

THE USE OF SIMULATION IN THE DESIGN OF AN INVERTER DRIVE

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Summary

The range of application of static inverter motor drives is rapidly increasing. As inverter drive configurations increase in complexity, development costs increase as well. The trial and error involved in proper design of an inverter drive for a specific application can be significantly reduced by a simulation of the proposed system.

In other areas, simulation by means of digital computers has proceeded rapidly. However, because of the switching-mode nature of the inverter supply, simulation of these systems require extensive computer runs which cannot yet be accomplished economically because of the small step size required. Alternatively, the analog computer is, in this case, less expensive to use and is readily programmable to accommodate even the most complicated configuration.

This paper illustrates how the analog computer has been utilized in the design of several modern inverter drives. The paper illustrates the utility of the analog computer as an aid in making trade-off decisions such as adjustable dc voltage versus pulse-width modulation; synchronized versus unsynchronized types of pulse-width modulation; and square versus sine wave shape of reference waveform. Also discussed is the use of simulation in the study of typical open loop and closed loop design problems. In particular, the effects of synchronous and asynchronous voltage modulation is discussed in detail. The use of an analog computer in the design of a closed loop compensator for an inverter synchronous-reluctance motor drive is given.

Introduction

This decade is witnessing a rapid increase in the range of application of inverter drives. The wide array of inverter configurations either being manufactured or proposed testifies to the vitality of the static power conversion field. During the same time span, we have also witnessed increases in development costs. As a result of these factors, it is no longer practical to investigate in detail each candidate inverter configuration at the hardware level. This paper illustrates areas where much of the trial and error involved in proper inverter design and application can be reduced by a simulation of the proposed system.

In other areas, development of simulation techniques by means of digital computers has proceeded at a rapid pace. A large number of simulation languages have evolved to assist the analyst in programming. Unfortunately, inverter supplies involve a basic switching frequency often containing a higher switching or chopping frequency of 250 - 1000 Hz. When the chopping frequency is locked to the fundamental motor frequency and when steady-state performance only is required, the problem can usually be recast as an algebraic one and

simulation avoided. These problems are, of course, readily solved with the digital computer. However, this "synchronous" mode of operation often applies to only a portion of the complete operating range of the drive. Asynchronous modes place much greater demands on the analyst. In addition, a steady-state analysis generally yields only a partial answer. Many questions inherently concern transient performance, and these problems generally require a reformulation of the problem, this time necessitating the solution of the system differential equations.

Since the chopping frequency is high, very small time steps must be used if the solution is to proceed using the digital computer. In contrast to these high frequency effects, the basic electrical and electromechanical time constants of the motor load are typically tenths of seconds or even seconds. Hence, a computer run may readily involve 20 cycles or more of the fundamental simply to generate a steady-state solution. Complete duty cycle runs can easily necessitate hundreds of cycles. These extensive computer runs cannot yet be accomplished economically with a digital computer using the small step size necessitated by the chopping frequency.

Alternatively, hourly rates for use of an analog computer are typically less than 1/10 that for a comparable digital computer. The analog computer is readily programmed to accommodate even the most complicated configuration. Computing time is reduced since all integration is accomplished in parallel. Although a computer time scale is generally required, a complete computer run is usually accomplished in less than one minute. The analyst is not, in this case, limited by computing time or computational cost. When the details of the commutating circuit are not considered, time scales of 10 to 30 are readily achievable, and one is, in practice, generally limited by the frequency response of his chart recorder and capacity of his computer facility. This paper will not dwell on the details of analog computer simulation. A number of approaches to simulation of inverter drives have been documented.¹⁻⁷ The approach used in this paper follows the basic studies initiated at the University of Wisconsin by Woloszyk^{8,9} and extended by Krause, Lipo, and others.^{10,11}

In applying an inverter drive to a specific task, many of the important decisions to be made center around three basic questions:

1. Which type of inverter shall I choose?
2. How do I optimize the open loop design of the inverter configuration selected?
3. What is the proper feedback configuration for satisfactory closed loop system performance?

In this paper, it is shown how the analog computer

can be used to answer some of the typical questions which arise in the design of an inverter drive.

