Stability Analysis of a Reluctance-Synchronous Machine

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Abstract—A stability study of a reluctance-synchronous machine (synchronous-induction machine) is performed by applying the Nyquist stability criterion to the equations which describe the behavior of the machine during small displacements about a steady-state operating point. This investigation reveals that machine instability can occur at low speeds (low frequencies) even though balanced, constant amplitude, sinusoidal voltages are applied to the stator terminals. Regions of machine instability are established from the results of a digital computer study. The results of an analog computer study are included to illustrate the modes of operation which occur within these regions. Also, regions of instability for different values of system parameters are given and discussed.

Introduction

With the improvements in the capabilities of controlled rectifiers, speed control of ac machines has become practical by controlling the frequency of the applied voltages. Wide-range, variable speed drives are practicable by incorporating a static frequency converter with either an induction machine, synchronous machine, or perhaps a reluctance-synchronous machine (synchronous-induction machine). In some applications it may be desirable to operate these drives at speeds as low as five or ten percent of the rated speed of the machine. The performance of the reluctance-synchronous machine at low speeds is the subject of this paper.

The equations which describe the behavior of the reluctance-synchronous machine are established by an analysis similar to that employed in synchronous machine theory. A stability study is then performed by applying the Nyquist stability criterion to the equations which describe the behavior of the machine during small excursions about a steady-state operating point. This study reveals that machine instability can occur at low speeds.

It is shown that the region of machine instability is dependent upon the amplitude of the applied voltage, the system inertia, and the electrical parameters of the machine. Moreover, these regions of instability occur with balanced, sinusoidal applied stator voltages which are independent of load current (zero-impedance source). Consequently, the presence of instability, in this study, is not due to an interaction between the machine and static converter.

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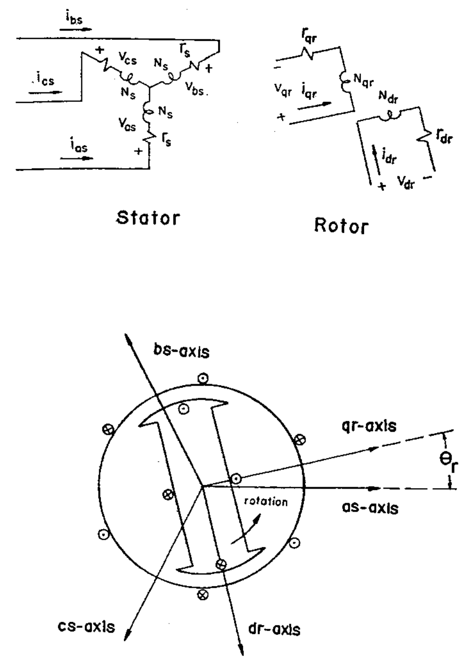

Fig. 1. Two-pole, 3-phase reluctance-synchronous machine.

Basic Equations

The equations which describe the behavior of an idealized reluctance-synchronous machine will be established by considering the elementary 2-pole, 3-phase machine shown in Fig. 1. The following assumptions are made in this development:

1) Each stator winding is distributed so as to produce a sinusoidal MMF wave along the air gap.
2) Stator slots produce negligible variations in the rotor inductances.
3) Saturation of the magnetic circuit is neglected.

These idealizing assumptions are commonly employed in analyzing transient and steady-state performance of salient-pole synchronous machines.[1]–[3]

With the appropriate subscript, that is, *as*, *bs*, *cs*, *dr*, or *qr*, the following voltage equation is applicable to each of the five windings shown in Fig. 1.

$$v = p\lambda + ri \tag{1}$$

where λ is the total flux linkages, r is the winding resistance, and p is the operator d/dt. The flux linkages may be expressed

$$\begin{bmatrix}\lambda_{as}\\ \lambda_{bs}\\ \lambda_{cs}\\ \lambda_{dr}\\ \lambda_{qr}\end{bmatrix} = \begin{bmatrix}L_{asas}L_{bsas}L_{csas}L_{dras}L_{qras}\\ L_{asbs}L_{bsbs}L_{csbs}L_{drbs}L_{qrbs}\\ L_{ascs}L_{bscs}L_{cscs}L_{drcs}L_{qrcs}\\ L_{asdr}L_{bsdr}L_{csdr}L_{drdr}L_{qrdr}\\ L_{asqr}L_{bsqr}L_{csqr}L_{drqr}L_{qrqr}\end{bmatrix} \times \begin{bmatrix}i_{as}\\ i_{bs}\\ i_{cs}\\ i_{dr}\\ i_{qr}\end{bmatrix}. \quad (2)$$

With the assumptions which have been set forth, the self-inductances of the rotor circuits (qr and dr) are constants, independent of the rotor angular displacement θ_r. However, the self-inductances of the stator windings and the mutual inductances between stator windings as well as between stator and rotor windings are sinusoidal functions of θ_r. The following equations describe the inductances which were given in (2).

Stator self-inductances:

$$L_{asas} = L_{ls} + L_{s0} - L_{sl} \cos 2\theta_r \quad (3)$$

$$L_{bsbs} = L_{ls} + L_{s0} - L_{sl} \cos 2(\theta_r - 2\pi/3) \quad (4)$$

$$L_{cscs} = L_{ls} + L_{s0} - L_{sl} \cos 2(\theta_r + 2\pi/3). \quad (5)$$

Rotor self-inductances:

$$L_{drdr} = L_{ldr} + L_{dr0} \quad (6)$$

$$L_{qrqr} = L_{lqr} + L_{qr0}. \quad (7)$$

Stator-to-stator mutual inductances:

$$L_{asbs} = L_{bsas} = -1/2L_{s0} - L_{sl} \cos 2(\theta_r - \pi/3) \quad (8)$$

$$L_{ascs} = L_{csas} = -1/2L_{s0} - L_{sl} \cos 2(\theta_r + \pi/3) \quad (9)$$

$$L_{bscs} = L_{csbs} = -1/2L_{s0} - L_{sl} \cos 2(\theta_r + \pi). \quad (10)$$

Rotor-to-rotor mutual inductances:

$$L_{qrdr} = L_{drqr} = 0. \quad (11)$$

Stator-to-rotor mutual inductances:

$$L_{asdr} = L_{dras} = L_{sdr} \sin \theta_r \quad (12)$$

$$L_{asqr} = L_{qras} = L_{sqr} \cos \theta_r \quad (13)$$

$$L_{bsdr} = L_{drbs} = L_{sdr} \sin (\theta_r - 2\pi/3) \quad (14)$$

$$L_{bsqr} = L_{qrbs} = L_{sqr} \cos (\theta_r - 2\pi/3) \quad (15)$$

$$L_{csdr} = L_{drcs} = L_{sdr} \sin (\theta_r + 2\pi/3) \quad (16)$$

$$L_{csqr} = L_{qrcs} = L_{sqr} \cos (\theta_r + 2\pi/3). \quad (17)$$

In the equations describing the self-inductances, the leakage inductance of the stator windings is denoted as L_{ls} whereas the leakage inductances of the dr and qr windings are denoted as L_{ldr} and L_{lqr}, respectively.

