{p}{\omega_b}(x_m i_{qr\gamma}) - V_{ds\alpha}\Big]\sin\omega_e t \end{aligned} \tag{32}$$

$$\begin{aligned} v^s_{ds} = {} & \frac{1}{2}\Big[-\frac{\omega_e}{\omega_b} x_m i_{qr\alpha} + \frac{p}{\omega_b}(x_m i_{qr\gamma}) + V_{ds\alpha}\Big]\cos\omega_e t \\ & - \frac{1}{2}\Big[\frac{\omega_e}{\omega_b} x_m i_{qr\gamma} + \frac{p}{\omega_b}(x_m i_{qr\alpha}) - V_{ds\gamma}\Big]\sin\omega_e t \\ & + \frac{1}{2}\Big[\frac{\omega_e}{\omega_b} x_m i_{qr\gamma} + \frac{p}{\omega_b}(x_m i_{qr\alpha}) + V_{ds\gamma}\Big]\sin\omega_e t \\ & - \frac{1}{2}\Big[-\frac{\omega_e}{\omega_b} x_m i_{qr\alpha} + \frac{p}{\omega_b}(x_m i_{qr\gamma}) - V_{ds\alpha}\Big]\cos\omega_e t \end{aligned} \tag{33}$$

Thus, the d^s_s-q^s_s voltages have again been separated into 4 balanced sets, two sets rotating in the positive direction, and two sets in the negative direction. Considering first only the two positively rotating sets, the d-q voltages in the +e frame, obtained by setting $\omega=\omega_e$ in (3) and (4) are

$$v^{+e}_{qs} = \frac{1}{2}\Big[\frac{\omega_e}{\omega_b} x_m i_{qr\gamma} + \frac{p}{\omega_b}(x_m i_{qr\alpha}) - V_{ds\gamma}\Big] \tag{34}$$

$$v^{+e}_{ds} = \frac{1}{2}\Big[-\frac{\omega_e}{\omega_b} x_m i_{qr\alpha} + \frac{p}{\omega_b}(x_m i_{qr\gamma}) + V_{ds\alpha}\Big] \tag{35}$$

Letting $\omega = -\omega_e$ and referring the two negatively rotating sets to the -e frame, the d^{e}_{s}, q^{e}_{s} voltages are

$$v^{-e}_{qs} = \frac{1}{2}\Big[\frac{\omega_e}{\omega_b} x_m i_{qr\gamma} + \frac{p}{\omega_b}(x_m i_{qr\alpha}) + V_{ds\gamma}\Big] \tag{36}$$

$$v_{ds}^{-e} = \frac{1}{2}\left[\frac{\omega_e}{\omega_b} x_m i_{qr\alpha} - \frac{p}{\omega_b}(x_m i_{qr\gamma}) + V_{ds\alpha}\right] \quad (37)$$

The d-q currents resulting from the positive rotating voltage sets can be solved by substituting (34) and (35) into the equations describing the machine in the +e frame of reference obtained by setting $\omega = +\omega_e$ in (1). Similarly, (36) and (37) are substituted in (1) where ω is set equal to $-\omega_e$. The resulting equations are

$$\begin{bmatrix} \frac{1}{2}\left(\frac{\omega_e}{\omega_b} x_m i_{qr\gamma} + \frac{p}{\omega_b} x_m i_{qr\alpha} - V_{ds\gamma}\right) \\ \frac{1}{2}\left(-\frac{\omega_e}{\omega_b} x_m i_{qr\alpha} + \frac{p}{\omega_b} x_m i_{qr\gamma} + V_{ds\alpha}\right) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} x_s & \frac{\omega_e}{\omega_b} x_s & \frac{p}{\omega_b} x_m & \frac{\omega_e}{\omega_b} x_m \\ -\frac{\omega_e}{\omega_b} x_s & r_s + \frac{p}{\omega_b} x_s & -\frac{\omega_e}{\omega_b} x_m & \frac{p}{\omega_b} x_m \\ \frac{p}{\omega_b} x_m & \frac{\omega_e - \omega_r}{\omega_b} x_m & r_r' + \frac{p}{\omega_b} x_r' & \frac{\omega_e - \omega_r}{\omega_b} x_r' \\ -\frac{\omega_e - \omega_r}{\omega_b} x_m & \frac{p}{\omega_b} x_m & -\frac{\omega_e - \omega_r}{\omega_b} x_r' & r_r' + \frac{p}{\omega_b} x_r' \end{bmatrix} \times \begin{bmatrix} i_{qs}^{+e} \\ i_{ds}^{+e} \\ i_{qr}^{\prime +e} \\ i_{dr}^{\prime +e} \end{bmatrix} \quad (38)$$