The Analog Computer as a Tool in Inverter Selection

Presently one is confronted with a bewildering array of various types of inverter configurations either being investigated or in manufacture. It is often very difficult to make an objective decision regarding which inverter best matches a given application. Among the trade-offs which must be evaluated are 120° vs. 180° firing logic, voltage source vs. current source inverters, pulse modulated vs. adjustable dc voltage inverters, sine wave vs. square wave modulated inverters, parallel SCR's vs. parallel inverters. Much of the information required to make a trade-off study requires only steady-state performance data. Such answers are readily obtainable from the analog computer simulation as a by-product of the transient simulation. Since the waveforms of all variables are instantly available and can be simultaneously plotted on a multi-channel chart recorder, a quick qualitative evaluation of the inverter behavior is readily obtained. Detailed data such as motor losses, torque pulsation harmonics, average thyristor currents, etc., are readily obtained by A/D sampling to a digital computer.

Figures 1 - 4 show typical analog computer runs taken for a system involving a 92 hp, 4 pole, 50 Hz, 350 V induction motor. In this application, power was supplied from a fixed dc voltage source. Since the source voltage is fixed, the proper ac fundamental voltage and frequency ($V/Hz = \text{const.}$) must be obtained by inverter switching techniques. Four methods of voltage adjustment were investigated. Figures 1 - 4 show typical results for a selected operating point. In all cases, the operating frequency was 50 Hz, the slip frequency was 1 Hz, and the load, 600 ft-lb. Fundamental output voltage is adjusted to maintain the same slip-load operating point.

In Fig. 1, the motor is fed from two series connected three phase inverters. In order to obtain voltage control, the thyristor firing signals fed to one of the three phase groups is time delayed relative to the other and, hence, the output voltages are phase shifted. In Fig. 1, the voltages have been phase shifted by 92° . At this operating point, smooth operation is apparent. However, as the low frequency end of the speed range is extended, the size of the coupling reactor increases. Increasingly larger components of harmonic voltages appear and, hence, harmonic losses and harmonic torques increase so that the practical speed range is rather limited.

Figure 2 demonstrates the level set method of pulse width modulation. In this case, a sine wave reference is compared with an adjustable voltage level A. Intersections of the sine wave with the three levels +A, zero, and -A cause a switching of the inverter output. The level A is used to adjust the fundamental volts. Additional levels can be provided to improve the output waveform and

extend the lower end of the speed range. For this particular operating point, using only three levels, a significant sixth harmonic pulsating torque is apparent.

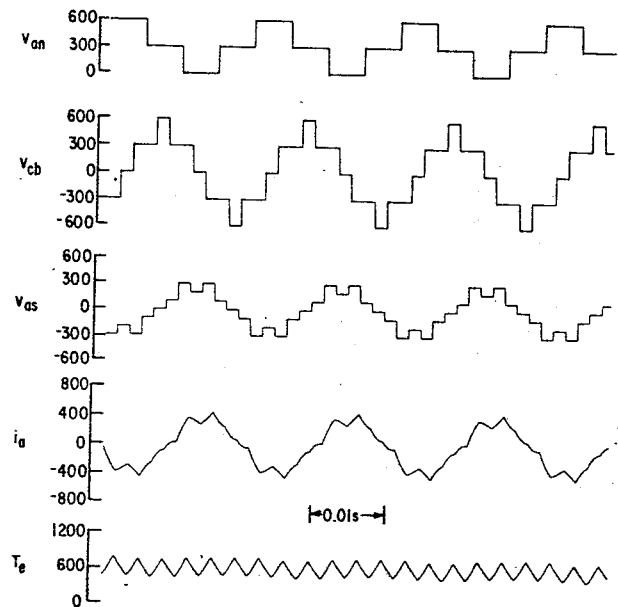


Fig. 1 Motor performance with two series connected, 3-phase inverters- 92° phase shift. Operation at 50 Hz, 0.02 slip, 600 ft. lb. Chart variables: v_{an} -line-to-negative dc bus voltage (volts); v_{cb} -c to b phase line-to-line voltage (volts); v_{as} -line a-to-motor neutral phase voltage (volts); i_a -a phase line current (amps); T_e -instantaneous torque (ft. lb.).

In Fig. 3, a different chopping scheme obtained by comparing a triangle wave to a three phase set of square waves was used. The triangle frequency (chopping frequency) is locked to six times the square wave frequency. In this case, the amplitude of the square wave is used as a means to adjust voltage. The triangle and square wave traces obtained from the simulation have been superimposed to more clearly show the switching instants. It can be noted that this approach produces a conventional square wave PWM waveshape except that the first and last pulse of each half-cycle is one-half the duration of the others. Again, a sixth harmonic torque pulsation is apparent although reduced in amplitude. Numerous other pulsing schemes based on this approach are possible.

