Stability Analysis for Variable Frequency Operation of Synchronous Machines

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Abstract—A stability study of a synchronous machine is performed by applying the Nyquist stability criterion to the equations which describe the behavior of the machine during small displacements about a steady-state operating point. This investigation reveals that, in some cases, machine instability can occur at low operating speeds. Regions of machine stability are established from the results of a digital computer study and the maximum resonance peak M_m is used as a criterion for determining relative stability when the response of the electromechanical system is damped. The effect on machine stability for changes in different system parameters is given. Also, the maximum resonance peak is used to establish an equivalent second-order electromechanical system. The response of this system is found to compare favorably to that of an idealized synchronous machine at any operating speed.

Introduction

THE ADVENT of the solid-state controlled rectifier has made speed control of ac machines practical. By controlling the frequency of the applied voltages, wide-range variable speed drives have been successfully implemented using induction machines and reluctance-synchronous machines. However, it has been shown that during low-frequency operation of both these devices the electromechanical system is lightly damped and, in the case of the reluctance-synchronous machine, the system may even become unstable.[1],[2]

In some applications where speed must be precisely controlled or where the electromechanical system must respond rapidly to changes in load torque, it may be desirable to employ a synchronous machine. In addition, since the synchronous machine can be conveniently operated at near unity power factor, the commutation problem and the resulting decrease in efficiency is alleviated. Therefore, in future applications where adverse operating environments are not encountered, an increased importance may be given to the synchronous machine.

To investigate the feasibility of the synchronous machine as a variable speed device, it is important to establish a set of equations which permit an analysis of machine stability at any speed. Although studies of free oscillation or instability of the synchronous machine have been conducted, the approximations made in these investigations are not applicable at low frequencies.[3],[4] The performance of the synchronous machine at low operating speeds is the subject of this paper.

Equations describing the behavior of the synchronous machine for small excursions about a steady-state operating point are developed using the method of small displacements. A stability analysis is then performed by employing the Nyquist stability criterion. The influence on the stability of the machine due to changes in machine parameters is summarized. Also, the effect of varying the field excitation and the amplitude of the applied voltages is discussed in some detail. This study reveals that a synchronous machine is lightly damped and may even be unstable at low speeds. However, a well-designed synchronous machine will generally be stable over all regions of operation.

For the regions of stable operation, the maximum resonance peak M_m is used as a measure of relative stability. Also, it is shown that the maximum resonance peak can be used to establish the constants of an equivalent second-order system which approximates the characteristic of an idealized synchronous machine at any frequency for which the machine is stable.

Application of Small Displacement Theory to Synchronous Machines

The equations that describe the dynamic behavior of a synchronous machine are well established. These equations are derived using the following assumptions:

1) Each stator winding is distributed so as to produce a sinusoidal MMF wave along the air gap.

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2) Stator slots produce negligible variations in the rotor inductances.

3) Saturation of the magnetic circuit and eddy currents are neglected.

In the derivation set forth by Park, the stator voltages and currents are transformed to a reference frame fixed in the rotor of the machine.[5] In addition, it is customary to refer the parameters of the rotor windings to the stator windings by a constant proportional to the turns ratio. The resulting voltage equations, with positive current selected as current flowing into the machine, are expressed[6],[12]

$$V_{qs} = \frac{1}{\omega_e}(p\psi_{qs} + \psi_{ds}p\theta_r) + R_sI_{qs} \qquad (1)$$

$$V_{ds} = \frac{1}{\omega_e}(p\psi_{ds} - \psi_{qs}p\theta_r) + R_sI_{ds} \qquad (2)$$

$$V_{fd} = \frac{p}{\omega_e}\psi_{fd} + R_{fd}I_{fd} \qquad (3)$$

$$0 = \frac{p}{\omega_e}\psi_{kq} + R_{kd}I_{kd} \qquad (4)$$

$$0 = \frac{p}{\omega_e}\psi_{kq} + R_{kq}I_{kq} \qquad (5)$$

where

$$\psi_{qs} = X_{ls}I_{qs} + X_{aq}(I_{qs} + I_{kq}) \qquad (6)$$

$$\psi_{ds} = X_{ls}I_{ds} + X_{ad}(I_{ds} + I_{fd} + I_{kd}) \qquad (7)$$

$$\psi_{fd} = X_{lf}I_{fd} + X_{ad}(I_{ds} + I_{fd} + I_{kd}) \qquad (8)$$

$$\psi_{kd} = X_{lkd}I_{kd} + X_{ad}(I_{ds} + I_{fd} + I_{kd}) \qquad (9)$$

$$\psi_{kq} = X_{lkq}I_{kq} + X_{aq}(I_{qs} + I_{kq}) \qquad (10)$$

and where p is the operator d/dt. These relationships can also be interpreted as a per unit set of equations if the original voltage equations are divided by a conveniently selected base voltage. Equations (1)–(10) suggest the equivalent circuit in Fig. 1.