$$\begin{bmatrix} \frac{1}{2}\left(\frac{\omega_e}{\omega_b} x_m i_{qr\gamma} + \frac{p}{\omega_b} x_m i_{qr\alpha} + V_{ds\gamma}\right) \\ \frac{1}{2}\left(\frac{\omega_e}{\omega_b} x_m i_{qr\alpha} - \frac{p}{\omega_b} x_m i_{qr\gamma} + V_{ds\alpha}\right) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} x_s & -\frac{\omega_e}{\omega_b} x_s & \frac{p}{\omega_b} x_m & -\frac{\omega_e}{\omega_b} x_m \\ \frac{\omega_e}{\omega_b} x_s & r_s + \frac{p}{\omega_b} x_s & \frac{\omega_e}{\omega_b} x_m & \frac{p}{\omega_b} x_m \\ \frac{p}{\omega_b} x_m & -\left(\frac{\omega_e + \omega_r}{\omega_b}\right) x_m & r_r' + \frac{p}{\omega_b} x_r' & -\left(\frac{\omega_e + \omega_r}{\omega_b}\right) x_r' \\ \frac{\omega_e + \omega_r}{\omega_b} x_m & \frac{p}{\omega_b} x_m & \frac{\omega_e + \omega_r}{\omega_b} x_r' & r_r' + \frac{p}{\omega_b} x_r' \end{bmatrix} \times \begin{bmatrix} i_{qs}^{-e} \\ i_{ds}^{-e} \\ i_{qr}^{\prime -e} \\ i_{dr}^{\prime -e} \end{bmatrix} \quad (39)$$

In their present form, these equations cannot yet be solved since the rotor current amplitudes $i_{qr\alpha}$ and $i_{qr\gamma}$ in the voltage vectors of (38), (39) are as yet unknown. Letting k=1 in (23) and (24), it is clear that $i_{qr\alpha}$ and $i_{qr\gamma}$ can be related to the synchronous frame d-q currents by

$$i_{qr\alpha} = i_{qr}^{+e} + i_{qr}^{-e} \quad (40)$$

$$i_{qr\gamma} = i_{dr}^{+e} - i_{dr}^{-e} \quad (41)$$

The machine matrix equations in the +e and -e reference frame can now be used to solve for the +e and -e d-q currents. However, since both +e and -e currents appear in both equations by virture of (40), (41), the matrix equations are coupled and cannot be solved independently.

Equations (38) and (39), together with (40) and (41) yield the single 8x8 matrix equation, Eq. (42). This equation is valid for both steady-state and for transients involving constant speed operation and is the final result using multiple reference frames. In the steady-state, the d-q currents in the +e and -e frames are constant and can be obtained by setting p=0 in (42). They are readily related back to the stator frame d-q currents by (23)-(26) and the analogous equations for stator currents where k=1. The stator phase currents are related to the stator frame d-q currents by the equations for currents analogous to (9)-(11).

It can be noted that solution of this problem using multiple reference frames apparently requires the solution of 8 coupled differential equations. However, two of these equations arise from incorporating the constraint $i_{as}=i_{qs}^{s}=0$. Hence, it follows that the cosine and sine components, $i_{qs\alpha}$ and $i_{qs\gamma}$ of