In Fig. 4, the square waves have been replaced by sine waves and the pulses then sine wave modulated. Here, it can be noted that symmetry does not exist between top and bottom halves of the line-to-line voltage. A significant fourth harmonic of voltage exists which results in an appreciable third harmonic torque pulsation. Various combinations of sine wave and triangle waves can be employed to change the harmonic structure and they can be investigated with relative ease on the analog computer.

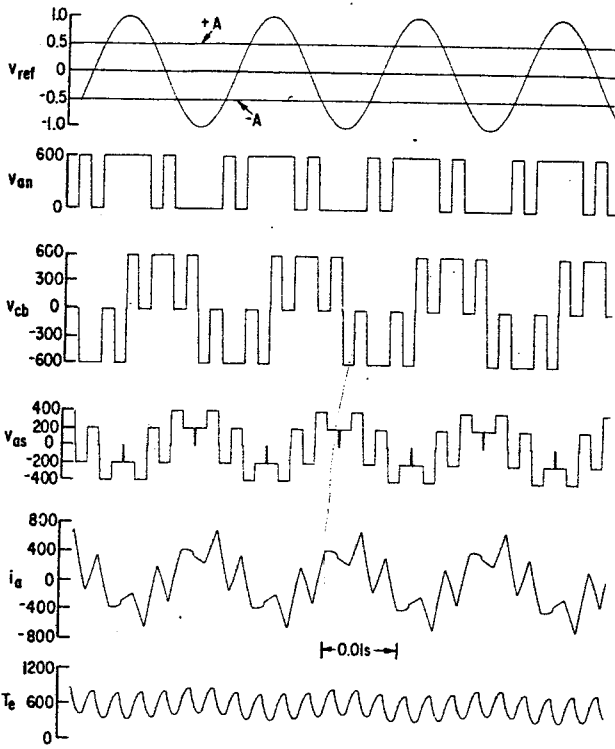


Fig. 2 Motor performance with level set voltage control. Operation at 50 Hz, 0.02 slip, 600 ft. lb. For description of chart variables, see Fig. 1.

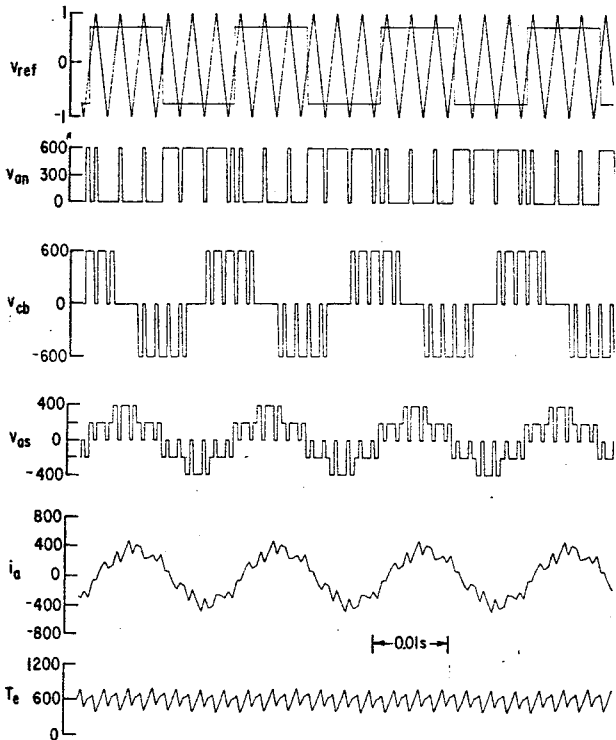


Fig. 3 Motor performance with square wave-triangle wave voltage modulation. Operation at 50 Hz, 0.02 slip, 600 ft. lb. For description of chart variables, see Fig. 1.