In the analysis of ac machinery, it is customary to employ a change of variables which formulates a transformation of stator and rotor variables to a common frame of reference.[1]–[3] Equations of transformation can be devised to transform these variables to a reference frame rotating at an arbitrary angular velocity.[4] If, however, either the stator or the rotor of a machine is unsymmetrical, time varying coefficients will appear in the voltage equations in all reference frames except for the one fixed in the machine where the asymmetry exists.[5] Therefore, in the case of a reluctance-synchronous machine it is convenient to select a reference frame fixed in the rotor. The stator variables are transformed to this reference frame by

$$f_{qs} = 2/3\ [f_{as} \cos \theta_r + f_{bs} \cos (\theta_r - 2\pi/3) + f_{cs} \cos (\theta_r + 2\pi/3)] \quad (18)$$

$$f_{ds} = 2/3\ [f_{as} \sin \theta_r + f_{bs} \sin (\theta_r - 2\pi/3) + f_{cs} \sin (\theta_r + 2\pi/3)] \quad (19)$$

$$f_{0s} = 1/3\ (f_{as} + f_{bs} + f_{cs}). \quad (20)$$

In the above equations the variable f can represent either voltage, current, or flux-linkage. The variable f_{0s} is incorporated since, in general, three independent variables are necessary. However, since the stator is a symmetrical, 3-wire system, the third substitute variable f_{0s} is zero.

If the above equations of transformation are used to transform the stator voltages and currents to a reference frame fixed in the rotor and if the rotor quantities are referred to the stator windings by the appropriate turns ratio, the voltage equations may be expressed

$$v_{qs} = p\lambda_{qs} + \lambda_{ds}p\theta_r + r_s i_{qs} \quad (21)$$

$$v_{ds} = p\lambda_{ds} - \lambda_{qs}p\theta_r + r_s i_{ds} \quad (22)$$

$$v_{qr}' = p\lambda_{qr}' + r_{qr}' i_{qr}' \quad (23)$$

$$v_{dr}' = p\lambda_{dr}' + r_{dr}' i_{dr}' \quad (24)$$

where

$$\lambda_{qs} = L_{ls} i_{qs} + L_{aq}(i_{qs} + i_{qr}') \quad (25)$$

$$\lambda_{ds} = L_{ls} i_{ds} + L_{ad}(i_{ds} + i_{dr}') \quad (26)$$

$$\lambda_{qr}' = L_{lqr}' i_{qr}' + L_{aq}(i_{qs} + i_{qr}') \quad (27)$$

$$\lambda_{dr}' = L_{ldr}' i_{dr}' + L_{ad}(i_{ds} + i_{dr}') \quad (28)$$

where

$$L_{ad} = (3/2)(L_{s0} + L_{sl}) = (3/2)(N_s/N_{dr})L_{sdr} = (3/2)(N_s/N_{dr})^2 L_{dr0} \quad (29)$$

$$L_{aq} = (3/2)(L_{s0} - L_{sl}) = (3/2)(N_s/N_{qr})L_{sqr} = (3/2)(N_s/N_{qr})^2 L_{qr0} \quad (30)$$

$$L_{ldr}' = (3/2)(N_s/N_{dr})^2 L_{ldr} \quad (31)$$

$$r_{dr}' = (3/2)(N_s/N_{dr})^2 r_{dr} \quad (32)$$

$$i_{dr}' = (2/3)(N_{dr}/N_s) i_{dr} \quad (33)$$

$$v_{dr}' = (N_s/N_{dr}) v_{dr}. \quad (34)$$

The equations relating qr and qr' quantities are similar to those relating dr and dr' quantities. Equations (21)

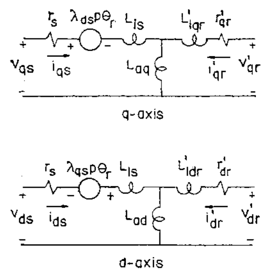

Fig. 2. The d and q equivalent circuits of a reluctance-synchronous machine—reference frame fixed in the rotor.

through (28) suggest the equivalent circuit shown in Fig. 2. In the case of a reluctance-synchronous machine the voltages v_{dr} and v_{qr} are zero.

Although a 2-pole, 3-phase machine has been used to establish the equations which describe the behavior of a reluctance-synchronous machine, the development can be extended to include a machine with any number of poles by multiplying the expression for torque by the number of pole pairs. Also, the equations which describe the 2-phase machine can be established by a procedure similar to that used for the 3-phase machine. In fact, a 2-phase machine could have been used in this development instead of the 3-phase machine without restricting the following analysis.

An expression for the instantaneous electromagnetic torque can be obtained by applying the principle of arbitrary displacement. This relation, which is positive for motor action, is

$$T_e = (n/2)(P/2)(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \tag{35}$$

where n is the number of phases and P is the number of poles.

Application of Small-Displacement Theory to Reluctance-Synchronous Machines

The small-displacement theory was initially employed in the analysis of electric machines by Park.[6] This theory enables one to establish linear relationships between the machine variables for small changes about an operating point. The small-displacement equations are not valid for large excursions of the variables, however, these relationships offer a means of investigating system stability when used in conjunction with either Nyquist's or Routh's criterion.[7]

In the case of a reluctance-synchronous machine the rotor circuits are short circuited. Also, during balanced, steady-state operation wherein the reluctance-synchronous machine rotates at an electrical angular velocity corresponding to that of the applied voltages, the rotor currents (i_{qr} and i_{dr}) are zero. It is clear, that in this mode of operation, the reluctance-synchronous machine operates as a reluctance machine. The equations which describe this mode of operation can be obtained directly from (21) through (28) by setting $v_{dr} = v_{qr} = i_{dr} = i_{qr} = 0$. The resulting equations are expressed as follows:

$$V \cos \delta_0 = X_{ds} f_R i_{ds0} + r_s i_{qs0} \tag{36}$$

$$V \sin \delta_0 = -X_{qs} f_R i_{qs0} + r_s i_{ds0}. \tag{37}$$

In the above equations the letter o has been added to the subscript so as to denote steady-state quantities. Also, V denotes the amplitude of the balanced, stator applied voltages and

$$X_{ds} = \omega_e (L_{ls} + L_{ad}) = X_{ls} + X_{ad} \tag{38}$$

$$X_{qs} = \omega_e (L_{ls} + L_{aq}) = X_{ls} + X_{aq} \tag{39}$$

$$\delta_0 = \theta_r - \omega_{r0} t \tag{40}$$

$$f_R = \omega_{r0}/\omega_e. \tag{41}$$

The steady-state rotor angle δ_0 is negative for motor action. The electrical angular velocity ω_e is a constant selected as base frequency convenient to the per-unit system employed. However, ω_{r0} is the steady-state electrical angular velocity of the rotor which is determined by the frequency of the applied stator voltages. It is clear that the ratio f_R is unity when the electrical angular velocity of the applied stator voltages is equal to ω_e.