$$\begin{bmatrix} -\frac{1}{2} V_{ds\gamma} \\ \frac{1}{2} V_{ds\gamma} \\ \frac{1}{2} V_{ds\alpha} \\ \frac{1}{2} V_{ds\alpha} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} x_s & 0 & \frac{\omega_e}{\omega_b} x_s & 0 & \frac{p}{2\omega_b} x_m & -\frac{p}{2\omega_b} x_m & \frac{\omega_e}{2\omega_b} x_m & \frac{\omega_e}{2\omega_b} x_m \\ 0 & r_s + \frac{p}{\omega_b} x_s & 0 & -\frac{\omega_e}{\omega_b} x_s & -\frac{p}{2\omega_b} x_m & \frac{p}{2\omega_b} x_m & -\frac{\omega_e}{2\omega_b} x_m & -\frac{\omega_e}{2\omega_b} x_m \\ -\frac{\omega_e}{\omega_b} x_s & 0 & r_s + \frac{p}{\omega_b} x_s & 0 & -\frac{\omega_e}{2\omega_b} x_m & \frac{\omega_e}{2\omega_b} x_m & \frac{p}{2\omega_b} x_m & \frac{p}{2\omega_b} x_m \\ 0 & \frac{\omega_e}{\omega_b} x_s & 0 & r_s + \frac{p}{\omega_b} x_s & -\frac{\omega_e}{2\omega_b} x_m & \frac{\omega_e}{2\omega_b} x_m & \frac{p}{2\omega_b} x_m & \frac{p}{2\omega_b} x_m \\ \frac{p}{\omega_b} x_m & 0 & \frac{\omega_e - \omega_r}{\omega_b} x_m & 0 & r_r' + \frac{p}{\omega_b} x_r' & 0 & \frac{\omega_e - \omega_r}{\omega_b} x_r' & 0 \\ 0 & \frac{p}{\omega_b} x_m & 0 & -\frac{\omega_e + \omega_r}{\omega_b} x_m & 0 & r_r' + \frac{p}{\omega_b} x_r' & 0 & -\frac{\omega_e + \omega_r}{\omega_b} x_r' \\ -\frac{\omega_e - \omega_r}{\omega_b} x_m & 0 & \frac{p}{\omega_b} x_m & 0 & -\frac{\omega_e - \omega_r}{\omega_b} x_r' & 0 & r_r' + \frac{p}{\omega_b} x_r' & 0 \\ 0 & \frac{\omega_e + \omega_r}{\omega_b} x_m & 0 & \frac{p}{\omega_b} x_m & 0 & \frac{\omega_e + \omega_r}{\omega_b} x_r' & 0 & r_r' + \frac{p}{\omega_b} x_r' \end{bmatrix} \times \begin{bmatrix} i_{qs}^{+e} \\ i_{qs}^{-e} \\ i_{ds}^{+e} \\ i_{ds}^{-e} \\ i_{qr}^{+e} \\ i_{qr}^{-e} \\ i_{dr}^{+e} \\ i_{dr}^{-e} \end{bmatrix} \quad (42)$$

i_{qs}^{s}, are also zero. From the expressions analogous to (23) and (24) for stator currents, the +e and -e currents must satisfy

$$i_{qs}^{+e} + i_{qs}^{-e} = i_{qs\alpha} = 0 \qquad (43)$$

$$i_{ds}^{+e} - i_{ds}^{-e} = i_{qr\gamma} = 0 \qquad (44)$$

Examination of the symmetry of (42) verifies these relationships. Thus, only six equations need be solved explicitly, and the order of the matrix to be solved can be reduce to six.

The instantaneous torque equation for the +e, -e currents incorporating (43) and (44) is

$$T_e = x_m \Big\{ i_{qs}^{+e}(i_{dr}^{\prime +e} - i_{dr}^{\prime -e}) - i_{ds}^{+e}(i_{qr}^{\prime +e} + i_{qr}^{\prime -e}) - \Big[i_{qs}^{+e}(i_{dr}^{+e} - i_{dr}^{-e}) + i_{ds}^{+e}(i_{qr}^{+e} + i_{qr}^{-e}) \Big] \cos 2\omega_e t + \Big[i_{qs}^{+e}(i_{qr}^{+e} + i_{qr}^{-e}) - i_{ds}^{+e}(i_{dr}^{+e} - i_{dr}^{-e}) \Big] \sin 2\omega_e t \Big\} \qquad (45)$$