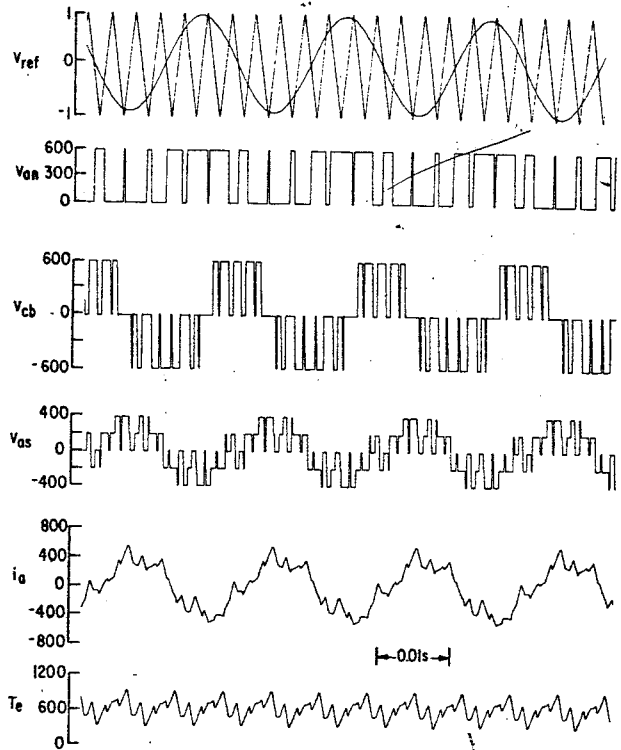


Fig. 4 Motor performance with sine wave-triangle wave voltage modulation. Operation at 50 Hz, 0.02 slip, 600 ft. lb. For description of chart variables, see Fig. 1.

For a complete study, a wide range of operating points must, of course, be investigated for complete evaluation of a given type of inverter. Numerous other control schemes also exist which must be evaluated. The computer can be used to obtain a quick visual picture of operation for any operating condition. Having once developed the basic drive simulation, it is a relatively simple matter to modify the simulation to any conceivable firing scheme.

The Analog Computer as a Tool in Inverter Open-Loop Design

Having selected the inverter configuration, many additional questions need to be answered. Typical questions which usefully employ a computer simulation in their resolution include: What is the practical low frequency limit?, Are dc filter components required, and if so, what are their values and how do they affect system performance?, Is the system stable for all operating conditions?, How does the system respond during regenerative and/or dynamic braking?, Can the motor reverse direction without cogging?, and so forth.

In this section, the triangular wave-square wave chopping scheme shown in Fig. 3 is examined in more detail. One of the most difficult design problems with this method of voltage control is programming of the chopping.

Generally, one has complete freedom over selection of the frequency of the triangular wave

relative to the square wave. The frequency of the triangular wave can be merely set to a fixed value in which case it runs asynchronously relative to the square wave, or it can be fixed as an integral multiple of the reference square wave. Although this integer is, in principle, arbitrary, as a practical consideration each integer produces its own characteristic harmonic spectrum. For a three phase system, the chopping frequency is set at an integer multiple of three times the operating frequency to eliminate even harmonics.

From the motor standpoint, it is generally advisable to set the chopping frequency as high as possible. However, a practical upper frequency limit exists on the pulse frequency. This frequency is set by state-of-the-art thyristor capabilities and losses in the commutation circuit. Presently, this limit is in the range from 1.0 to 2.0 kHz. For example, if it is assumed that a particular inverter design must operate up to a frequency of 150 Hz and that the ratio of chopping frequency to fundamental is selected at 6 to 1, the maximum chopping frequency is then set at 900 Hz. If the pulsing frequency is synchronized to the operating frequency at this ratio and the operating frequency is reduced, the chopping frequency is reduced proportionally. As a result of the reduction in chopping frequency, motor performance deteriorates to an extent where a minimum frequency is reached at which point motor and drive performance is no longer acceptable.

In order to offset this effect while maintaining synchronous operation, the ratio between the operating frequency and the chopping frequency should be set so that the chopping frequency remains as high as possible. Generally, these changes are made in 2 to 1 steps so that if the ratio were initially 6/1, the chopping ratio would be changed to 12/1, 24/1, 48/1, etc. at selected points in the operating frequency range. Clearly, there is a limit to the number of step changes which can be incorporated into an economical inverter design.

If the maximum ratio is limited to 48/1 and the inverter is required to produce a low or zero frequency output, then difficulties are encountered. Losses in the motor rise rapidly; and at very low frequencies, the drive begins to respond to the chopping frequency as a consequence of the high peak/low average voltage pulses occurring at the chopping frequency. Finally, since the minimum commutating time of the main thyristors is fixed, the minimum voltage amplitude is fixed; and the ideal volts/Hz characteristic may eventually no longer be supplied by the inverter.

