Analysis and Simplified Representations of a Rectifier-Inverter Induction Motor Drive

PAUL C. KRAUSE, SENIOR MEMBER, IEEE, AND THOMAS A. LIPO, MEMBER, IEEE

Abstract—**Simplified representations of a rectifier-inverter induction motor drive system are established and verified by comparing the results obtained from a computer study using these representations to those obtained using a detailed simulation of the system. It is shown that when all harmonic components are neglected the static drive system may be conveniently represented in the synchronously rotating reference frame. The computer simulation resulting from this type of representation can be readily implemented, and in many cases it will predict the system performance with sufficient accuracy. Also, in the analysis leading to these simplified representations, the operation of the inverter is analytically expressed in the synchronously rotating reference frame with the harmonic components due to the switching in the inverter included. These equations of transformation may be used to advantage in describing the interaction between the filter and the induction motor.**

INTRODUCTION

CYCLOCONVERTERS, inverters, and rectifier-inverter systems are being used with ac machines in an increasing number of variable speed applications. There are many aspects of these static drive systems which offer interesting and challenging problems. However, most variable frequency drive systems are quite complex, and it is often difficult to predict their dynamic performance without the aid of a computer. Consequently, the design of control systems associated with static drive systems may be facilitated by appropriate use of either an analog or a digital computer.

In earlier publications the analog computer simulation of a rectifier-inverter induction motor drive system was developed and verified [1], [2]. This simulation was recently used to verify the stability analysis of the static drive system at low frequencies [3]. However, this analog computer simulation is involved and requires a substantial amount of computing equipment. Therefore, it is desirable to establish simplified representations which yield a more direct means of simulating the drive system while maintaining the salient features which determine its performance. The analytical development and the verification of simplified representations of the rectifier-inverter induction motor drive system is the subject of this paper.

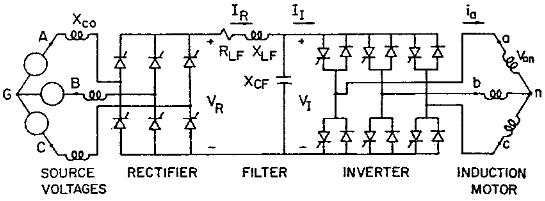


Fig. 1. System studied.

The first simplified system representation is developed by neglecting the harmonic components due to the switching in the rectifier. Next, the operation of the inverter and induction motor is expressed in a reference frame which rotates in synchronism with the fundamental component of the stator applied voltages. This analysis yields equations of transformation for the inverter and establishes an equivalent circuit which describes the static drive system in a synchronously rotating reference frame with only the harmonic components due to the switching in the rectifier neglected. This equivalent circuit conveniently describes the interaction between the filter and the motor. Moreover, a markedly simplified representation results from this equivalent circuit if the harmonic components due to the switching in the inverter are also neglected.

The analysis set forth in this paper yields two simplified system representations which may be simulated on the analog computer more directly than the detailed computer simulation of the complete system. As mentioned, the first of these result from the system representation wherein the harmonic components due to rectifier switching are neglected. The second is obtained by neglecting all harmonic components and employing the representation of the system in the synchronously rotating reference frame. The validity and limitations of each of these simplified representations are established by comparing the results obtained from the analog computer simulation of these representations with those obtained from the detailed analog computer simulations of the complete system.

SYSTEM DESCRIPTION AND BASIC EQUATIONS

A simplified diagram of the rectifier-inverter drive system is given in Fig. 1. Similar systems were studied in two previous papers and in a companion paper [1], [3], [4]. This system consists of a three-phase power source, a six-phase rectifier, a filter, an inverter, and a three-phase symmetrical induction machine. Although there are various converter configurations and control systems being employed, the system shown in Fig. 1 forms the basis of many of the present-day rectifier-inverter drive systems.

The equations which describe the symmetrical induction machine in the arbitrary reference frame may be expressed [5]–[7] as follows:

Paper 68 TP 640-PWR, recommended and approved by the Rotating Machinery Committee of the IEEE Power Group for presentation at the IEEE Summer Power Meeting, Chicago, Ill., June 23–28, 1968. Manuscript submitted February 12, 1968; made available for printing May 6, 1968. This work was supported by the U. S. Army Mobility Equipment Research and Development Center, Fort Belvoir, Va., under Contract DAAK02-67-C-0073.

P. C. Krause is with the Department of Electrical Engineering, University of Wisconsin, Madison, Wis. 53706.

T. A. Lipo was with the Department of Electrical Engineering, University of Wisconsin, Madison, Wis. He is now with the Department of Electrical and Electronic Engineering, University of Manchester Institute of Science Technology, Manchester, England.

