

STABILITY IMPROVEMENT OF INVERTER DRIVEN INDUCTION MOTORS
BY USE OF FEEDBACK

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SUMMARY

An analytical procedure for designing feedback control loops used to improve the performance of induction motor drives, supplied from static inverters, is presented. A cost functional is derived using the differential equations that describe behavior of the system state variables about a periodic steady-state operating point.

The cost functional is used to derive the best gains for a specific feedback structure. This technique is applied to a drive system, previously shown to exhibit instability over a wide speed range.

It is demonstrated, by means of a detailed analog computer simulation, that the feedback coefficients calculated by the cost functional technique indeed results in near optimum, well-damped operation.

ZUSAMMENFASSUNG

Ein analytisches Verfahren für den Entwurf von rückgekoppelten Regelsystemen zur Leistungsverbesserung von Drehstrom-Motorantrieben ist hier beschrieben. Das Regelsystem wird hierbei von Thyristor-Invertern gespeist. Ein analytischer Ausdruck zur Beschreibung des erforderlichen Aufwandes wird entwickelt, wobei die Differentialgleichungen zur Beschreibung der Verhaltensweise der System-Zustandsvariablen eines Betriebspunktes für den periodisch stationären Zustand aufgestellt werden.

Der analytische Ausdruck zur Beschreibung des erforderlichen Aufwandes wird benutzt, um den besten Verstärkungsfaktor eines speziellen Rückkopplungs-Systemes zu bestimmen. Diese Methode findet ihre Anwendung an einem Antriebssystem, das zuvor ein instabiles Verhalten über einen weiten Drehzahlbereich anzeigte.

Durch Anwendung einer ins Einzelne gehenden analogen Computer-Modellierung wird gezeigt, dass die Rückkopplungskoeffizienten, die mit Hilfe der analytischen Aufwands-Funktionsmethode berechnet wurden, tatsächlich einen fast optimalen Betrieb mit guter Dämpfungscharakteristik ergeben.

INTRODUCTION

The advent of static adjustable-frequency, adjustable-voltage inverters has made the application of AC drives capable of a wide speed range an attractive alternative to DC drives. The wide range of operating voltages and frequencies encountered in such drives has resulted in effects not normally experienced at fixed 50 or 60 Hz. operation. One effect, of particular concern, is the onset of continuous steady-state speed oscillations which occur when any type of AC machine is used in an adjustable-frequency application /1,2,3/.

Perhaps, one of the most widely used configurations is the rectifier-inverter drive wherein the three-phase AC source voltage is rectified to variable amplitude direct voltage by means of a controlled rectifier bridge, processed through a filter and subsequently inverted by means of a square wave inverter. Unfortunately, such a system has been found to exhibit continuous oscillations over a speed range typically between 6 and 30 Hz. /4/. Proper choice of filter parameters has been found to improve performance to the point where the instability is eliminated. However, the size of the components required is often impractical. Also, system damping, although positive, typically remains poor so that a supplementary damping means is highly desirable.

One such damping technique is to utilize the frequency command to the inverter as a control input /5/. This method of control, when used in conjunction with a square wave inverter, is limited in value since the inverter constitutes a sampled-data system, fixed by the switching frequency of the inverter. Hence, the effectiveness of the control diminishes as the output frequency of the inverter decreases. In addition, continuous adjustment of the inverter frequency can lead to undesirable motor speed perturbations.

Another input that can be utilized as a means of control is the command signal to the rectifier bridge (delay angle). In this case the motor phase voltage is perturbed rather than frequency. Although this input is also sampled-data in nature, the delay time is fixed by the source frequency and, hence, can be more readily compensated. Also, this input has an additional advantage in that disturbances in load torque, source voltage, etc., can be minimized without materially affecting the steady-state motor speed.

Because of the wide variety of variables that can be utilized as the feedback signal, including power, speed, VAR's, current, flux, etc., it is highly desirable to establish techniques that can be used for a quantitative evaluation. One recently developed approach employs transfer function analysis /7/. However, since the method relies upon root

loci in order to set proper gains, the analysis can be time consuming if numerous feedback variables are to be investigated. This paper describes an alternative analytical method using state space techniques. A cost functional is derived using the differential equations which describe system behavior around a steady-state operating point. It is shown that the cost functional can be approximated in terms of the feedback coefficients and the steady-state solution for zero feedback. The cost functional insures that all change to any disturbance is bounded and, for increasingly negative values, the time required to reach a nominal operating point decreases. The approach is applied to a practical inverter drive which has previously been shown to be unstable. In particular, the feedback of the instantaneous motor line current is selected as the feedback variable of interest. The optimum gains calculated by this technique are shown to result in good, well damped performance in the previously unstable region.