The steady-state torque for a 2-pole, 3-phase machine is expressed

$$T_{e0} = (3/2)(1/\omega_e)(X_{ds} - X_{qs}) i_{ds0} i_{qs0} \tag{42}$$

If all variables are allowed to change by small amounts about an initial operating point (21) through (28) may be expressed, in matrix form, as follows:

$$\begin{bmatrix} V \cos(\delta_0 + \Delta\delta) \\ V \sin(\delta_0 + \Delta\delta) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + (p/\omega_e)X_{qs} & X_{ds}(f_R + \Delta\omega_r/\omega_e) & (p/\omega_e)X_{aq} & X_{ad}(f_R + \Delta\omega_r/\omega_e) \\ -X_{qs}(f_R + \Delta\omega_r/\omega_e) & r_s + (p/\omega_e)X_{ds} & -X_{aq}(f_R + \Delta\omega_r/\omega_e) & (p/\omega_e)X_{ad} \\ (p/\omega_e)X_{aq} & 0 & r_{qr}' + (p/\omega_e)X_{qr}' & 0 \\ 0 & (p/\omega_e)X_{ad} & 0 & r_{dr}' + (p/\omega_e)X_{dr}' \end{bmatrix} \times \begin{bmatrix} i_{qs0} + \Delta i_{qs} \\ i_{ds0} + \Delta i_{ds} \\ \Delta i_{qr}' \\ \Delta i_{dr}' \end{bmatrix} \tag{43}$$

where

$$\Delta\omega_r = p\Delta\delta \quad (44)$$

$$X_{dr}' = X_{ldr}' + X_{ad} = \omega_e(L_{ldr}' + L_{ad}) \quad (45)$$

$$X_{qr}' = X_{lqr}' + X_{aq} = \omega_e(L_{lqr}' + L_{aq}). \quad (46)$$

For small changes in the rotor angle δ_0 the following approximations are valid:

$$V \cos (\delta_0 + \Delta\delta) \doteq V \cos \delta_0 - V \sin \delta_0 \Delta\delta \quad (47)$$

$$V \sin (\delta_0 + \Delta\delta) \doteq V \sin \delta_0 + V \cos \delta_0 \Delta\delta. \quad (48)$$

If (47) and (48) are substituted into (43) it is possible to eliminate the terms in (43) which describe the steady-state mode of operation as given in (36) and (37). Also, it is clear from (43), that the rotor currents may be expressed in terms of the stator currents. Therefore, if second order differences are neglected (i.e., $\Delta\Delta$ terms) and if (47) and (48) are substituted into (43) and the terms describing steady-state operation are eliminated, and if the rotor currents are expressed in terms of stator currents the following small-displacement equations are obtained:

$$\begin{bmatrix} [-V \sin \delta_0 - X_{ds} i_{ds0}(p/\omega_e)]\Delta\delta \\ [V \cos \delta_0 + X_{qs} i_{qs0}(p/\omega_e)]\Delta\delta \end{bmatrix} = \begin{bmatrix} r_s + (p/\omega_e)X_{qs} - \dfrac{(p/\omega_e)^2 X_{aq}^2}{r_{qr}' + (p/\omega_e)X_{qr}'} & X_{ds} f_R - \dfrac{(p/\omega_e) f_R X_{ad}^2}{r_{dr}' + (p/\omega_e)X_{dr}'} \\ -X_{qs} f_R + \dfrac{(p/\omega_e) f_R X_{aq}^2}{r_{qr}' + (p/\omega_e)X_{qr}'} & r_s + (p/\omega_e)X_{ds} - \dfrac{(p/\omega_e)^2 X_{ad}^2}{r_{dr}' + (p/\omega_e)X_{dr}'} \end{bmatrix} \times \begin{bmatrix} \Delta i_{qs} \\ \Delta i_{ds} \end{bmatrix}. \quad (49)$$

The torque expression during small changes about an operating point becomes

$$T_{e0} + \Delta T_{e0} = (3/2\omega_e)(X_{ds} - X_{qs})(i_{ds0} + \Delta i_{ds})(i_{qs0} + \Delta i_{qs}) + (3/2\omega_e)X_{ad}(i_{qs0} + \Delta i_{qs})\Delta i_{dr}' - (3/2\omega_e)X_{aq}(i_{ds0} + \Delta i_{ds})\Delta i_{qr}'. \quad (50)$$

If second-order differences are neglected, and if the expression for the steady-state torque described in (42) is eliminated from (50) and if the rotor currents are expressed in terms of stator currents, the small-displacement torque equation becomes

$$\Delta T_e = (3/2\omega_e)(X_{ds} - X_{qs})(i_{ds0}\Delta i_{qs} + i_{qs0}\Delta i_{ds}) - \frac{(3/2\omega_e)X_{ad}^2 i_{qs0}(p/\omega_e)}{r_{dr}' + (p/\omega_e)X_{dr}'}\Delta i_{ds} + \frac{(3/2\omega_e)X_{aq}^2 i_{ds0}(p/\omega_e)}{r_{qr}' + (p/\omega_e)X_{qr}'}\Delta i_{qs}. \quad (51)$$

If (49) is solved for Δi_{qs} and Δi_{ds} and the results substituted into (51), the following expression can be obtained

$$\Delta T_e = G(p)\Delta\delta. \quad (52)$$

Although the procedure is straightforward, it is a formidable task to solve for $G(p)$. This expression will not be given, instead $G(j\nu)$ will be given when, in the following frequency response analysis, p is set equal to $j\nu$. Since, $p = j\nu$, $p^2 = -\nu^2$, etc., (49) and (51) can be expressed more compactly and consequently the expression for $G(j\nu)$ is obtained more directly than $G(p)$.