It would appear that the solution to this dilemma is asynchronous operation since the chopping frequency can then be maintained high thereby minimizing torque pulsations and motor losses. However, this approach is not without its own set of problems. Further investigation has revealed that asynchronous operation can cause serious torque pulsations if the ac motor load is lightly damped over some operating frequency range. This effect is particularly pronounced when using a

salient pole type of machine. These torque pulsations, which are caused by difference frequency components, are normally most severe in the middle or upper frequency range of operation.

As a consequence of this effect, an inverter designed to operate over a wide frequency range generally employs a combination of both synchronous and asynchronous operating modes. An analog computer simulation can be useful in studying synchronous and asynchronous operation over a wide range of conditions in order to determine what compromise between synchronization and asynchronous operation should be used.

To emphasize these points and to illustrate the utility of the analog computer in studying the problem, typical results are given for a PWM inverter with a six step square wave envelope as was illustrated in Fig. 3. In this case, the study was conducted for a 40 hp, 6 pole, 250 V synchronous-reluctance motor. Rated frequency is 68 Hz. Results given in this section are in per unit using rated frequency, rated line voltage, and rated output power as base quantities. To illustrate the need for asynchronous operation, it is helpful to examine first the synchronous case. Figures 5 and 6 show the line-to-line voltage and air gap torque for a synchronous condition with two different frequencies. The pulse-to-fundamental frequency ratio was 6/1 in Fig. 5 and 12/1 in Fig. 6. It is clear from Fig. 6 that performance has seriously deteriorated and a higher pulse ratio is needed to improve harmonic content.

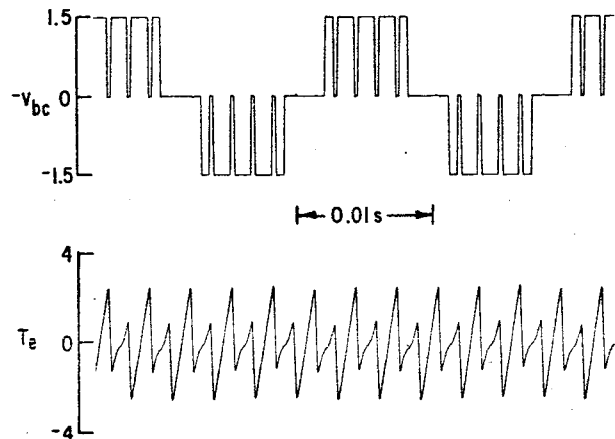


Fig. 5 Synchronous-reluctance motor performance at 54.4 Hz, no load with rated air gap flux. Chopping frequency synchronized to six times fundamental.

In order to operate asynchronously, the motor frequency can be increased (decreased) by a small amount relative to the fixed chopping frequency. As a result, the line-to-line voltage pulses appear to move to the left (right) with respect to the waveform envelope. In so doing, a phase modulation of the fundamental voltage component results. It can be shown that the waveform of the phase modulation for this basic inverter type is of triangular form as shown in Fig. 7.

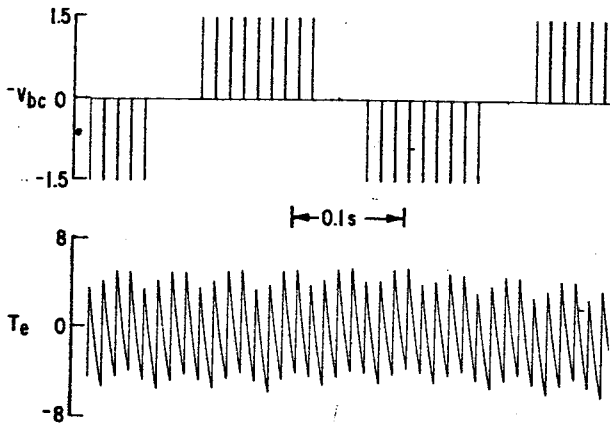


Fig. 6 Synchronous-reluctance motor performance at 3.4 Hz, no load with rated air gap flux. Chopping frequency synchronized to twelve times fundamental.