An expression for the instantaneous electromagnetic torque is obtained by employing the principle of arbitrary displacement. The per unit torque can be expressed

$$T_e = \psi_{ds}I_{qs} - \psi_{qs}I_{ds}. \qquad (11)$$

This quantity is positive for motor action.

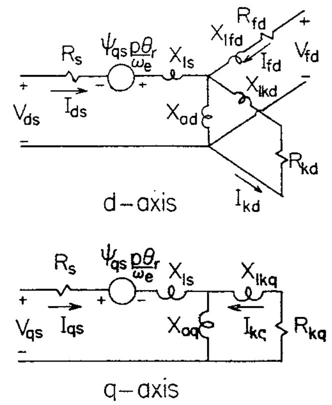

Fig. 1. The d- and q-axis equivalent circuits of a synchronous machine-reference frame fixed in the rotor.

During balanced, steady-state operation the rotor currents in the damper windings I_{kd} and I_{kq} become zero and the field current is constant. In this mode of operation, since the machine rotates at a constant speed, the time rate of change in rotor angle $p\theta_r$ is a constant, ω_{r0}. The equations which describe balanced, steady-state operation are expressed as

$$V_{qs0} = V\cos\delta_0 = f_R(X_{ds}I_{ds0} + X_{ad}I_{fd0}) + R_sI_{qs0} \qquad (12)$$

$$V_{ds0} = V\sin\delta_0 = -f_RX_{qs}I_{qs0} + R_sI_{ds0} \qquad (13)$$

$$V_{fd0} = I_{fd0}R_{fd} \qquad (14)$$

$$T_{e0} = (X_{ds} - X_{qs})I_{ds0}I_{qs0} + X_{ad}I_{qs0}I_{fd0} \qquad (15)$$

where the subscript 0 has been added so as to denote steady-state quantities. The steady-state rotor angle δ_0 and the frequency ratio f_R are defined

$$\delta_0 = \theta_r - \omega_{r0}t \qquad (16)$$

$$f_R = \omega_{r0}/\omega_e. \qquad (17)$$

The rotor angle δ_0 is assumed negative for motor action. The base electrical angular velocity ω_e is a constant selected so as to conveniently calculate per unit reactances. In this study ω_e = 377 rad/s. It is clear that the frequency ratio f_R depends upon the value of the electrical angular velocity of the applied stator voltages and that f_R is unity when this angular velocity equals ω_e.

If all variables are permitted to change by a small amount about an initial operating point, (1)–(5) become, in matrix form

$$\begin{bmatrix} V\cos(\delta_0+\Delta\delta) \\ V\sin(\delta_0+\Delta\delta) \\ 0 \\ V_{fd0} \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + \frac{p}{\omega_e}X_{qs} & X_{ds}\left(f_R + \frac{\Delta\omega_r}{\omega_e}\right) & \frac{p}{\omega_e}X_{aq} & X_{ad}\left(f_R + \frac{\Delta\omega_r}{\omega_e}\right) & X_{ad}\left(f_R + \frac{\Delta\omega_r}{\omega_e}\right) \\ -X_{qs}\left(f_R + \frac{\Delta\omega_r}{\omega_e}\right) & R_s + \frac{p}{\omega_e}X_{ds} & -X_{aq}\left(f_R + \frac{\Delta\omega_r}{\omega_e}\right) & \frac{p}{\omega_e}X_{ad} & \frac{p}{\omega_e}X_{ad} \\ \frac{p}{\omega_e}X_{aq} & 0 & R_{kq} + \frac{p}{\omega_e}X_{kq} & 0 & 0 \\ 0 & \frac{p}{\omega_e}X_{ad} & 0 & R_{fd} + \frac{p}{\omega_e}X_{fd} & \frac{p}{\omega_e}X_{ad} \\ 0 & \frac{p}{\omega_e}X_{ad} & 0 & \frac{p}{\omega_e}X_{ad} & R_{kd} + \frac{p}{\omega_e}X_{kd} \end{bmatrix} \times \begin{bmatrix} I_{qs0}+\Delta I_{qs} \\ I_{ds0}+\Delta I_{ds} \\ \Delta I_{kq} \\ I_{fd0}+\Delta I_{fd} \\ \Delta I_{kd} \end{bmatrix} \qquad (18)$$

where

$$X_{kq} = X_{lkq} + X_{aq} \quad (19)$$

$$X_{kd} = X_{lkd} + X_{ad} \quad (20)$$

$$X_{fd} = X_{lfd} + X_{ad} \quad (21)$$

$$\Delta\omega_r = p\Delta\delta. \quad (22)$$

Using the approximation

$$V \cos(\delta_0 + \Delta\delta) \doteq V \cos \delta_0 - V \sin \delta_0 \, \Delta\delta \quad (23)$$