Equation (52) formulates a linear, small-displacement relationship between torque and rotor angle. The equation has been derived assuming conventional units, however, it is a simple task to convert to a per unit system. This conversion can be accomplished by selecting a frequency and kilovolt-ampere base as well as a base voltage to correspond to the rating of the machine.

An additional small-displacement equation can be obtained by considering the dynamics of the mechanical system during small changes about an initial operating speed. In the steady-state, the load torque T_{L0} (including friction and windage losses) is equal to the steady-state electromagnetic torque T_{e0}. For small changes in all variables the following per unit relationship can be written

$$T_{L0} + \Delta T_L = T_{e0} + \Delta T_e - (2H/\omega_e)p^2\Delta\delta \quad (53)$$

where H is the inertia constant of the machine expressed in seconds. Since $T_{L0} = T_{e0}$ the desired small-displacement equation is obtained directly from (53), that is

$$\Delta T_L = \Delta T_e - (2H/\omega_e)p^2\Delta\delta. \quad (54)$$

Methods of Determining Machine Stability

In the past, the theory of small-displacements has been used extensively in studying the dynamic behavior of systems involving synchronous and induction machines.[8]–[11] The analysis wherein damping and synchronizing torques are used as well as the methods employing Routh's or Nyquist's criteria to determine system stability, all incorporate the theory of small-displacements in a manner similar to that set forth here. The purpose of this paper, however, is to establish equations which can be used to determine the stability of a reluctance-synchronous machine over a wide frequency range of applied stator voltages.

Equations (52) and (54) suggest the closed-loop system shown in Fig. 3. This system is linear. Thus, the stability of the reluctance-synchronous machine can be investigated as if it were a linear feedback system. In this paper, the Nyquist criterion will be used to determine stability, however, either the Routh stability criterion or the root-locus method could be employed.[7] Nyquist's criterion was used by Aldred and Shackshaft in studying synchronous machine stability.[10],[11]

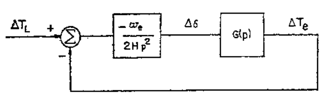

Fig. 3. Small-displacement, closed-loop system.

From Fig. 3, the open-loop transfer function is expressed

$$F(p) = \omega_e G(p)/2Hp^2. \tag{55}$$

If p is set equal to $j\nu$, the open-loop transfer function becomes

$$F(j\nu) = -\omega_e[a(\nu) + jb(\nu)]/2H\nu^2 \tag{56}$$

where

$$G(j\nu) = a(\nu) + jb(\nu). \tag{57}$$

The stability of the closed-loop system can be determined from the locus of $F(j\nu)$ as ν varies from $-\infty$ to $+\infty$. In particular, Nyquist's criterion states that a feedback system is stable if and only if the locus of $F(j\nu)$ does not pass through the $(-1,0)$ point and the number of counterclockwise encirclements about the $(-1,0)$ point equals the number of poles of $F(p)$ with positive real parts.[7] In the case of a reluctance-synchronous machine, it can be shown that $F(p)$ does not have poles with positive real parts, therefore, the feedback system is stable if and only if the locus of $F(j\nu)$ does not pass through or encircle the $(-1,0)$ point.

The expression for $G(j\nu)$ is given in the Appendix. The ratio f_ν is used in the expression; this ratio is defined as

$$f_\nu = \nu/\omega_{r0}. \tag{58}$$

Stability Studies

The equations which have been developed offer a convenient means of predicting the behavior of reluctance-synchronous machines with applied voltages of any frequency. A change in steady-state operating speed (change in frequency) can be taken into account by appropriate changes in the values of f_R and f_ν. In this study, 60 Hz is assumed to be rated frequency and the per unit system is based on operation at rated frequency, ($\omega_e = 377$ rad/s). Since $f_R = \omega_{r0}/\omega_e$, it may be preferable to interpret this ratio as the steady-state operating speed expressed in per unit.

In variable speed systems the amplitude of the applied voltages is decreased as frequency decreases. In this study it is assumed that the amplitude of the sinusoidal stator applied voltages decreases linearly with frequency. That is

$$V = f_R V_m \tag{59}$$

where V_m corresponds to rated voltage (1.0 per unit voltage).

In order to investigate completely the stability of the reluctance-synchronous machine it is necessary to consider the locus of $F(j\nu)$, as ν is varied, at all operating speeds and all practical steady-state rotor angles, δ_0. Such a task is prohibitive without the aid of a computer. If, however, the digital computer is programmed appropriately, regions of machine instability can be determined quite readily. A region of instability for a 60-Hz, 2-pole, 3-phase reluctance-synchronous machine is shown in Fig. 4. The per unit parameters of the machine are given in Table 1. The dashed line shown in Fig. 4 indicates the pull-out or maximum steady-state torque at the various operating speeds. Negative torque output denotes generator action.

The continuous curve (contour) shown in Fig. 4 forms the boundary between stable and unstable regions of operation. More specifically, the contour connects all initial operating points for which the locus of $F(j\nu)$ passes through the $(-1,0)$ point. The Nyquist diagrams shown in Fig. 5 help to explain further the significance of this contour. The four plots of $F(j\nu)$ shown in Fig. 5 correspond to the initial operating points indicated in Fig. 4 as (a), (b), (c), and (d). These operating points occur at a machine speed of $0.1\omega_e$ electrical radians per second; in this case, the stator applied voltages would vary at 6 Hz. Also, when $T_{e0} = 0.3$ p.u., $\delta_0 = -0.199$ rad; $T_{e0} = 0.24$ p.u., $\delta_0 = -0.074$ rad; $T_{e0} = 0.18$, $\delta_0 = 0.013$ rad; $T_{e0} = 0$, $\delta_0 = 0.21$ rad. With $T_{e0} = 0.24$ p.u., point (b), the locus of $F(j\nu)$ shown in Fig. 5, passes through the $(-1, 0)$ point as ν varies from $-\infty$ to $+\infty$. Therefore, point (b) locates a point on the contour shown in Fig. 4. At the initial operating conditions wherein $T_{e0} = 0$, point (d), and $T_{e0} = 0.18$ p.u., point (c), the plot of $F(j\nu)$ encircles the $(-1, 0)$ point. The system is unstable at these operating conditions. However, with $T_{e0} = 0.3$ p.u., point (a), the plot of $F(j\nu)$ fails to encircle or pass through the $(-1, 0)$ point; the system is stable. It is clear that in order to establish a complete contour it is necessary to determine the condition of loading, at each operating speed, which will cause the locus of $F(j\nu)$ to pass through the $(-1, 0)$ point.

The method of determining machine stability by applying the Nyquist criterion to the small-displacement equations is straightforward and quite convenient.