SYSTEM ANALYZED

A simplified diagram of the basic system studied is shown in Figure 1. The system is comprised of a three-phase power source, a six-phase controlled rectifier with filter, an inverter, and a three-phase induction motor. In addition, a feedback of motor variables is introduced through a high-pass filter with feedback gains K_1 and K_2 . Although the analysis technique to be described is no way limited, two components will be discussed at a later stage. The damping voltage derived from the feedback is added to the nominal rectifier input voltage to form the commanded value of bridge current.

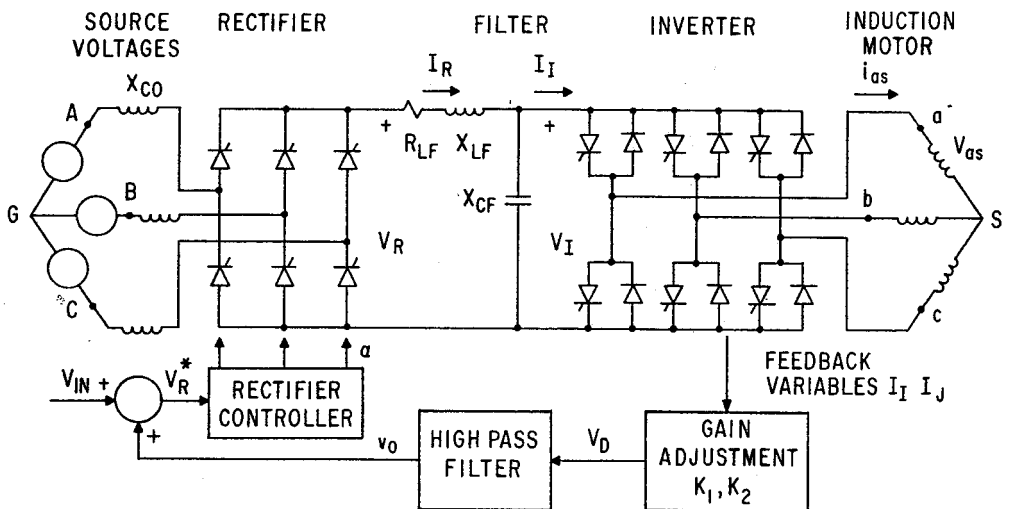


FIGURE 1: System Considered

The equations which describe open loop behavior of the electrical system are, in per unit and matrix form /4/.

(1)

$$\begin{bmatrix} v_{qs}^s \\ v_{ds}^s \\ 0 \\ 0 \\ I_I \\ V_R \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} X_s & 0 & \frac{p}{\omega_b} X_m & 0 & 0 & 0 \\ 0 & r_s + \frac{p}{\omega_b} X_s & 0 & \frac{p}{\omega_b} X_m & 0 & 0 \\ \frac{p}{\omega_b} X_m & -\frac{\omega_r}{\omega_b} X_m & r_r + \frac{p}{\omega_b} X_r & -\frac{\omega_r}{\omega_b} X_r & 0 & 0 \\ \frac{\omega_r}{\omega_b} X_m & \frac{p}{\omega_b} X_m & \frac{\omega_r}{\omega_b} X_r & r_r + \frac{p}{\omega_b} X_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \frac{p}{\omega_b X_{CF}} \\ 0 & 0 & 0 & 0 & R_{LF} + \frac{p}{\omega_b} X_{LF} & 1 \end{bmatrix} \times \begin{bmatrix} i_{qs}^s \\ i_{ds}^s \\ i_{qr}^s \\ i_{dr}^s \\ I_R \\ V_I \end{bmatrix}$$

The electromechanical equation of motion is, in per unit:

$$\frac{p \omega_r}{\omega_b} = \frac{X_m}{2H} (i_{dr}^s i_{qs}^s - i_{qr}^s i_{ds}^s) - \frac{T_L}{2H} \quad (2)$$

In (1), the motor is characterized in a d-q axis fixed in the stator, wherein, by convention, the q-axis is aligned with the as phase and the d-axis is oriented clockwise from the q-axis by 90° /6/. Also, X_s , X_r , and X_m are the per unit stator self-, rotor self- (referred to the stator) and magnetizing reactance in per unit. The quantities r_s and r_r are the per unit stator and rotor resistance referred to the stator. The reactances are expressed in terms of base frequency ω_b . Since ω_r is the rotor angular velocity in equivalent electrical radians/second, the quantity ω_r/ω_b (also written as v_r in this paper) can be considered as the rotor speed in per unit. The identity of the filter variables are shown in Figure 1.