$$V \sin(\delta_0 + \Delta\delta) \doteq V \sin \delta_0 - V \cos \delta_0 \, \Delta\delta \quad (24)$$

and eliminating the steady-state terms defined by (12)–(15), the previous matrix equation can be solved for the Δ quantities only. The resulting set of linear differential equations is

$$\begin{bmatrix} -\left(V \sin\delta_0\Delta\delta + X_{ds}I_{ds0}\frac{p\Delta\delta}{\omega_e} + X_{ad}I_{fd0}\frac{p\Delta\delta}{\omega_e}\right) \\ V \cos\delta_0\Delta\delta + X_{qs}I_{qs0}\frac{p\Delta\delta}{\omega_e} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + \frac{p}{\omega_e}X_{qs} & f_R X_{ds} & \frac{p}{\omega_e}X_{aq} & f_R X_{ad} & f_R X_{ad} \\ -f_R X_{qs} & R_s + \frac{p}{\omega_e}X_{ds} & -f_R X_{aq} & \frac{p}{\omega_e}X_{ad} & \frac{p}{\omega_e}X_{ad} \\ \frac{p}{\omega_e}X_{aq} & 0 & R_{kq} + \frac{p}{\omega_e}X_{kq} & 0 & 0 \\ 0 & \frac{p}{\omega_e}X_{ad} & 0 & R_{fd} + \frac{p}{\omega_e}X_{fd} & \frac{p}{\omega_e}X_{ad} \\ 0 & \frac{p}{\omega_e}X_{ad} & 0 & \frac{p}{\omega_e}X_{ad} & R_{kd} + \frac{p}{\omega_e}X_{kd} \end{bmatrix} \times \begin{bmatrix} \Delta I_{qs} \\ \Delta I_{ds} \\ \Delta I_{kq} \\ \Delta I_{fd} \\ \Delta I_{kd} \end{bmatrix} \quad (25)$$

The linearized torque equation valid for small variations about an operating point is

$$\Delta T_e = T_e - T_{e0} = (X_{ds} - X_{qs})(I_{ds0}\Delta I_{qs} + I_{qs0}\Delta I_{ds}) + X_{ad}I_{fd0}\Delta I_{qs} + X_{ad}I_{qs0}(\Delta I_{fd} + \Delta I_{kd}) - X_{aq}I_{ds0}\Delta I_{kq}. \quad (26)$$

It is evident from the form of (25) that the three rotor currents can be expressed in terms of the stator currents. If the resulting 2 by 2 matrix is solved for ΔI_{ds} and ΔI_{qs} in terms of $\Delta\delta$ and the results substituted into (26), it is clear that one can obtain an expression which has the form

$$\Delta T_e = G(p)\Delta\delta. \quad (27)$$

The expression for the transfer function $G(p)$ is included in the Appendix. In order to completely describe the behavior of the machine about a steady-state operating point, the dynamics of the mechanical system must be considered. In the steady state, the load torque T_{L0} is equal to the electromagnetic torque T_{e0}. If ΔT_L is a small, positive change from the steady-state load torque then

$$T_{L0} + \Delta T_L = T_{e0} + \Delta T_e - \left(\frac{2H}{\omega_e}\right) p^2\Delta\delta \quad (28)$$

where H is the inertia constant of the synchronous machine and connected load. Eliminating the steady-state terms yields

$$\Delta T_L = \Delta T_e - \left(\frac{2H}{\omega_e}\right) p^2\Delta\delta. \quad (29)$$

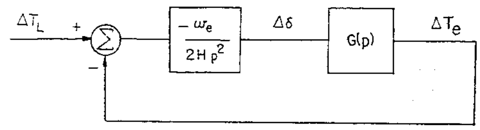

Fig. 2. Small displacement, closed-loop system.

Equations (27) and (29) suggest the block diagram representation shown in Fig. 2. By employing the method of small displacements, the problem of determining synchronous machine stability has been recast in the form of a simple feedback control system. Consequently, the problem of machine stability is amenable to any of the approaches commonly employed in linear feedback control theory. Stability could be established employing the Routh test or the root locus method; however, a Nyquist criterion will be used in this development. It is clear from Fig. 2 that the open-loop transfer function is

$$F(p) = \frac{\omega_e G(p)}{2Hp^2}. \quad (30)$$

The stability of the closed-loop system can be determined by setting $p = j\nu$ and observing the locus of $F(j\nu)$ as ν varies from $-\infty$ to $+\infty$. Since $F(p)$ does not have poles with positive real parts, the closed-loop system is stable if and only if the locus of $F(j\nu)$ does not pass through or encircle the $(-1,0)$ point.[7]