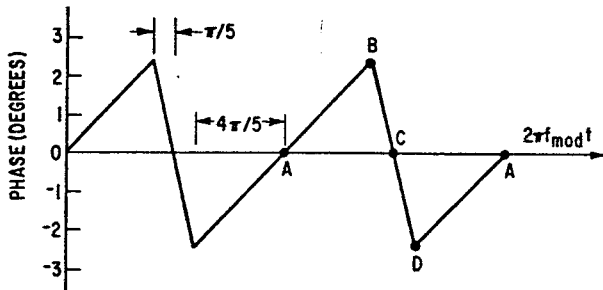


Fig. 7 Instantaneous phase of the fundamental component of motor voltage relative to a frequency reference fixed at 0.8 pu. Chopping frequency $f_c=4.8$ pu. Operating frequency f_e slightly greater than 0.8 pu.

Near the synchronous operating condition of Fig. 5, the frequency of the fundamental component of the modulation is equal to twice the magnitude of the difference between the chopping frequency and six times the operating frequency, or

$$f_{\text{mod}} = 2 |f_c - 6 f_e| \quad (1)$$

As the operating frequency range moves away from the synchronous condition, the modulation frequency rises until it eventually becomes greater than the operating frequency. However, as the operating frequency continues to change, the modulation frequency begins to decrease until another synchronous condition is eventually reached where a synchronous condition can exist (i.e., 9/1 or 12/1 frequency ratios). At this point, the modulation or difference frequency returns to zero and another modulation equation similar to (1) and valid for operation near this new chopping ratio can be defined.

Figure 8 illustrates a computer solution of a no-load asynchronous operating condition corresponding to the phase modulation diagram of Fig. 7. In particular, the motor frequency is set at 0.8125 pu. Synchronous chopping corresponds to operation at 0.8 pu (Fig. 5). One complete period of the modulation frequency has been shown.

The points labeled A, B, C and D correspond to points similarly labeled in Fig. 7. The relative movement of the pulses of the line-to-line voltage with respect to the fundamental square wave can be observed. A large torque pulsation resulting from the modulation at the difference frequency is clearly evident.

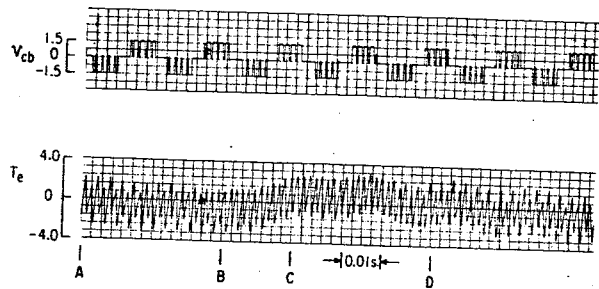


Fig. 8 Asynchronous operation near a synchronous frequency of 0.8 pu (54.4 Hz). Line frequency $f_e=0.8125$ pu, chopping frequency $f_c=4.8$ pu.

One can readily study effects of the asynchronous operation with the analog computer by observing these torque pulsations produced as the operating frequency is gradually lowered. Figure 9 shows a portion of such a study. In this figure, the fundamental frequency has been reduced linearly from 0.8 pu to 0.72 pu, while the chopping frequency has been maintained constant at 4.8 pu. Again, synchronous operation corresponds to 0.8 pu. The air gap torque has been filtered to remove the effects of the high frequency chopping and square wave voltage envelope. This allows the pulsations resulting from the difference frequency modulation to be more clearly observed. The difference or modulation frequency in Hertz has been marked along the time axis. Two distinct peaks can be noted in the response, the first occurring at approximately 5 Hz and the second near 50 Hz. A linearized analysis verifies that these peaks are caused by resonances predicted by the open loop response of the machine to the input voltage modulation signal. It is well known that any ac machine demonstrates low damping over a range of operating frequencies.¹²⁻¹⁶ Although the resultant instability problem has been investigated in depth, other phenomena arising from this same cause such as asynchronous phase modulation is less well known. It is apparent that torque pulsations produced by the asynchronous chopping can increase dramatically near points of resonance in the open loop response of the machine.