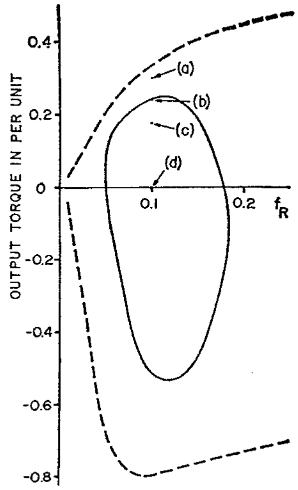

Fig. 4. Region of instability of a reluctance-synchronous machine.

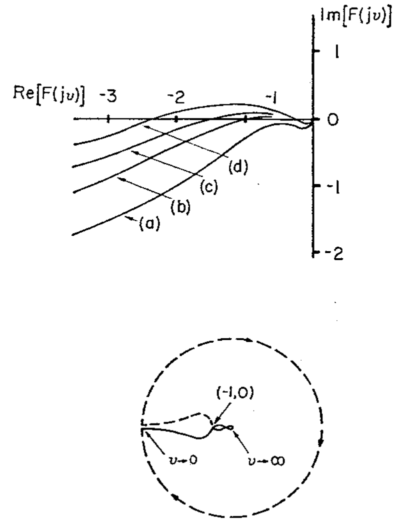

Fig. 5. Nyquist diagrams—plot of $F(j\nu)$, $f_R = 0.1$.

TABLE I

Figure	r_s p.u.	X_{ls} p.u.	r_{dr}' p.u.	X_{ldr}' p.u.	r_{qr}' p.u.	X_{lqr}' p.u.	X_{ad} p.u.	X_{aq} p.u.	H s	V_m p.u.
4,5,6	0.045	0.10	0.03	0.10	0.015	0.10	2.0	0.50	1.0	1.0
7	0.045	0.10	0.03	0.10	0.015	0.10	2.0	0.50	1.0	varied
8	0.045	0.10	0.03	0.10	0.015	0.10	2.0	0.50	varied	1.0
9	0.045	0.10	0.03	0.10	0.015	0.10	varied	varied	1.0	1.0
10	varied	0.10	0.03	0.10	0.015	0.10	2.0	0.50	1.0	1.0
11	0.045	0.10	varied	0.10	varied	0.10	2.0	0.50	1.0	1.0
12	0.045	varied	0.03	varied	0.015	varied	2.0	0.50	1.0	1.0
13	0.045	varied	0.03	0.10	0.015	0.10	2.0	0.50	1.0	1.0

However, one might question the validity of using the small-displacement theory to determine machine stability since a loss of stability can not occur without large excursions in machine variables. Fortunately, an interpretation of the region of instability as well as a verification of the method used to determine machine stability can be obtained from a direct simulation of the machine equations, (21) through (28) and (35). This type of a simulation can easily be represented on either a digital or an analog computer. Moreover, it provides a means of studying the dynamic behavior of the reluctance-synchronous machine which is independent of the theory of small-displacements.

The equations which describe the behavior of the reluctance-synchronous machine, (21) through (28) and (35), were simulated on the analog computer. Several of the modes of operation considered in Figs. 4 and 5 were also considered in the analog computer study. The computer traces of the electromagnetic torque T_e and rotor angle δ during load torque switching are shown in Fig. 6. In Fig. 6(a) the machine is operating initially with 0.3 p.u. load torque. The load torque is suddenly switched to 0.24 p.u. and held constant at this value. This switching corresponds to a change from point (a), a stable point of operation, to point (b), an unstable point of operation (Fig. 4). The traces shown in Fig. 6(a) illustrate the sustained oscillations which occur with the load torque of 0.24 p.u. The traces shown in Fig. 6(b) demonstrate the machine response when the load torque is increased from 0.24 p.u. back to 0.3 p.u. The recordings of output torque and rotor angle given in Fig. 6(b) illustrate the damping which occurs in the stable region. The computer traces of torque output and rotor angle shown in Fig. 6(c) show the performance of the machine when the load is switched from 0.3 p.u. to zero. This load torque switching corresponds to a change from point (a) to point (d) in Fig. 4. The amplitudes of the sustained oscillations in output torque and rotor angle are larger for a load torque of zero than for one of 0.24 p.u.

The results of the stability study based on the theory of small-displacements indicate that sustained oscillations will occur only at operating points on the contour; oscillations will be damped for operation outside the region of instability, and negative damping will exist in the unstable region. Therefore, one might interpret a point within the region of instability as a condition which would cause a loss of synchronism. If it is assumed that steady operation could be established within the region of instability, any small disturbance would cause a larger disturbance. However, as the amplitude of the disturbance (oscillations) increases the behavior of the machine can not be predicted by the small-displacement equations. The equations used to simulate the machine on the analog computer are not restricted to small excursions about an

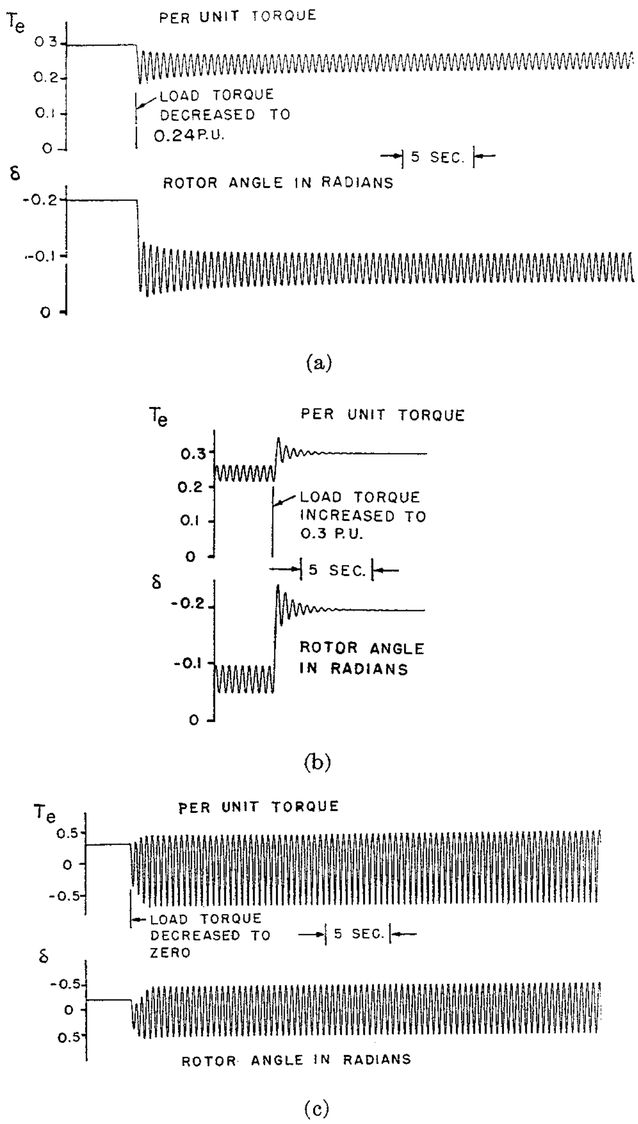

Fig. 6. Load torque switching-analog computer study.