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr}' \\ v_{dr}' \end{bmatrix} = \begin{bmatrix} r_s + (p/\omega_b)X_s & (\omega/\omega_b)X_s & (p/\omega_b)X_m & (\omega/\omega_b)X_m \\ -(\omega/\omega_b)X_s & r_s + (p/\omega_b)X_s & -(\omega/\omega_b)X_m & (p/\omega_b)X_m \\ (p/\omega_b)X_m & (\omega - \omega_r/\omega_b)X_m & r_r' + (p/\omega_b)X_r' & (\omega - \omega_r/\omega_b)X_r' \\ -(\omega - \omega_r/\omega_b)X_m & (p/\omega_b)X_m & -(\omega - \omega_r/\omega_b)X_r' & r_r' + (p/\omega_b)X_r' \end{bmatrix} \times \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr}' \\ i_{dr}' \end{bmatrix} \tag{1}$$

where

$$X_s = X_{ls} + X_m \tag{2}$$

$$X_r' = X_{lr}' + X_m. \tag{3}$$

The electromagnetic torque, expressed positive for motor action, is

$$T = \left(\frac{n}{2}\right)\left(\frac{P}{2}\right)\left(\frac{X_m}{\omega_b}\right)(i_{qs}i_{dr}' - i_{ds}i_{qr}') \tag{4}$$

where

p operator d/dt
r_s stator resistance
r_r' rotor resistance (referred to the stator winding)
X_{ls} stator leakage reactance
X_{lr}' rotor leakage reactance
n number of phases
P number of poles.

The base electrical angular velocity ω_b is introduced in the machine equations for the purpose of converting inductances to inductive reactances whereupon a per unit system may be conveniently employed. It is clear that $v_{qr}' = v_{dr}' = 0$ if the machine is singly fed.

In (1) the electrical angular velocity of the arbitrary rotating reference frame is denoted as ω. If it is desirable to express the equations of the induction machine in a stationary reference frame, ω is set equal to zero. For a reference frame fixed in the rotor ω is set equal to the electrical angular velocity of the rotor ω_r. The equations in the synchronously rotating reference frame are obtained by setting ω equal to the electrical angular velocity of the fundamental frequency components of the applied stator voltages herein denoted as ω_e.

The notation employed in this paper differs slightly from that used in earlier publications involving induction machines [5], [8]. In these papers, as in this paper, ω_e is used to denote the speed of the synchronously rotating reference frame which corresponds to the frequency of the stator applied voltages. During constant rated frequency operation ω_e would generally be equal to ω_b. However, during variable frequency operation ω_e varies with the frequency of the applied voltages. The notation employed in [5] was selected without consideration for variable frequency operation. That is, ω_e was used in the induction machine equations to denote the constant base electrical angular velocity (herein denoted as ω_b), and it was also used to denote the speed of the synchronously rotating reference frame.

In the discussion of Jordan's paper, it is demonstrated that if the applied stator voltages form a balanced-set variable-frequency operation can be simulated in the synchronously rotating reference frame by continuously changing the speed of the reference frame to correspond to the frequency of the applied voltages [8]. The previous use of ω_e as a constant to convert an inductance to an inductive reactance is misleading during variable frequency operation. However, this situation may be easily resolved if all ω_e appearing in (54)–(68), (79), (80), and (103)–(126), as well as in the computer representation shown in Fig. 6 of [5] are changed to ω_b. The computer representation of the symmetrical machine in the arbitrary reference frame convenient for variable-frequency applications is given in Fig. 2. Therein

$$\psi_{qs} = X_{ls}i_{qs} + \psi_{mq} \tag{5}$$

$$\psi_{ds} = X_{ls}i_{ds} + \psi_{md} \tag{6}$$

$$\psi_{qr}' = X_{lr}'i_{qr}' + \psi_{mq} \tag{7}$$

$$\psi_{dr}' = X_{lr}'i_{dr}' + \psi_{md} \tag{8}$$

$$\psi_{mq} = X_{mq}\left(\frac{\psi_{qs}}{X_{ls}} + \frac{\psi_{qr}'}{X_{lr}'}\right) \tag{9}$$

$$\psi_{md} = X_{md}\left(\frac{\psi_{ds}}{X_{ls}} + \frac{\psi_{dr}'}{X_{lr}'}\right) \tag{10}$$

$$X_{ls} = \omega_b L_{ls}, \text{ etc.} \tag{11}$$

$$X_{mq} = X_{md} = \frac{1}{1/X_m + 1/X_{ls} + 1/X_{lr}'}. \tag{12}$$

The torque is expressed

$$T = \left(\frac{n}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)(\psi_{qr}'i_{dr}' - \psi_{dr}'i_{qr}'). \tag{13}$$

Hereafter, ω_e will be reserved to denote only the electrical angular velocity of the synchronously rotating reference frame.