It can be noted that (1) is not in the proper form since the inverter dc voltage V_I and the motor terminal voltages v_{qs}^s and v_{ds}^s are not independent. Similarly, the dc and ac side inverter currents are related to the switching of the inverter.

Equations which relate the dc side quantities to the ac side d-q quantities are summarized in Figure 2. It can be noted that the current I_I is simply the inverter dc input current. The current I_J can be considered the component orthogonal to I_I , ("reactive" component) during each switching interval. In general, the linking equations differ for each interval and unless symmetry is in some way utilized, a steady-state solution requires a piecewise analysis over six successive intervals. This problem can be avoided by proper selection of the reference frame.

State	Connection	Equations
1		$I_I = i_{qs}^s \quad v_{qs}^s = \frac{2}{3}V_I$ $I_Q = i_{ds}^s \quad v_{ds}^s = 0$
2		$I_I = \frac{1}{2}i_{qs}^s - \frac{\sqrt{3}}{2}i_{ds}^s \quad v_{qs}^s = V_I/3$ $I_Q = \frac{\sqrt{3}}{2}i_{qs}^s + \frac{1}{2}i_{ds}^s \quad v_{ds}^s = V_I/\sqrt{3}$
3		$I_I = \frac{1}{2}i_{qs}^s - \frac{\sqrt{3}}{2}i_{ds}^s \quad v_{qs}^s = -\frac{1}{3}V_I$ $I_Q = \frac{\sqrt{3}}{2}i_{qs}^s - \frac{1}{2}i_{ds}^s \quad v_{ds}^s = V_I/\sqrt{3}$
4		$I_I = i_{qs}^s \quad v_{qs}^s = -\frac{2}{3}V_I$ $I_Q = -i_{ds}^s \quad v_{ds}^s = 0$
5		$I_I = -\frac{1}{2}i_{qs}^s + \frac{\sqrt{3}}{2}i_{ds}^s \quad v_{qs}^s = -\frac{1}{3}V_I$ $I_Q = -\frac{\sqrt{3}}{2}i_{qs}^s - \frac{1}{2}i_{ds}^s \quad v_{ds}^s = V_I/\sqrt{3}$
6		$I_I = -\frac{1}{2}i_{qs}^s - \frac{\sqrt{3}}{2}i_{ds}^s \quad v_{ds}^s = \frac{1}{3}V_I$ $I_Q = -\frac{\sqrt{3}}{2}i_{qs}^s + \frac{1}{2}i_{ds}^s \quad v_{ds}^s = V_I/\sqrt{3}$

FIGURE 2:

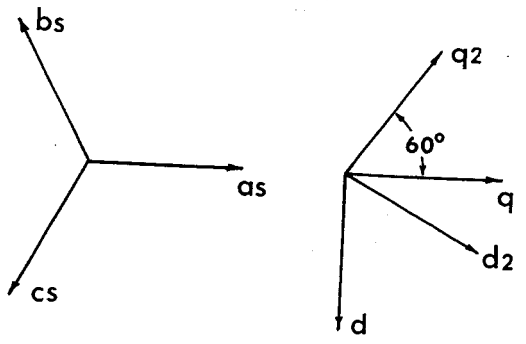
Inverter States and Resulting d-q Equations

Consider a second set of axes, q2-d2, oriented 60° from the original d-q axes as shown in Figure 3