Stability Studies

The equations that have been developed offer a convenient means of predicting the behavior of synchronous machines. A change in frequency (steady-state speed) is taken into account by a change in the value of f_R. Throughout this study, 60 Hz is assumed to be rated frequency and the per unit system employed is based on operation at this frequency. Thus, the frequency ratio f_R can be interpreted as the steady-state operating speed expressed in per unit. By utilizing the equations developed in the previous section, a digital computer can easily be programmed to compute Nyquist contours for the entire range of possible operating conditions in a manner similar to that carried out for the reluctance-synchronous machine.[2] If the particular machine studied is unstable, the region of instability can be readily determined. A region of instability for a 60 Hz, 2-pole, 3-phase synchronous machine is shown in Fig. 3. The per unit parameters of the machine corresponding to Fig. 3, as well as Figs. 4–8, 10, 12–14, are $R_s = 0.09$, $R_{kd} = 0.1$, $R_{kq} = 0.05$, $R_{fd} = 0.01$, $X_{ls} =$

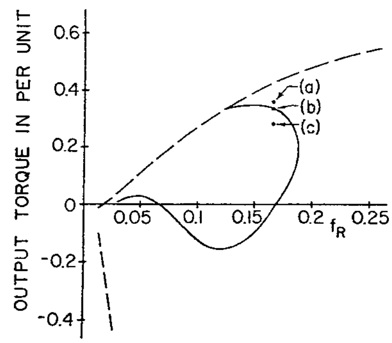

Fig. 3. Region of instability of a synchronous machine.

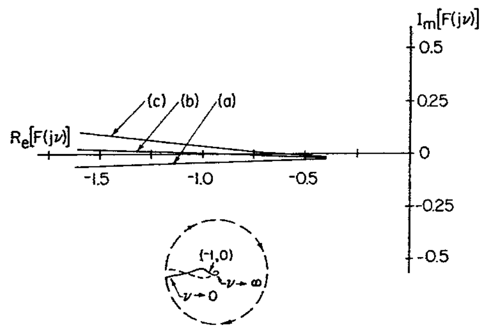

Fig. 4. Nyquist contours–plot of $F(j\nu)$, $f_R = 0.166$.

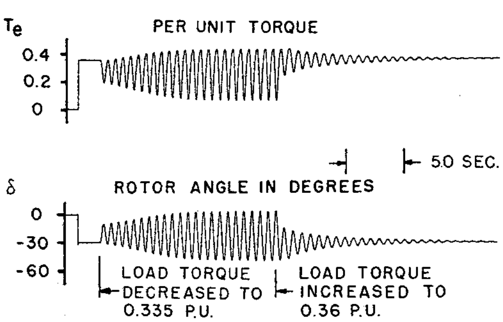

Fig. 5. Load torque switching near region of instability.

0.14, $X_{lkd} = 0.1$, $X_{lkq} = 0.05$, $X_{lfd} = 0.4$, $X_{ad} = 1.5$, $X_{aq} = 0.71$, and $H = 1.0$. In Fig. 3, the amplitude of the applied voltages decreases linearly with frequency. That is to say

$$V = f_R V_m \tag{31}$$

where V_m corresponds to rated voltage (1.0 per unit). The voltage E_I is defined as the per unit exciter voltage and expressed as

$$E_I = \frac{V_{fd0} X_{ad}}{R_{fd}}. \tag{32}$$

With the machine operating at base or unit speed, unit voltage is induced in the stator winding if E_I is unity. In Fig. 3, E_I is fixed at 1.0 per unit. The dashed line shown in Fig. 3 indicates the pull-out or maximum steady-state torque at the various operating speeds. Negative torque output denotes generator action. The closed contour in Fig. 3 forms the boundary between stable and unstable regions of operation. The Nyquist contours sketched in Fig. 4 indicate the significance of this contour. The plots of $F(j\nu)$ in Fig. 4 correspond to initial operating points (a), (b), and (c) shown in Fig. 3. These operating points occur at a machine speed of 0.166 ω_e or $f_R = 0.166$. In this case, the stator applied voltages would vary at 10 Hz. Also, when $T_{e0} = 0.36$ per unit, $\delta_0 = -29.4°$; $T_{e0} = 0.335$ pu, $\delta_0 = -26°$; $T_{e0} = 0.28$ pu, $\delta_0 = -20°$. With $T_{e0} = 0.335$ pu or point (b) on Fig. 3 the locus of $F(j\nu)$ shown in Fig. 4 passes through the $(-1,0)$ point. Hence, point (b) locates a point on the contour shown in Fig. 3. At the operating condition $T_{e0} = 0.28$ pu point (c), the $F(j\nu)$ contour encircles the $(-1,0)$ point and the system is unstable at this condition. With $T_{e0} = 0.36$, point (a), the $F(j\nu)$ plot does not encircle the $(-1,0)$ point and the system is stable.