The voltage equation for the filter during continuous operation ($I_R > 0$) may be written

$$V_R = V_I + [(p/\omega_b)X_{LF} + R_{LF}]I_R \tag{14}$$

$$V_I = (\omega_b/p)X_{CF}(I_R - I_I). \tag{15}$$

It is clear that the rectifier current I_R cannot be negative. When V_I exceeds V_R and I_R is forced to zero, V_R becomes equal to V_I, that is, when

$$I_R = 0 \tag{16}$$

$$V_R = V_I. \tag{17}$$

A detailed analog computer simulation of the complete system shown in Fig. 1 is described in [1] and [2], and results obtained from a computer study are compared with test results. This detailed simulation will not be repeated. However, the computer results obtained using this simulation will be compared with those obtained using the simplified representations presented in the following sections.

In regard to the development of the detailed computer simulation, it is important to mention that C. H. Thomas, G. E. Gareis, and R. A. Hedin were instrumental in the development of the simulation of the rectifier. Also L. T. Woloszyk contributed to the simulation of the rectifier-filter-inverter combination [1], [2].

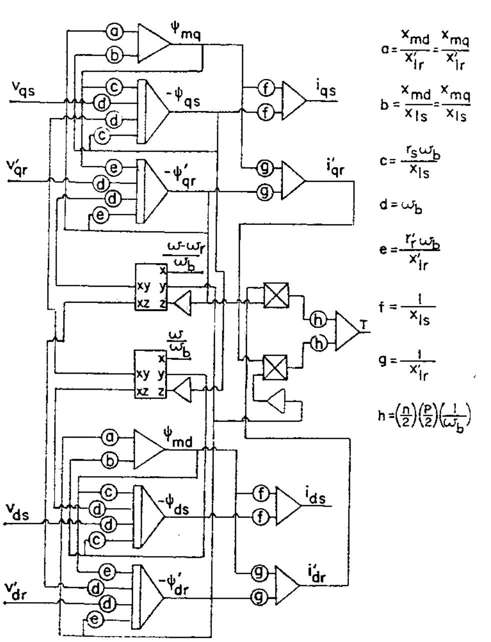


Fig. 2. Computer representation of symmetrical induction machine; arbitrary reference frame.

SIMPLIFIED REPRESENTATION OF RECTIFIER WITH MACHINE SIMULATED IN STATIONARY REFERENCE FRAME

The complete computer simulation of the static drive system is rather involved and requires a substantial amount of computing equipment [1], [2]. However, by employing a functional representation for the rectifier the complexity of the simulation may be reduced while retaining many of the salient features of the actual system. A suitable functional representation of the rectifier was reported in a recent paper [9]. In particular, an equation which expresses the average output voltage of a six-phase converter in a d–q axis is set forth and verified. This relationship was established by substituting equations of transformation into the equations which describe the operation of the converter. If the commutating reactance X_{co} is small compared to the filter reactance X_{LF}, the average output voltage V_R may be expressed as follows:

$$V_R = (3\sqrt{3}/\pi)V_S \cos\alpha - (3/\pi)X_{co}I_R \tag{18}$$

where V_S is the magnitude of the line-to-neutral source voltage and α the delay angle. It is clear that (18) is the familiar expression for the average output voltage of a six-phase converter. However, a slightly modified interpretation is possible. That is, if the ac voltages at the rectifier are appropriately transformed to a synchronously rotating reference frame which corresponds to the frequency of these voltages, the q-axis voltage is equal to V_S while the d-axis voltage is maintained at zero. This transformation can be readily simulated if the source voltages (e_{GA}, e_{GB}, and e_{GC}) are independent of load current. However, if it is necessary to include source impedance other than the commutating reactance, the simulation is more involved [9]. Source impedances other than the commutating reactance will not be considered in this paper.

The control system which establishes the delay angle α may also be incorporated using techniques described in [9]. Consequently (18) describes the average rectifier voltage which may be related to the reference frame which rotates in synchronism with the electrical angular velocity of the source voltages e_{GA}, e_{GB}, and e_{GC}. It is clear that all variables (voltages and currents) of the dc system (filter) may be interpreted directly, without transformation.