FIGURE 3:
Axis Orientation

It is clear that variables in the new set of axes are related to those in the old by the algebraic relations

$$f_{qs}^2 = \frac{1}{2} f_{qs}^s - \frac{\sqrt{3}}{2} f_{ds}^s \quad (3)$$



$$f_{ds}^{s2} = \frac{\sqrt{3}}{2} f_{qs}^s + \frac{1}{2} f_{ds}^s \quad (4)$$

where the symbol "f" corresponds either to voltage "v" or current "i". Using these substitute variables, a table similar to Figure 2 can be assembled. In particular, it can be shown that the voltage and current connection equations during State 2 will be

$$\begin{aligned} I_I &= i_{qs}^{s2} & V_{qs}^{s2} &= \frac{2}{3} V_I \\ I_J &= i_{ds}^{s2} & V_{ds}^{s2} &= 0 \end{aligned} \quad (5)$$

which is identical in form to those in State 1 with the original d-q variables. This process can clearly be continued for all six inverter states. Hence, it is apparent that if a different set of d-q axes are chosen for each state or equivalently, if a single d-q axes is rotated by 60° at the instant of each change in inverter state, then the equations defining inverter operation will remain the same. This corresponds to selecting a reference frame speed = $\xi(t)$ where $\xi(t)$ is the set of impulse functions of strength $\pi/3$

$$\xi(t) = \frac{\pi}{3} \sum_{n=0}^{\infty} \delta(t-nT) \quad (6)$$

and $T = \pi/3 \omega_e$, corresponds to the basic switching frequency of the inverter.

When expressed in terms of this new reference frame, the system equations are

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_R \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} X_s & \frac{\xi}{\omega_b} X_s & \frac{p}{\omega_b} X_m & \frac{\xi}{\omega_b} X_m & -\frac{2}{3} & 0 \\ -\frac{\xi}{\omega_b} X_s & r_s + \frac{p}{\omega_b} X_s & -\frac{\xi}{\omega_b} X_m & \frac{p}{\omega_b} X_m & 0 & 0 \\ \frac{p}{\omega_b} X_m & \frac{\xi - \omega}{\omega_b} r_{Xm} & r_r + \frac{p}{\omega_b} X_r & \frac{\xi - \omega}{\omega_b} r_{Xr} & 0 & 0 \\ -\frac{\xi - \omega}{\omega_b} r_{Xm} & \frac{p}{\omega_b} X_m & -\frac{\xi - \omega}{\omega_b} r_{Xr} & r_r + \frac{p}{\omega_b} X_r & 0 & 0 \\ 1 & 0 & 0 & 0 & \frac{p}{\omega_b X_{CF}} & -1 \\ 0 & 0 & 0 & 0 & 1 & R_{LF} + \frac{p}{\omega_b} X_{LF} \end{bmatrix} \begin{bmatrix} i_{qs}^{su} \\ i_{ds}^{su} \\ i_{qr}^{su} \\ i_{dr}^{su} \\ V_I \\ I_R \end{bmatrix} \quad (7)$$

where the current i_{qs}^{su} corresponds to the stator q-axis current in the stationary reference frame defined for the n-th interval. A similar interpretation applies to the other three motor currents.

It can be noted that the number of inputs to the electric system equations has now been reduced to a single input V_R . Referring to Figure 1, it is apparent that this remaining input is not yet an independent variable since it is constrained by a feedback equation

$$V_R^* = V_{in} + v_D \quad (8)$$

where V_R^* is the command input to the rectifier, v_D is the damping signal from the output of the high pass filter and V_{in} is an external input which hence can be considered as independent. In principle command signal V_R^* is related to the actual dc bridge voltage V_R by a sampled data type of transfer function. However, since the inverter frequencies of interest are generally low, this effect will be neglected in this paper so that $V_R = V_R^*$.

In general, the damping signal

$$v_D = V_D - \hat{V}_D \quad (9)$$

where \hat{V}_D is the average value of the input to the high pass filter and

$$V_D = K_1 i_{qs}^{su} + K_2 i_{ds}^{su} \quad (10)$$

In order to avoid defining additional state variables for the dynamics of the filter, a fictitious feedback signal V_{in}' is defined such that

$$V_{in}' = V_{in} - V_D$$

Equation (8) can then be written in terms of the feedback signal V_D as

$$V_R = V_{in}' + V_D = V_{in}' + K_1 i_{qs}^{su} + K_2 i_{ds}^{su} \quad (11)$$

When (11) is incorporated into (7) and the result expressed in standard state variable form, the system equations, including the effect of feedback can be written in the form

$$\frac{d\bar{z}}{dt} = \bar{A}(v_R) + \xi(t)\bar{\omega} \bar{z} + \bar{u} \quad (12)$$

$$\frac{dv_r}{dt} = \frac{X_m}{2H}(z_1 z_4 - z_2 z_3) - \frac{T_L}{2H} \quad (13)$$