In order to determine the actual performance of an idealized synchronous machine in this region, the equations which describe the complete behavior of the synchronous machine, (1)–(10), were simulated on the analog computer. The computer traces of the electromagnetic torque T_e and the rotor angle δ during load torque switching are shown in Fig. 5. With the machine initially operating at 0.36 pu torque, the load is suddenly switched to 0.28 pu, held constant at this value, and then switched back to 0.36 pu load torque. This switching corresponds to a change from point (a), the stable operating point shown in Fig. 3, to point (c), an unstable point, and back to point (a). The recordings portray the sustained oscillations which occur in the unstable region, point (c), and the damping at the stable operating point (a).

One might suspect that since, at point (c), the $(-1,0)$ point has been encircled, the system is completely unstable and the oscillations should be unbounded. However, it is recalled that the original derivation progressed upon the assumption of small displacements. As the oscillations become large, the theory of small displacement no longer applies and the differential equations become nonlinear. However, the results of the analog computer study reveal that the magnitude of the oscillation indeed does reach a maximum. When the oscillations become large, the nonlinearity of the differential equations precludes any rigorous analysis. However, this effect can be explained from a quasi-steady-state point of view. As the oscillation increases, operating points are included which provide positive damping. When a condition is reached where the positive damping over one cycle equals the negative damping, then the net damping is zero and the sustained oscillations occur. It was found that at $f_R = 0.15$ the positive damping over a cycle is insufficient to produce a sustained oscillation and the machine pulls out of synchronism for any positive load torque.

The contours shown in Fig. 6 illustrate that stability of this same machine is improved as a result of an increase in the amplitude of the stator applied voltages. In all cases the applied stator voltage V is decreased linearly with f_R. That is $V = f_R V_m$, where the voltage V_m is normally 1.0 per unit. Fig. 6 shows the contours for $V_m = 1.0$, 1.1, 1.2, and 1.3 per unit.

The increase in the region of instability due to an increase in field excitation is shown in Fig. 7. Contours are given for $E_I = 0.75$, 1.0, and 1.25 pu. With an excitation of $E_I = 0.5$ this machine is stable. It is noted that an increase in terminal voltage serves to stabilize the machine whereas an increase in field excitation tends to make the machine less stable.

The stability of several typical synchronous machines was investigated. It was found that most well-designed synchronous machines do not exhibit the type of sustained oscillation shown in Fig. 5. For example, if the field resistance of the synchronous

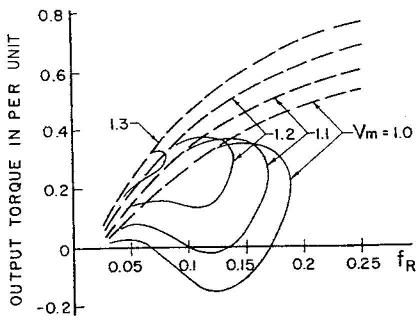

Fig. 6. Regions of instability for changes in the amplitude of the stator applied voltages.

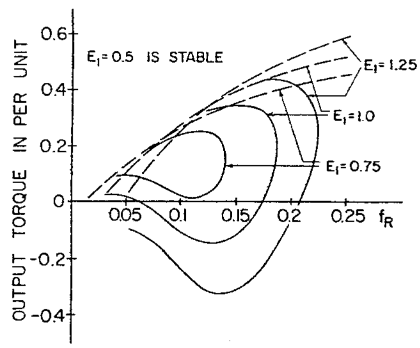

Fig. 7. Regions of instability for changes in field excitation.

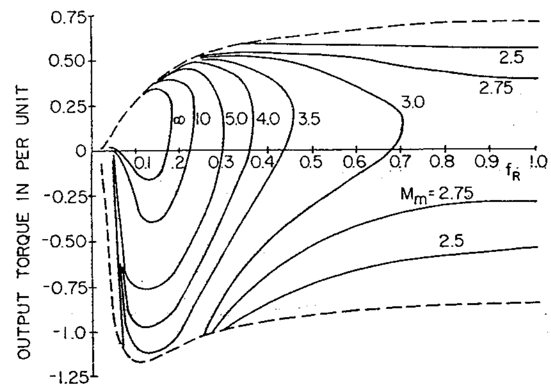

Fig. 8. Contours of constant M_m, $R_f = 0.01$ per unit.

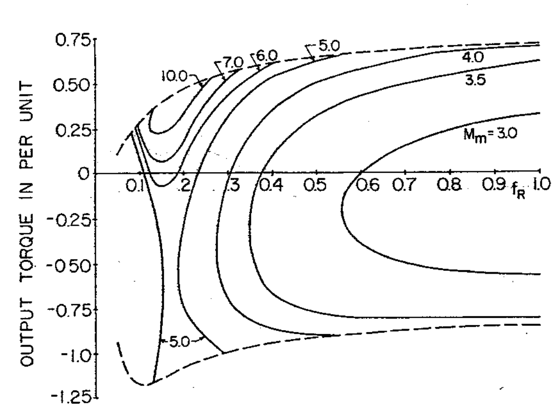

Fig. 9. Contours of constant M_m, $R_f = 0.001$ per unit.

machine corresponding to Fig. 5 is reduced to $R_f = 0.001$ per unit, the machine is lightly damped in some regions of operation but, in fact, stable over all regions.