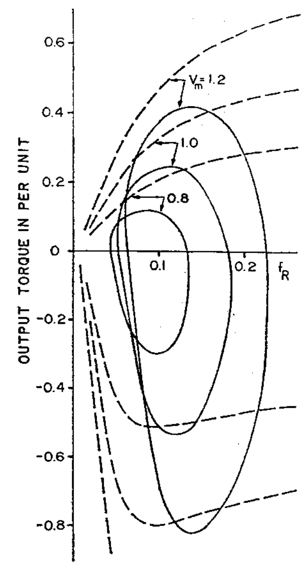

Fig. 7. Regions of instability for changes in amplitude of applied stator voltages.

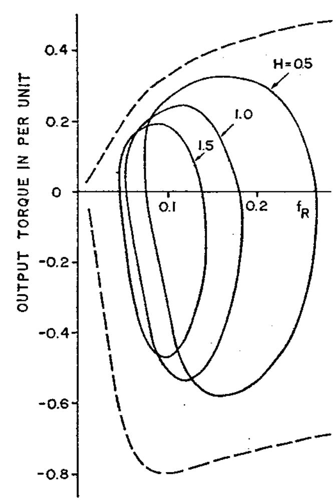

Fig. 8. Regions of instability for different values of inertia.

operating point, therefore, the traces shown in Fig. 6 reveal the complete dynamic characteristics of the reluctance-synchronous machine. It can be concluded from this study, that stable operation can not be achieved within the region of machine instability, however, sustained oscillations of the machine variables may occur rather than a complete loss of synchronism.

Although the nonlinear characteristics of the machine equations prevent a rigorous analysis of sustained oscillations, one might explain this mode of operation from a quasi steady-state point of view. During the part of the oscillation which includes operating points outside the region of instability, positive damping occurs; negative damping occurs during the part of the oscillation which is inside the region of instability. Sustained oscillations occur when the equivalent damping over a complete cycle is zero. It follows, that the magnitude of oscillation will be largest for values of load torque near the center of the region of instability. Also, a loss of synchronism may occur if the region of instability is large.

The contours shown in Figs. 7 through 13 illustrate the effect upon machine stability of changes in system and machine constants. To facilitate a direct comparison, the contour given in Fig 4 is also included in Figs. 7 through 13. The values of the system parameters and the quantities which were varied in each case are given in Table I.

Variation in the region of instability due to a change in the amplitude of the stator applied voltages are shown in Fig. 7. The applied stator voltage V is decreased linearly with f_R. That is $V = f_R V_m$, wherein the voltage V_m is normally 1.0 p.u. Figure 7 shows the contours for V_m = 0.8 p.u., 1.0 p.u., and 1.2 p.u.

The effect upon machine stability due to a change in the inertia of the machine or machine-load combination is given in Fig. 8. Regions of instability are shown for H = 0.5 s, 1.0 s, and 1.5 s.

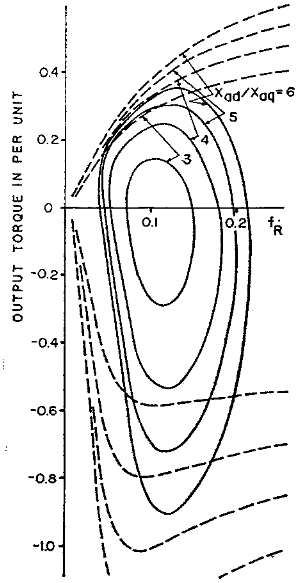

Fig. 9. Regions of instability for various X_{ad}/X_{aq} ratios.

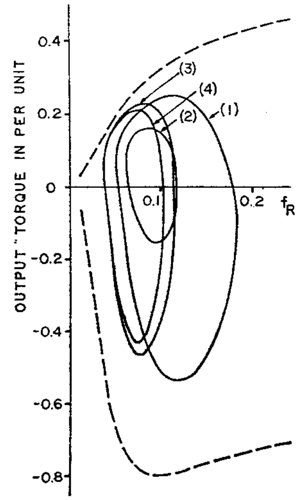

Fig. 11. Regions of instability for different rotor resistances.
1) $r_{dr}' = 0.030$ p.u.; $r_{qr}' = 0.015$ p.u.
2) $r_{dr}' = 0.150$ p.u.; $r_{qr}' = 0.030$ p.u.
3) $r_{dr}' = 0.060$ p.u.; $r_{qr}' = 0.075$ p.u.
4) $r_{dr}' = 0.075$ p.u.; $r_{qr}' = 0.060$ p.u.

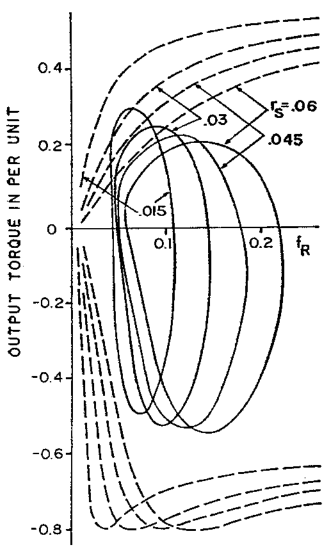

Fig. 10. Regions of instability for change in stator resistance.

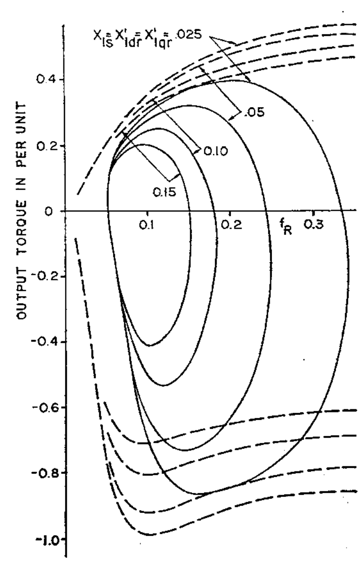

Fig. 12. Regions of instability for several values of stator and rotor leakage reactances.

The contours given in Fig. 9 show an increase in the region of instability with an increase in the ratio X_{ad}/X_{aq}. In other words, an increase in the maximum steady-state torque is accompanied by a larger region of instability. With the machine parameters given in Table I, it was found that instability did not occur with $X_{ad}/X_{aq} = 2$.