The variables z_1 -- z_4 are the first four elements of the state vector \bar{z} defined by

$$\bar{z} = \left[i_{qs}^{su} \quad i_{ds}^{su} \quad i_{qr}^{su} \quad i_{dr}^{su} \quad y_1 \quad y_2 \quad y_3 \quad y_4 \right]^T \quad (14)$$

The input \bar{u} is

$$\bar{u} = \frac{2\omega_b^2}{3X^2} \begin{bmatrix} X_r V_{in}' & 0 & -X_m V_{in}' & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (15)$$

The matrices \bar{A} and $\bar{\Omega}$ are defined as

$$\bar{A} = \begin{bmatrix} \begin{array}{c|cccc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ \hline \omega_b & 0 & 0 & 0 & \sigma & & -\omega & & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega & & \sigma & & 0 & 0 \\ 0 & \omega_b & 0 & 0 & 0 & & 0 & & \sigma & -\omega \\ 0 & 0 & 0 & 0 & 0 & & 0 & & \omega & \sigma \end{array} & \begin{array}{c} \gamma X_r X_{LF} \\ 0 \\ -\gamma X_m X_{LF} \\ 0 \end{array} & \begin{array}{c} \gamma X_r \left(\frac{\sigma}{\omega} + \frac{\omega_b}{\omega} \frac{K_1}{X_{LF}} \right) \\ 0 \\ -\gamma X_m \left(\frac{\sigma}{\omega} + \frac{\omega_b}{\omega} \frac{K_1}{X_{LF}} \right) \\ 0 \end{array} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} & \begin{array}{c} \gamma \frac{\omega_b}{\omega} \frac{X_r}{X_{LF}} K_2 \\ 0 \\ -\gamma \frac{\omega_b}{\omega} \frac{X_r}{X_{LF}} K_2 \\ 0 \end{array} \end{bmatrix} \quad (16)$$

$$\bar{\Omega} = \begin{bmatrix} 0 & -1 & 0 & 0 & | & 1 \\ 1 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & -1 & | & 0 \\ 0 & 0 & 1 & 0 & | & 0 \\ \hline & & \bar{0} & & | & \bar{0} \end{bmatrix} \quad (17)$$

where

$$\bar{A}_1 = \frac{\omega_b}{X^2} \begin{bmatrix} -X_r R_s & X_m^2 v_r & X_m R_r & X_m X_r v_r \\ -X_m^2 v_r & -X_r R_s & -X_m X_r v_r & X_m R_r \\ X_m R_s & -X_m X_s v_r & -X_s R_r & -X_r X_s v_r \\ X_m X_s v_r & -X_m R_s & X_r X_s v_r & -X_s R_r \end{bmatrix} \quad (18)$$

Also in these equations

$$X^2 = X_s X_r - X_m^2 \quad (19)$$

$$\gamma = \frac{2}{3} \frac{X_{CF} \omega_b}{X^2 X_{LF}} \quad (20)$$

$$v_r = \frac{\omega_r}{\omega_b} \quad (21)$$

$\bar{0}$ is a 4 x 4 matrix of zeros and σ and ω are the real and imaginary parts of the 2-pole filter. The pairs of state variables $(Y_1 Y_2)$ and $(Y_3 Y_4)$ represent the two variables necessary to specify the output of the two-pole filter for each of the two feedback elements i_{ds}^{su} and i_{qs}^{su} . These auxiliary state variables have been defined such that the real and quadrature components of the filter poles appear explicitly in the state equation.

STEADY STATE SOLUTION

As presently constituted, the state equations defined by (12) and (13) are time-varying due to the impulse-type function $\xi(t)$ and non-linear since the per unit speed v_r is not constant. However, if it is assumed that the inertia is very large then the rotor speed can be assumed essentially constant during steady-state operation. With this assumption, the remaining time dependent function in the plant matrix is the impulse series. It has been established /8/ that a convergent, solution is possible even in the presence of such a series in the \bar{A} matrix. Although the assumption of constant speed may appear artificial it will be shown that the speed deviation will be as small as possible. Once the solution has been found for fixed speed, it will be possible to consider small changes of all variables from their steady-state trajectory.