A method of determining the relative stability of synchronous machines would be of importance in a stability analysis. Although a general performance criterion for systems greater than second-order is not available, the maximum resonance peak M_m of the closed-loop frequency response has been used extensively by control engineers as a measure of the relative stability for higher order systems.[7] The maximum resonance peak M_m is defined as the maximum value of the magnification curve M where M is the magnitude of the closed loop transfer function. That is

$$M = \left| \frac{F j(\nu)}{1 + F(j\nu)} \right|. \quad (33)$$

It is clear that if $F(j\nu)$ passes through the $(-1,0)$ point, the magnification curve becomes infinite and M_m is infinite for this case. A plot of constant M loci superimposed up a rectangular plot of magnitude and phase of the transfer function $F(j\nu)$ form the basis of the Nichols chart which is commonly employed as a means of analyzing a higher-order system.[8] When the system is of second order, the maximum resonance peak is related to the maximum overshoot resulting from a step input into the system.[7] This fact will be discussed in more detail in the next section.

Contours of constant M_m for two different synchronous machines are shown in Figs. 8 and 9. The machine parameters for the machine used in Fig. 8 are the same as those in Fig. 3. The locus of points where $M_m = \infty$ corresponds to the region of instability given in Fig. 3. The parameters of the machine in Fig. 9 are identical to Fig. 8 except that the field resistance R_f has been reduced to 0.001 per unit. Although the machine is stable, the loci of constant M_m indicates clearly the region for which the machine is lightly damped. A decrease of field resistance improves the stability of the machine. Although an investigation of the effects of other machine parameters upon stability was carried out using the previous procedure, it would be impractical to discuss each in detail here. However, a summary of this investigation showing the overall influence upon stability due to changes of the more important parameters is given in Table I. In each case the change considered is about a nominal set of machine parameters corresponding to those used in Fig. 3.

TABLE I

Parameter Increased	Relative Stability Increased	Relative Stability Decreased
E_I		X
V	X	
X_{aq}	X	
R_s		X
X_{ls}	X	
R_{kd}	X	
R_{fd}		X
R_{kq}		X
H		X

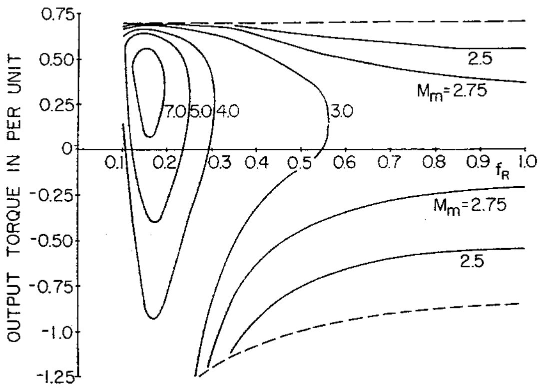

Fig. 10. Contours of constant M_m with terminal voltage adjusted for constant pull-out torque, $R_f = 0.01$ per unit.

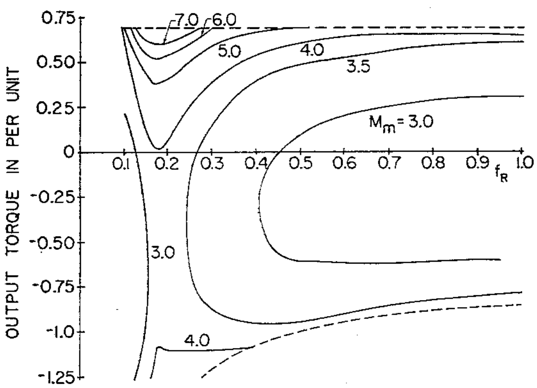

Fig. 11. Contours of constant M_m with terminal voltage adjusted for constant pull-out torque, $R_f = 0.001$ per unit.

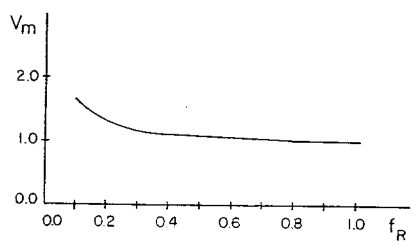

Fig. 12. Change in V_m to maintain a constant pull-out torque.