Adding and subtracting corresponding rows of (38) and (39) generates another form of the solution which more directly is related to the variables of interest. If this operation is carried out, the current vector of the resulting equation is

$$[i_{qs}^{+e} + i_{qs}^{-e}, i_{qs}^{+e} - i_{qs}^{-e}, i_{ds}^{+e} + i_{ds}^{-e}, i_{ds}^{+e} - i_{ds}^{-e}, \ldots, i_{dr}^{+e} - i_{dr}^{-e}]^T$$

From (43) and (44), it is clear that the first and fourth rows of this modified 8x8 matrix equation are not required in the solution. The remaining sum and differences can be related back to stator d-q reference frame currents by (23)-(26). Eliminating the first and fourth rows, the reduced 6x6 matrix equation, expressed in terms of stator d-q reference frame variables is

$$\begin{bmatrix} V_{ds\alpha} \\ V_{ds\gamma} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} x_s & \frac{\omega_e}{\omega_b} x_s & \frac{p}{\omega_b} x_m & \frac{\omega_e}{\omega_b} x_m & 0 & 0 \\ -\frac{\omega_e}{\omega_b} x_s & r_s + \frac{p}{\omega_b} x_s & -\frac{\omega_e}{\omega_b} x_m & \frac{p}{\omega_b} x_m & 0 & 0 \\ \frac{p}{\omega_b} x_m & \frac{\omega_e}{\omega_b} x_m & r_r' + \frac{p}{\omega_b} x_r' & \frac{\omega_e}{\omega_b} x_r' & \frac{\omega_r}{\omega_b} x_r' & 0 \\ -\frac{\omega_e}{\omega_b} x_m & \frac{p}{\omega_b} x_m & -\frac{\omega_e}{\omega_b} x_r' & r_r' + \frac{p}{\omega_b} x_r' & 0 & \frac{\omega_r}{\omega_b} x_r' \\ -\frac{\omega_r}{\omega_b} x_m & 0 & -\frac{\omega_r}{\omega_b} x_r' & 0 & r_r' + \frac{p}{\omega_b} x_r' & \frac{\omega_e}{\omega_b} x_r' \\ 0 & -\frac{\omega_r}{\omega_b} x_m & 0 & -\frac{\omega_r}{\omega_b} x_r' & -\frac{\omega_e}{\omega_b} x_r' & r_r' + \frac{p}{\omega_b} x_r' \end{bmatrix} \times \begin{bmatrix} i_{ds\alpha} \\ i_{ds\gamma} \\ i_{dr\alpha} \\ i_{dr\gamma} \\ i_{qr\alpha} \\ i_{qr\gamma} \end{bmatrix} \qquad (46)$$

The corresponding torque equation is

$$T_e = \frac{x_m}{2} \Big[-i_{ds\alpha} i_{dr\alpha} - i_{ds\gamma} i_{qs\gamma} + (i_{ds\gamma} i_{qr\gamma} - i_{ds\alpha} i_{qr\alpha}) \cos 2\omega_e t + (i_{ds\alpha} i_{qr\gamma} + i_{qr\alpha} i_{ds\gamma}) \sin 2\omega_e t \Big] \qquad (47)$$

Conclusion

In this paper a simplified presentation of Krause's multiple reference frames was given. The method was then extended to include the case of a single phase open circuit on a three-phase, three-wire induction motor supplied by a three-phase, single frequency source. The technique can be readily extended to the case of source voltages containing multiple frequencies.

The significance of this analysis is clearly not the solution itself which was obtained in the past by methods involving complex components. The analysis was undertaken to indicate how multiple reference frames may be extended to the study of impedance unbalances. The solution demonstrates that the method of multiple reference frames which has in the past been applied only to balanced two-or three-phase connections is entirely suited to the study of open-circuited ac machines, an area traditionally solved by symmetrical components. It is now anticipated that the technique can be further extended to other types of impedance unbalances. Hence, it appears that the method of multiple reference frames is a valid alternative to symmetrical components in the study of ac machinery having either impedance or voltage unbalances.

d-q axis method provide the analyst with a flexible tool with which nonlinear transients (non-constant rotor speed), linear transients (constant rotor speed) as well as steady-state unbalances can be studied. In addition, d-q methods have recently been applied to steady-state situations where symmetrical components have not been successful.[6] Because of the flexibility of the method, it is suggested that additional emphasis be placed on its use in the future.

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