In a specific application all parameters which appear in the plant \bar{A} matrix are fixed, except for the per unit speed v_r and input voltage V_{in} . Hence, any small change in the steady-state operating point can be expressed approximately as

$$\Delta \bar{Z}_0 = \frac{\partial \bar{Z}_0}{\partial v_r} \Delta v_r + \frac{\partial \bar{Z}_0}{\partial V_{in}} \Delta V_{in} \quad (22)$$

In general, changes in speed are associated with the output and hence sensitivity of the system to changes in this variable are of more importance than input changes. If it is assumed that $\Delta V_{in} = 0$, then for small changes (14) can be written

$$\frac{d\Delta v_r}{dt} = \frac{X_m}{H} (\bar{Z}_0^T \bar{M} \frac{\partial \bar{Z}_0}{\partial v_r}) \Delta v_r + \frac{\Delta T_L}{2H} \quad (23)$$

where

$$\bar{M} = \frac{1}{2} \left[\begin{array}{cccc|c} 0 & 0 & 0 & 1 & \bar{0} \\ 0 & 0 & -1 & 0 & \bar{0} \\ 0 & -1 & 0 & 0 & \bar{0} \\ 1 & 0 & 0 & 0 & \bar{0} \\ \hline & & & \bar{0} & \bar{0} \end{array} \right] \quad (24)$$

The subscript 0 is used to denote the nominal steady-state solution. The solution of (23) is

$$\Delta v_r(t) = \left(\exp \int_0^t \frac{X_m}{H} \bar{Z}_0^T \bar{M} \frac{\partial \bar{Z}_0}{\partial v_r} d\tau \right) \cdot \Delta v_r \quad (25)$$

$$+ \left(\exp \int_0^t \frac{X_m}{2H} \bar{Z}_0^T \bar{M} \frac{\partial \bar{Z}_0}{\partial v_r} d\tau \right) * \frac{\Delta T_L}{2H}$$

In (25), the asterisk denotes the convolution of two functions. In cases where the load torque remains constant then $\Delta T_L = 0$ and it is necessary that, for sufficiently large t , Δv_r approach zero so that the operating point remain unchanged. If the load torque does not change, it is desired that a new steady-state operating point be reached when t is large. This requirement, in turn, implies that the resulting change in speed, Δv_r , must be bounded.

For the case where T_L is constant, it is apparent that if the change in speed is to ultimately approach zero the quantity

$$\exp \int_0^t \frac{X_m}{2H} \bar{Z}_0^T \bar{M} \frac{\partial \bar{Z}_0}{\partial v_r} d\tau \quad (26)$$

must approach zero for large values of t . If the load changes, that is if ΔT_L has a magnitude greater than zero, then by nature of the convolution of two functions, it is seen that the change in speed can remain bounded only if the above term approaches zero for large t . Hence a necessary (and clearly sufficient) condition for stability is that the coefficient of Δv_r in (25) approach zero for large t .

For convenience let

$$f(t) = \frac{X_m}{H} \bar{Z}_0^T \bar{M} \frac{\partial \bar{Z}_0}{\partial v_r} \quad (27)$$

If an arbitrary time t is selected such that $t = nT + \sigma$, n an integer, then clearly

$$\int_0^t f(\tau) d\tau = \int_0^{nT} f(\tau) d\tau + \int_{nT}^{nT+\sigma} f(\tau) d\tau \quad (28)$$

It must now be assumed that the eigenvalues of \bar{A} are in the left half plane otherwise \bar{Z}_0 is unbounded as $t \rightarrow \infty$ and it would be useless to consider small changes in \bar{Z}_0 . With this assumption, then \bar{Z}_0 exists so that for some integer n , large enough, and any integer $m \geq 0$

$$\bar{Z}(t - (n+m)T) = \bar{Z}(t - nT)$$

which implies that

$$\left. \frac{\partial \bar{Z}}{\partial v_r} \right|_{t - (n+m)T} = \left. \frac{\partial \bar{Z}}{\partial v_r} \right|_{t - nT} \quad (29)$$

Hence, for the integral of $f(t)$

$$\int_{nT}^{(n+1)T} f(\tau) d\tau = \int_{(n+m)T}^{(n+m+1)T} f(\tau) d\tau \quad (30)$$

at the time $t = (n+m+1)T$ (28) can be written as

$$\int_0^{(n+m+1)T} f(\tau) d\tau = \int_0^{nT} f(\tau) d\tau + m \int_{nT}^{(n+1)T} f(\tau) d\tau \quad (31)$$

so that (26) can be expressed as

$$\exp \int_0^t f(\tau) d\tau = \left[\exp \int_0^{nT} f(\tau) d\tau \right] \left[\exp m \int_{nT}^{(n+1)T} f(\tau) d\tau \right] \quad (32)$$