Another set of M_m loci are plotted in Figs. 10 and 11. The machine parameters are identical to those used in Figs. 8 and 9, respectively; however, in this case the terminal voltages have been adjusted so as to maintain the same positive pull-out torque for all frequencies. This torque corresponds to the maximum possible torque the machine exerts operating as a motor for the condition $V_m = E_I = f_R = 1.0$. The pull-out torque for this condition and for the machine parameters chosen is 0.702 per unit. Again the terminal voltage is adjusted so that $V = f_R V_m$. However, in this case, the quantity V_m varies as shown in Fig. 12 so as to keep the pull-out torque constant. The region below $f_R = 0.1$ in Figs. 10 and 11 is not included since the assumption of no saturation is almost certainly violated below this point. It is noted that the machine is stable for either value of field resistance. The region of least damping is again immediately apparent.

The Synchronous Machine as a Second-Order System

In the previous section, the method of small displacements was employed to determine the stability of a synchronous machine at low frequencies. In the regions where the machine is stable the maximum resonance peak M_m has been utilized as a means of indicating relative stability. In regions of stable operation the machine responds to a step change in load torque in a manner characteristic of an underdamped second-order system. This feature has been recognized for some time and has been used to approximate the behavior of the machine for the study of synchronizing phenomena as well as the analysis of forced oscillations caused by pulsating torques originating external to the machine.[9],[10] However, the study of these problems is accompanied by a series of simplifying approximations which become invalid as the frequency of the applied stator voltages is reduced.

Although the loci of maximum resonance peak M_m has been used in the previous section to indicate the relative stability of a synchronous machine at any frequency, this quantity takes on an added significance if the behavior of the synchronous machine can be approximated by a second-order system. If the machine behaves as a second-order system at all frequencies, the maximum resonance peak can then be employed as a rapid and accurate means of determining the response of the machine to a change in load torque. Since none of the usual simplifying assumptions have been made in the foregoing analysis, this approach is an accurate means of determining the response of the machine and is valid regardless of the frequency of the applied voltages. This assumption is justified by a comparison between the resulting equivalent second-order system and an exact analog computer representation of the machine. Referring to Fig. 10, it is noted that in regions of relatively stable operation the loci of constant M_m at any frequency ratio do not vary rapidly unless the electromagnetic torque of the machine is near its pull-out value. Should the electromagnetic torque be near the pull-out value or approach a region of instability, M_m varies rapidly and the second-order system will no longer approximate the performance of the actual machine. If one selects a nominal value for the maximum resonance peak M_m and the corresponding resonant frequency ω_m, the undamped natural frequency ω_n and the damping ratio ζ of an equivalent second-order system for any operating point is readily found since

$$M_m = \frac{1}{2\zeta\sqrt{1 - \zeta^2}} \tag{34}$$

and

$$\omega_m = \omega_n \sqrt{1 - 2\zeta^2}. \tag{35}$$

The differential equation of the equivalent second-order system is

$$\frac{1}{{\omega_n}^2} \frac{d^2T_e}{dt^2} + \frac{2\zeta}{\omega_n} \frac{dT_e}{dt} + T_e = \Delta T_L(t). \tag{36}$$

The forcing function ΔT_L of the differential equation is the change in load torque from any initial operating condition. Provided the change in load torque ΔT_L is small the response of the synchronous machine to any type of load torque switching is readily determined.

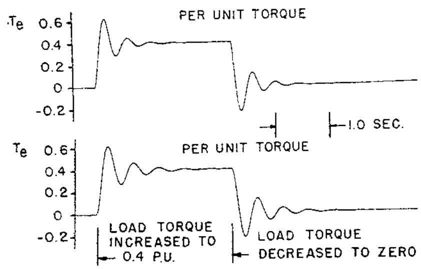

Fig. 13. Response of a synchronous machine to a step change in load torque, $f_R = 1.0$. Top trace—analog computer simulation of synchronous machine. Bottom trace—equivalent second-order system.

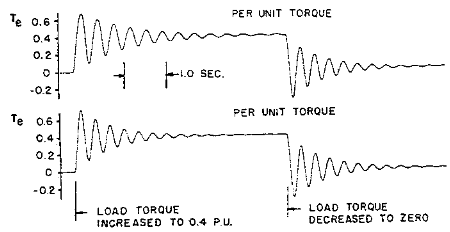

Fig. 14. Response of a synchronous machine to a step change in load torque, $f_R = 0.2$. Top trace—analog computer simulation of synchronous machine. Bottom trace—equivalent second-order system.

An analog computer study was conducted in which the response of the equivalent second-order system was compared to a complete analog computer representation of the synchronous machine.[12] The synchronous machine studied corresponded to that of Fig. 11. In this study, the stator terminal voltage was adjusted so as to maintain a constant pull-out torque. Fig. 13 shows the response of the analog computer simulation of the synchronous machine and the equivalent second-order system to a step change in load torque from zero to 0.4 per unit at a frequency ratio $f_R = 1.0$. In this case, the maximum resonance peak $M_m = 2.84$ and the resonant frequency $\omega_m = 14.06$ rad/s. The corresponding system parameters are $\zeta = 0.179$ and $\omega_n = 13.6$ rad/s.