The increase in the region of instability due to an increase in stator resistance is shown in Fig. 10. The influence of the rotor resistances, r_{qr}' and r_{dr}', on the region of instability is shown in Fig. 11.

Regions of instability for several values of machine leakage reactances are shown in Fig. 12. In this study the leakage reactances of the machine were maintained equal, that is, $X_{ls} = X_{ldr}' = X_{lqr}'$. The contours shown in Fig. 12 illustrate that the region of instability decreases as the leakage reactances of the machine increase. This characteristic is also demonstrated in Fig. 13. In this study, however, the leakage reactances of the rotor circuits were maintained at 0.10 p.u. and machine stability was investigated for several values of stator leakage reactance. The contours given in Fig. 13 show that the

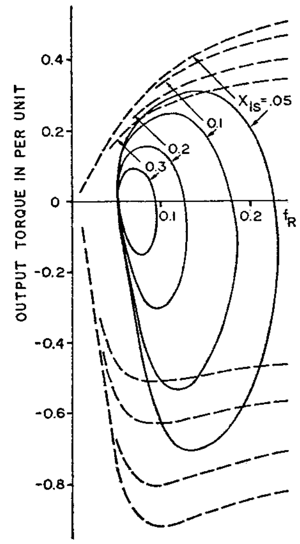

Fig. 13. Regions of instability for changes in stator leakage reactance.

region of instability is decreased as the stator leakage reactance increases. With the machine parameters given in Table I, it was found that instability did not occur with $X_{ls} = 0.4$ p.u. It is clear that the stator leakage reactance can be increased by placing an inductance in series with each of the stator windings. Therefore, incorporating an external inductance in series with each stator winding for the purpose of improving machine stability may be of practical importance. However, it should be pointed out that an increase in the stator leakage reactance by this method is accompanied by an increase in the stator resistance. Also, when series inductances are placed in each of the stator phases, the maximum steady-state torque is decreased.

Conclusion

The equations which describe the behavior of a reluctance-synchronous machine have been established and the theory of small displacements has been employed to linearize these equations. By applying Nyquist's stability criterion to these small-displacement equations it was found that the reluctance-synchronous machine may demonstrate instability at low operating speeds (low frequency).

Regions of instability predicted by the method employing Nyquist's criterion were obtained from a digital computer study. Results of an analog computer study provide a means of interpreting the modes of operation which occur within these regions of machine instability. The regions of instability were found to be dependent upon the amplitude of the applied stator voltages, system inertia, and the machine parameters. All studies were performed assuming zero-impedance voltage sources rather than a static frequency converter, therefore, machine instability occurs due to insufficient damping within the machine.

Appendix

The following quantities will be used to express $G(j\nu)$:

$$l = (X_{ds} - X_{qs}) r_{qr}' i_{ds0}$$

$$m = f_p f_R [X_{qr}'(X_{ds} - X_{qs}) + X_{aq}^2] i_{ds0}$$

$$\begin{aligned} n = {} & -[r_s r_{dr}' - f_p^2 f_R^2 (X_{ds} X_{dr}' - X_{ad}^2)](r_{qr}' V \sin \delta_0 - \\ & f_p^2 f_R^2 X_{ds} X_{qr}' i_{ds0}) - \\ & f_R r_{dr}' X_{ds} (r_{qr}' V \cos \delta_0 - f_p^2 f_R^2 X_{qs} X_{qr}' i_{qs0}) + \\ & f_p^2 f_R^2 (r_{dr}' X_{ds} + r_s X_{dr}')(X_{qr}' V \sin \delta_0 + \\ & X_{ds} r_{qr}' i_{ds0}) + \\ & f_p^2 f_R^3 (X_{ds} X_{dr}' - X_{ad}^2)(X_{qr}' V \cos \delta_0 \\ & + X_{qs} r_{qr}' i_{qs0}) \end{aligned}$$

$$\begin{aligned} p = {} & -f_p f_R [r_s r_{dr}' - f_p^2 f_R^2 (X_{ds} X_{dr}' - X_{ad}^2)] \\ & (X_{qr}' V \sin \delta_0 + X_{ds} r_{qr}' i_{ds0}) - \\ & f_p f_R^2 r_{dr}' X_{ds} (X_{qr}' V \cos \delta_0 + X_{qs} r_{qr}' i_{qs0}) - \\ & f_p f_R (r_{dr}' X_{ds} + r_s X_{dr}')(r_{qr}' V \sin \delta_0 - \\ & f_p^2 f_R^2 X_{ds} X_{qr}' i_{ds0}) - \\ & f_p f_R^2 (X_{ds} X_{dr}' - X_{ad}^2)(r_{qr}' V \cos \delta_0 - \\ & f_p^2 f_R^2 X_{qs} X_{qr}' i_{qs0}) \end{aligned}$$

$$q = (X_{ds} - X_{qs}) r_{dr}' i_{qs0}$$

$$r = f_p f_R [X_{dr}'(X_{ds} - X_{qs}) - X_{ad}^2] i_{qs0}$$

$$\begin{aligned} s = {} & [r_s r_{qr}' - f_p^2 f_R^2 (X_{qs} X_{qr}' - X_{aq}^2)](r_{dr}' V \cos \delta_0 - \\ & f_p^2 f_R^2 X_{qs} X_{dr}' i_{qs0}) - \\ & f_R r_{qr}' X_{qs} (r_{dr}' V \sin \delta_0 - f_p^2 f_R^2 X_{ds} X_{dr}' i_{ds0}) - \\ & f_p^2 f_R^2 (r_{qr}' X_{qs} + r_s X_{qr}')(X_{dr}' V \cos \delta_0 + X_{qs} \\ & r_{dr}' i_{qs0}) + \\ & f_p^2 f_R^3 (X_{qs} X_{qr}' - X_{aq}^2)(X_{dr}' V \sin \delta_0 + \\ & X_{ds} r_{dr}' i_{ds0}) \end{aligned}$$

$$\begin{aligned} t = {} & f_p f_R (r_{qr}' X_{qs} + r_s X_{qr}')(r_{dr}' V \cos \delta_0 - f_p^2 f_R^2 X_{qs} \\ & X_{dr}' i_{qs0}) - \\ & f_p f_R^2 (X_{qs} X_{qr}' - X_{aq}^2)(r_{dr}' V \sin \delta_0 - \\ & f_p^2 f_R^2 X_{ds} X_{dr}' i_{ds0}) + \\ & f_p f_R [r_s r_{qr}' - f_p^2 f_R^2 (X_{qs} X_{qr}' - X_{aq}^2)](X_{dr}' V \cos \delta_0 + \\ & X_{qs} r_{dr}' i_{qs0}) - \\ & f_p f_R^2 r_{qr}' X_{qs} (X_{dr}' V \sin \delta_0 + X_{ds} r_{dr}' i_{ds0}) \end{aligned}$$