In this simulation the inverter is represented as described in [1] and [2]. In particular, the operating frequency of the inverter is established from a variable-frequency sine–cosine oscillator. This oscillator is used to establish a three-phase set of voltage signals displaced 120 electrical degrees. These voltage signals are then used to operate three comparator relays which simulate the switching of the machine terminals to the appropriate capacitor terminal. Commutating circuitry is not included in the simulation of the inverter, that is, it is assumed that commutation occurs instantaneously. In this type of static drive system, the applied phase voltages of the induction machine will be of stepped waveform. When the applied voltages of the induction machine are not a balanced sinusoidal three-phase set, it is desirable to represent the machine in the stationary reference frame ($\omega = 0$) [5].

In summary, this method of simulation differs from the complete simulation of the system in that the rectifier is replaced by a functional representation. The filter, inverter, and the induction machine are simulated using the same techniques employed in the complete simulation [1], [2].

REPRESENTATION IN THE SYNCHRONOUSLY ROTATING REFERENCE FRAME

The stepped voltages applied to the stator phases of the induction machine may be approximated by Fourier series expansions. For example, during normal balanced operation, phase A of the three-phase set of stepped voltages may be expressed [6] as follows:

$$v_{as} = (2V_I/\pi)(\cos\omega_e t + \tfrac{1}{5}\cos 5\omega_e t - \tfrac{1}{7}\cos 7\omega_e t - \cdots). \tag{19}$$

It is clear that during normal operation the stator-phase voltages may be considered as a series of three-phase sets formed by the fundamental and the 5th, 7th, 11th, 13th, ... harmonic components. Moreover, the amplitude of each of these balanced three-phase sets is determined by the instantaneous value of the capacitor voltage V_I. That is, the fundamental and the harmonic components of the phase voltages each may be considered as sinusoidal functions modulated by the instantaneous value of V_I.

If the stepped-phase voltages are transformed to a reference frame which rotates in synchronism with the fundamental frequency of the applied stator voltages v_{qs}^e and v_{ds}^e become [3], [6]

$$v_{qs}^e = (2V_I/\pi)(1 + \tfrac{2}{35}\cos 6\omega_e t - \tfrac{2}{143}\cos 12\omega_e t + \cdots) \tag{20}$$

$$v_{ds}^e = (2V_I/\pi)(\tfrac{12}{35}\sin 6\omega_e t - \tfrac{24}{143}\sin 12\omega_e t + \cdots). \tag{21}$$

In the above equations the angular relationship between the q axis and the magnetic axes of the stator and rotor phases has been selected so that these axes coincide at time zero [5]–[7]. It is clear that the speed of the synchronously rotating reference frame ω_e is determined by the frequency of the fundamental component of the applied voltages which in turn is determined by the frequency at which switching (commutation) is caused to occur in the inverter.

If it assumed that there is no power loss in the inverter,

$$V_I I_I = \tfrac{3}{2}(v_{qs}^e i_{qs}^e + v_{ds}^e i_{ds}^e). \tag{22}$$

If (20) and (21) are substituted into (22), the inverter current may be expressed as follows:

$$\begin{aligned} I_I = {} & (3/\pi) i_{qs}^e (1 + \tfrac{2}{35}\cos 6\omega_e t - \tfrac{2}{143}\cos 12\omega_e t + \cdots) \\ & + (3/\pi) i_{ds}^e (\tfrac{12}{35}\sin 6\omega_e t - \tfrac{24}{143}\sin 12\omega_e t + \cdots). \end{aligned} \tag{23}$$

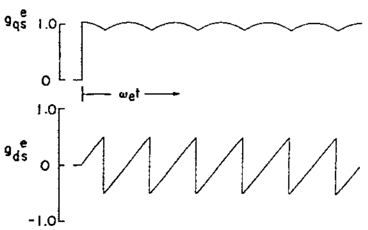


Fig. 3. g_{qs}^e and g_{ds}^e.

It is convenient to define

$$g_{qs}^e = 1 + \tfrac{2}{35}\cos 6\omega_e t - \tfrac{2}{143}\cos 12\omega_e t + \cdots \tag{24}$$

$$g_{ds}^e = \tfrac{12}{35}\sin 6\omega_e t - \tfrac{24}{143}\sin 12\omega_e t + \cdots. \tag{25}$$

Equations (20), (21), and (23) may now be written

$$v_{qs}^e = (2/\pi)V_I g_{qs}^e \tag{26}$$

$$v_{ds}^e = (2/\pi)V_I g_{ds}^e \tag{27}$$

$$I_I = (3/\pi)(i_{qs}^e g_{qs}^e + i_{ds}^e g_{ds}^e). \tag{28}$$

The above expressions of g_{qs}^e and g_{ds}^e are Fourier series expansions of the functions shown in Fig. 3. The maximum value of g_{qs}^e and g_{ds}^e is $\pi/3$ and $\pi/6$, respectively. It is interesting to note that the waveform of g_{qs}^e is analogous to the output of an ideal six-phase rectifier which is operating without phase delay and with zero commutating reactance. Similarly, the waveform of g_{ds}^e is analogous to the negative of the continuous output of an ideal six-phase rectifier with a 90-degree phase delay and with zero commutating reactance.