Another disturbing feature of this type of pulsating torque is that the entire frequency range is swept from zero on up past the operating frequency. Hence, torsional mechanical resonances are also a potential problem. This computer study suggests that a careful analysis be made to evaluate the effect of torque pulsations in regions where asynchronous chopping is contemplated. For example, Fig. 10 shows that with an operating frequency of 14 Hz (0.206 pu) with a frequency ratio of approximately 24/1, the torque pulsations are still large, indicating the inverter must remain synchronized until a much lower frequency range is

reached. The 6th harmonic torque pulsation resulting from the square wave envelope and chopping pulsations has not been filtered out in this figure and can also be observed. At a sufficiently low frequency, the torque due to the predominant fifth and seventh voltage harmonics supplant the modulation frequency as the dominant frequencies of interest. These harmonics, in turn, can also excite the motor at its resonant frequencies and a computer investigation of these amplitude modulated effects is warranted.

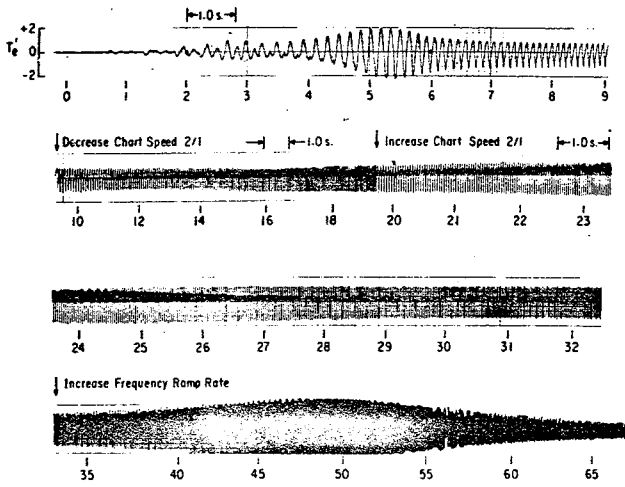


Fig. 9 Phase modulation torque pulsations for a linear decrease in motor frequency f_e from 0.8 to 0.72 pu. Chopping frequency f_c fixed at 4.8 pu.

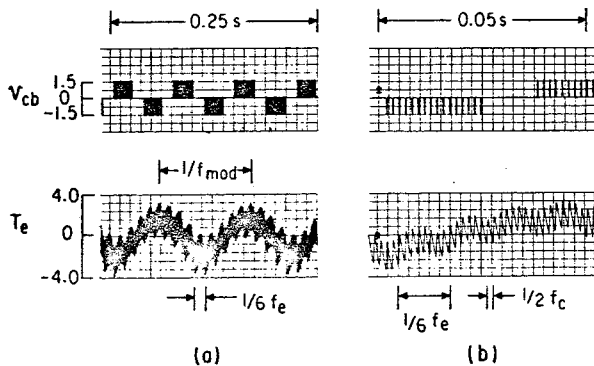


Fig. 10 (a) Asynchronous operation at 14 Hz (0.206 pu), $f_c=4.8$ pu. (b) Same as (a) with expanded time scale.

The Analog Computer as a Tool in Closed-Loop System Design

Closed loop system design affords another area where simulation is effective. In the design of any drive system, it is often necessary to examine numerous feedback configurations as well as ranges of compensator gains and break frequencies. The analog computer is ideally suited to this design procedure. A problem which has been recently solved is that of stabilizing an otherwise unstable or lightly damped synchronous-reluctance motor by feedback control. The particular motor, chosen here for purposes of illus-

tration, has a low frequency instability region.^{12,13} Within this low frequency range, the motor is unstable in the steady state. A steady-state operating frequency of 0.1125 pu, located inside the instability region, was chosen as the design point.

Utilizing a 1 pu stepload-1 pu step unload torque response of the inverter/motor system as a design criterion, more than 150 separate response runs were made in a short period. Three alternate feedback control approaches were investigated and feedback network parameters were varied over a sufficiently wide range to evaluate each approach.

Typical results for the first approach are shown in Fig. 11. All computer traces were taken at an operating frequency where the motor was normally unstable at both no load and one per unit load. The high frequency (6th harmonic and chopping frequency) torque pulsations were again filtered out in order to more easily evaluate the low frequency damping provided by the feedback control. The two parameters adjusted in this figure are the feedback network break frequency and the network gain factor. The conclusion reached with method one was that while the motor could be stabilized, there is a limited range of the parameter settings over which stable operation can be achieved.