$$
\begin{aligned}
u &= r_s^2 r_{qr}' r_{dr}' + f_R^2 r_{qr}' r_{dr}' X_{ds} X_{qs} - f_p^2 f_R^2 r_s r_{dr}' \\
&\quad (X_{qs} X_{qr}' - X_{aq}^2) - \\
&\quad f_p^2 f_R^2 r_s r_{qr}' (X_{ds} X_{dr}' - X_{ad}^2) - f_p^2 f_R^2 \\
&\quad (r_s X_{qr}' + r_{qr}' X_{qs})(r_s X_{dr}' + r_{dr}' X_{ds}) - \\
&\quad f_p^2 f_R^4 (1 - f_p^2)(X_{qs} X_{qr}' - X_{aq}^2)(X_{ds} X_{dr}' - X_{ad}^2) \\
v &= f_p f_R r_s r_{dr}' (r_s X_{qr}' + r_{qr}' X_{qs}) + f_p f_R r_s r_{qr}' \\
&\quad (r_s X_{dr}' + r_{dr}' X_{ds}) + \\
&\quad f_p f_R^3 r_{dr}' X_{ds} (X_{qs} X_{qr}' - X_{aq}^2) + f_p f_R^3 r_{qr}' X_{qs} \\
&\quad (X_{ds} X_{dr}' - X_{ad}^2) - \\
&\quad f_p^3 f_R^3 (r_s X_{dr}' + r_{dr}' X_{ds})(X_{qs} X_{qr}' - X_{aq}^2) - \\
&\quad f_p^3 f_R^3 (r_s X_{qr}' + r_{qr}' X_{qs})(X_{ds} X_{dr}' - X_{ad}^2) \\
w &= r_{qr}' \\
x &= f_p f_R X_{qr}' \\
y &= r_{dr}' \\
z &= f_p f_R X_{dr}'.
\end{aligned}
$$

The expression for $G(j\nu)$ can be written as

$$G(j\nu) = \frac{(1 + jm)(n + jp)}{(u + jv)(w + jx)} + \frac{(q + jr)(s + jt)}{(u + jv)(y + jz)}.$$

It is clear that by employing the conventional rules of complex numbers, the expression for $G(j\nu)$ can be written

$$G(j\nu) = a(\nu) + jb(\nu).$$

Acknowledgment

The digital computer study was performed on the computer at the University of Wisconsin–Milwaukee, Milwaukee. The analog computer study was done at Allis-Chalmers, Milwaukee, Wis.

References

[1] C. Concordia, *Synchronous Machines*. New York: Wiley, 1951.

[2] A. C. Fitzgerald and C. Kingsley, *Electrical Machinery*. 2nd ed. New York: McGraw Hill, 1961.

[3] D. C. White and H. H. Woodson, *Electromechanical Energy Conversion*. New York: Wiley, 1959.

[4] P. C. Krause and C. H. Thomas, "Simulation of symmetrical induction machinery," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-84, pp. 1038–1053, November 1965.

[5] P. C. Krause, "Simulation of unsymmetrical two-phase induction machines," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-84, pp. 1025–1037, November 1965.

[6] R. H. Park, "Two-reaction theory of synchronous machines: II," *Trans. AIEE*, vol. 52, pp. 352–355, June 1933.

[7] G. J. Thaler and R. G. Brown, *Analysis and Design of Feedback Control Systems*. 2nd. ed. New York: McGraw Hill, 1960.

[8] C. Concordia, "Synchronous machine damping and synchronizing torques," *Trans. AIEE (Power Apparatus and Systems)*, vol. 69, pp. 1550–1553, 1950.

[9] ——,"Induction motor damping and synchronizing torques," *Trans. AIEE (Power Apparatus and Systems)*, vol. 71, pp. 364–366, January 1952.

[10] A. S. Aldred and G. Shackshaft, "Frequency response analysis of the stabilizing effect of a synchronous machine damper," *Proc. IEE (London)*, vol. 108, pt. C, pp. 58–63, 1961.

[11] ——,"A frequency response analysis of the stabilizing effect of a synchronous machine damper," *Proc. IEE (London)*, vol. 107, pt. C, pp. 2–10, 1960.

[12] G. J. Rogers, "Linearised analysis of induction-motor transients," *Proc. IEE (London)*, vol. 112, pp. 1917–1926, 1965.

Discussion

Howard E. Jordan (Reliance Electric & Engineering Company, Cleveland, Ohio): The authors are to be commended on a very complete analysis of the subject of inherent stability in reluctance-synchronous motors. They have demonstrated for the first time, to the best of my knowledge, that the reluctance-synchronous machine can be unstable under certain operating conditions.

It seems worthwhile to emphasize a point mentioned in the paper that this analysis covers instability which can occur in a machine excited with balanced, sinusoidal applied voltages from a zero impedance source. In actual practice, increasing numbers of reluctance-synchronous machines are being applied in variable frequency applications with a static frequency converter as the power supply. These converters cannot always be accurately represented as a zero impedance source. Further, the voltage waveforms supplied by static converters are not sinusoidal but are often rich in time harmonics which can produce additional speed fluctuations, particularly during low-frequency operation. These speed oscillations would be superimposed upon the ones described by the authors and could produce additional problems in applying this type of system.

The authors have made an important contribution in isolating and analyzing one aspect of the problem of stable operation of reluctance-synchronous motors at low frequencies. The extension of this analysis to cover the combined effects of inherent instability plus converter source impedance and time harmonics provides a significant area for future study.

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T. A. Lipo and **P. C. Krause**: We wish to thank Dr. Jordan for his interesting discussion of our paper. In this paper we have analyzed what we believe to be one of the two major sources of machine instability at low frequencies, that of inherent instability of the machine itself. Dr. Jordan points out the need for additional study of the second source of instability, that of interaction between the electrical machine and the frequency converting system. This indeed is an important area for future investigation.

In closing, we wish to point out that the effects of saturation have not been included in this paper. Since saturation decreases the effective direct axis magnetizing reactance X_{ad}, it reduces the X_{ad}/X_{aq} ratio. As mentioned in the paper a decrease in saliency has the effect of reducing the region of instability with increasing terminal voltage, as increase in terminal voltage sufficient to saturate the direct axis may result in a net decrease in the instability region.

Manuscript received September 12, 1966.