In Fig. 14 the step change in load torque is repeated for the same machine but at a frequency ratio $f_R = 0.2$. In this case $M_m = 7.6$, $\omega_m = 15.83$ rad/s and the corresponding system parameters are $\zeta = 0.066$, $\omega_n = 15.7$ rad/s.

The response of the second-order system is similar to that of the synchronous machine. In particular, it is interesting to note that although the change in load torque and hence the resulting torque angle change is appreciable, the second-order system obtained by assuming only infinitesimal changes in torque angle is still an acceptable model of the synchronous machine. A determination of the exact extent to which this result is valid and the means of predicting the error in such an assumption would necessitate an investigation beyond the scope of this paper. It is clear that since the property of steady-state stability has not been incorporated into the second-order system approximation, the model becomes invalid as the pull-out torque is approached.

Conclusion

A linearized set of equations which describe the behavior of a synchronous machine for small changes about an equilibrium point have been derived and a transfer function developed which characterizes the behavior of the electromechanical system at any speed. By applying Nyquist's stability criterion to this transfer function, it was found that the synchronous machine may in some cases demonstrate instability at low operating speeds. Regions of instability predicted by Nyquist's stability criterion were obtained from a digital computer study. Results of an analog computer study correlate with those obtained from the digital computer and provide a means of interpreting the modes of operation which occur near these regions of instability.

In cases where the machine is stable, the maximum resonance peak M_m was employed as a criterion for relative stability. It was also shown that the maximum resonance peak is an accurate means of calculating the constants of an equivalent second-order system which closely approximates the electromechanical response of an idealized synchronous machine.

Appendix

$$Z_1 = R_s + \frac{p}{\omega_e} X_{qs} - \frac{\left(\frac{p}{\omega_e}\right)^2 X_{aq}^2}{R_{kq} + \frac{p}{\omega_e} X_{kq}}$$

$$Z_2 = f_R X_{ds} - \frac{\frac{p}{\omega_e} f_R X_{ad}^2 \left[R_{fd} + R_{kd} + \frac{p}{\omega_e}(X_{fd} + X_{kd} - 2X_{ad}) \right]}{\left\{ \left[R_{kd} + \frac{p}{\omega_e}(X_{kd} - X_{ad}) \right] \left[R_{fd} + \frac{p}{\omega_e} X_{fd} \right] + \left[R_{fd} + \frac{p}{\omega_e}(X_{fd} - X_{ad}) \right] \frac{p}{\omega_e} X_{ad} \right\}}$$

$$Z_3 = -f_R X_{qs} + \frac{\frac{p}{\omega_e} f_R X_{aq}^2}{R_{kq} + \frac{p}{\omega_e} X_{kq}}$$

$$Z_3 = R_s + \frac{p}{\omega_e} X_{ds} - \frac{\left(\frac{p}{\omega_e}\right)^2 X_{ad}^2 \left[R_{fd} + R_{kd} + \frac{p}{\omega_e}(X_{fd} + X_{kd} - 2X_{ad}) \right]}{\left\{ \left[R_{kd} + \frac{p}{\omega_e}(X_{kd} - X_{ad}) \right] \left[R_{fd} + \frac{p}{\omega_e} X_{fd} \right] + \left[R_{fd} + \frac{p}{\omega_e}(X_{fd} - X_{ad}) \right] \frac{p}{\omega_e} X_{ad} \right\}}$$

$$V_1 = -\left[V \sin \delta_0 + \frac{p}{\omega_e}(X_{ds} I_{ds0} + X_{ad} I_{fd0}) \right]$$

$$V_2 = V \cos \delta_0 + \frac{p}{\omega_e} X_{qs} I_{qs0}$$

$$V_3 = (X_{ds} - X_{qs}) I_{ds0} + X_{ad} I_{fd0} + \frac{\frac{p}{\omega_e} X_{aq}^2 I_{ds0}}{R_{kq} + \frac{p}{\omega_e} X_{kq}}$$

$$V_4 = (X_{ds} - X_{qs})I_{qs0} + X_{ad}I_{qs0} - \frac{\frac{p}{\omega_e} X_{ad}{}^2 I_{qs0}\left[R_{fd} + R_{kd} + \frac{p}{\omega_e}(X_{fd} + X_{kd} - 2X_{ad})\right]}{R_{kd}R_{fd} + \frac{p}{\omega_e}(R_{kd}X_{fd} + R_{fd}X_{kd}) + \left(\frac{p}{\omega_e}\right)^2 (X_{fd}X_{kd} - X_{ad}{}^2)}.$$

The expression for $G(p)$ can be written as

$$G(p) = \frac{(V_1Z_4 - V_2Z_2)V_3}{Z_1Z_4 - Z_2Z_3} + \frac{(V_2Z_1 - V_1Z_3)V_4}{Z_1Z_4 - Z_2Z_3}.$$

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