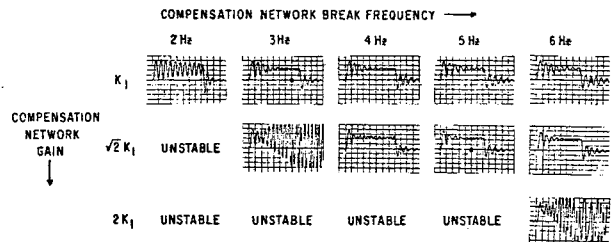


Fig. 11 First stabilization method. Step load torque response.

In Fig. 12 are shown typical results for a second feedback scheme. This approach resulted in considerable improvement over the first, in that significantly more damping could be introduced and stable operation occurred over a wider range of feedback network parameters. However, it was observed for both these approaches that the system went into steady-state oscillation when the network gain was set too high.

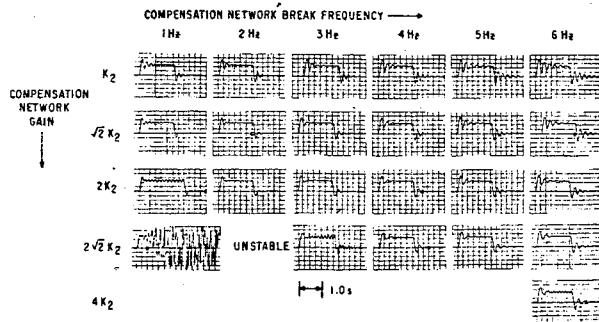


Fig. 12 Second stabilization approach. Step load torque response.

Computer traces for a third approach are given in Fig. 13. In this case, the damping appears to increase without limit when increasing the network gain constant. The setting of the break frequency was found to be not critical.

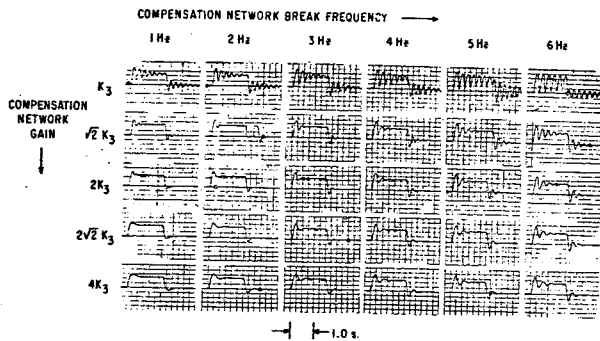


Fig. 13 Third stabilization method. Step load torque response.

Since the study was conducted entirely at an operating frequency of 0.1125 pu, a check was made to determine the effectiveness of the feedback method at other operating frequencies. Typical results are shown in Fig. 14. This figure can be used to compare the 1.0 pu step load and unload

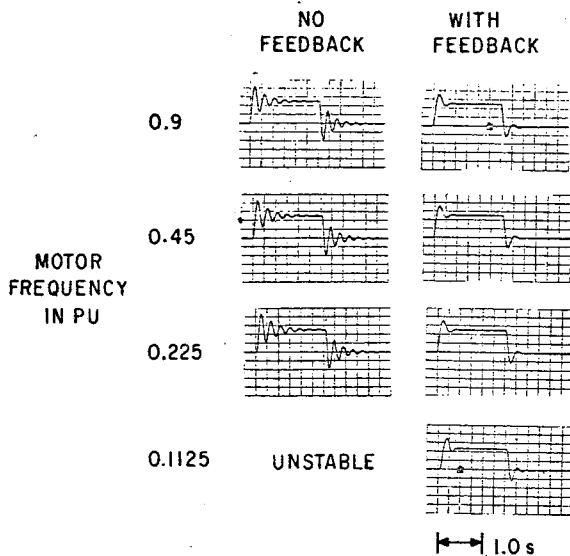


Fig. 14 Comparison of torque response with and without third feedback stabilization approach.

response of the machine with and without feedback control. These computer traces demonstrate that the feedback damping technique is very effective at all operating frequencies and produces a transient response which is essentially independent of motor frequency.

Conclusions

This paper has presented typical applications of analog computation in the design of static inverter drives. Discussed at length were two spe-

cific problems involving asynchronous pulsating torques and closed-loop compensation network design. It is not the intention of this paper to suggest that an analog computer solution is always superior to digital simulation, closed form analytic solutions, or linearized small signal approaches. Indeed, when the scope of static drive application has stabilized to the point where analyses become routine, other approaches can yield special advantages not easily realized with a simulation. However, until this time, simulation by means of an analog computer will continue to be a flexible, accurate tool for drive system analysis.

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