The first term of (32) is not equal to zero since the integral of $f(t)$ is clearly not minus infinity. Hence a necessary and sufficient condition for (32) to approach zero for large t is that

$$\int_{nT}^{(n+1)T} \frac{X_m}{H} \bar{Z}_0^{-T} \bar{M} \frac{\partial \bar{Z}_0}{\partial v_r} d\tau < 0 \quad (33)$$

a steady-state solution of the state equations exists only if the eigenvalues of \bar{A} lie in the left half plane so that a practical evaluation of the effects of feedback requires a simple method for finding the $\lambda_i(\bar{A})$ for any value of feedback vector \bar{K} . It can be shown that the closed loop characteristic polynomial can be expressed in terms of the polynomial without feedback which need be calculated only once for a specified operating point, and the feedback vector \bar{K} /8/. It is also possible to reduce the eighth degree polynomial to a sixth degree polynomial since both λ_1 and λ_2 ; the eigenvalues of the filter, are single roots of $|\lambda \bar{I} - \bar{A}|$. It can be demonstrated that K_1 affects three coefficients of the remaining sixth degree polynomial and K_2 affects only one /8/. Hence, stability can be efficiently evaluated using Routh's Criterion.

THE COST FUNCTIONAL

It is clearly desirable that the per unit speed v_r approach its new value as rapidly as possible for small changes in system input. Examination of (25) and (33) indicates that the means to insure a rapid change is to minimize the functional

$$\text{cost} = \frac{2X_m}{T} \int_0^T \bar{Z}_0^{-T} \bar{M} \frac{\partial \bar{Z}_0}{\partial v_r} d\tau \quad (34)$$

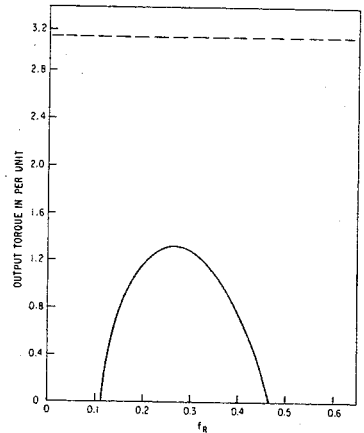
since this quantity causes the expression

$$\exp \int_0^t \frac{X_m}{H} \bar{Z}_0^{-T} \bar{M} \frac{\partial \bar{Z}_0}{\partial v_r} d\tau$$

to be reduced by the factor $\exp[T \times \text{cost}/2H]$ over each inverter switching interval when t is large. This criterion is consistent with the stability criterion which requires that the cost functional be less than zero.

instability region.

Figure 5 shows the transient response for the specified operating point obtained by solving the system equations (12, 13) assuming the per unit rotor speed v_r as constant. It can be noted that the zero feedback case is poorly damped but stable as required when speed is constant. Expansion of the traces indicate that feedback of the power component yields better transient response than the quadrature component for this operating point. Although the response for the actual system wherein speed changes occur cannot be calculated due to the nonlinearity of the equations this effect has been found to introduce a small



influence on the proper gain setting. Figure 6 shows a typical result from a detailed analog computer simulation wherein the system was represented in considerable detail with the actual nonlinearities of the motor equations represented. In this case the system is oscillating without feedback, at the same operating frequency as predicted by the stability analysis; at the time denoted by an arrow, feedback corresponding to $K_1 = 0.00$, $K_2 = -0.13$ was suddenly introduced. A quick damped response to a stable operating condition is readily apparent. Similar results are obtained over a wide range of operating conditions.

SUMMARY

This paper has described a new analytical approach to the design of feedback compensations for the purpose of damping static inverter-induction motor drive systems. A cost functional is defined which ensures rapid recovery of rotor speed to small disturbances. Since the cost functional can be approximated in terms of feedback coefficients and the steady state solution for zero feedback the method is computationally efficient and requires only algebra after the solution for zero feedback is obtained. The approach is applied to a rectifier-inverter induction motor drive system and it is shown that the computed feedback gains result in near optimum response both for infinite and finite rotor inertia. It appears that the technique described in this paper can be readily extended to include a wide variety of drive configurations.

References

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