It is desirable to incorporate the following substitute quantities which, in effect, refers the variables to the stator winding of the induction machine.

$$V_I' = (2/\pi)V_I \tag{29}$$

$$I_I' = (\pi/3)I_I \tag{30}$$

$$R_{LF}' = (6/\pi^2)R_{LF} \tag{31}$$

$$X_{LF}' = (6/\pi^2)X_{LF} \tag{32}$$

$$X_{CF}' = (6/\pi^2)X_{CF}. \tag{33}$$

Substituting (29) and (30) into (22) yields

$$\tfrac{3}{2}V_I'I_I' = \tfrac{3}{2}(v_{qs}^e i_{qs}^e + v_{ds}^e i_{ds}^e). \tag{34}$$

The following substitute variables are introduced for the purpose of referring the rectifier variables to the stator winding of the induction machine:

$$V_R' = (2/\pi)V_R \tag{35}$$

$$I_R' = (\pi/3)I_R. \tag{36}$$

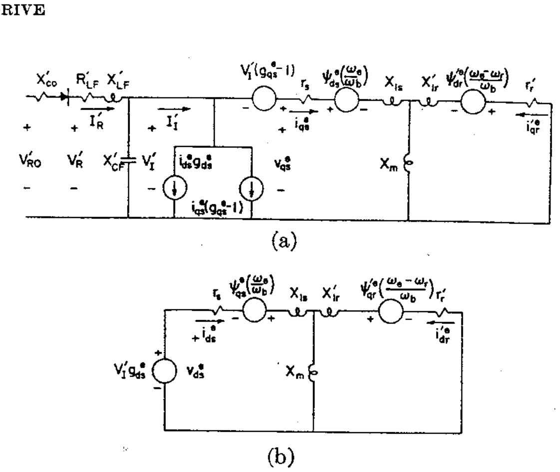
Thus (14) and (15) may be written

$$V_R' = V_I' + [(p/\omega_b)X_{LF}' + R_{LF}']I_R' \tag{37}$$

$$V_I' = (\omega_b/p)X_{CF}'(I_R' - I_I'). \tag{38}$$

The simplified representation for the rectifier developed in the previous section is also used to describe the operation of the rectifier in the representation developed in this section. With the appropriate substitute variables introduced, (18) may be expressed

$$V_R' = V_{RO}' - X_{co}'I_R' \tag{39}$$



(a)

(b)

Fig. 4. Equivalent circuit in synchronously rotating reference frame with harmonic components due to rectifier switching neglected. (a) q axis. (b) d axis.

where

$$V_{RO}' = (6\sqrt{3}/\pi^2)V_S\cos\alpha \tag{40}$$

$$X_{co}' = (18/\pi^3)X_{co}. \tag{41}$$

Incorporating (24)–(41) with the equation which describes the induction machine in the synchronously rotating reference frame yields the equivalent circuit shown in Fig. 4. This equivalent circuit describes the operation of the rectifier-inverter induction motor drive system in the synchronously rotating reference frame with the harmonic components due to the rectifier switching neglected. The electrical angular velocity of the synchronously rotating reference frame ω_e corresponds to the frequency of the fundamental component of the applied voltages which is determined by the frequency at which switching occurs in the inverter. Thus changing ω_e in Fig. 4 corresponds to changing the frequency at which switching occurs in the inverter. In Fig. 4 it is clear that

$$\psi_{ds}^e = X_{ls}i_{ds}^e + X_m(i_{ds}^e + i_{dr}'^e)\cdots. \tag{42}$$

Although the equivalent circuit shown in Fig. 4 is of importance in that it permits one to describe conveniently the interaction between the filter and the motor, it does not yield a computer simulation which is more convenient or more readily implemented than the one described in the preceding section. If, however, the harmonic components due to the switching of the inverter are neglected, the equivalent circuit and thus the computer representation are markedly simplified. That is, if

$$g_{qs}^e = 1 \tag{43}$$

$$g_{ds}^e = 0 \tag{44}$$

then

$$v_{qs}^e = V_I' \tag{45}$$

$$v_{ds}^e = 0 \tag{46}$$

$$I_I' = i_{qs}^e. \tag{47}$$

Equations (45)–(47) describe the operation of an idealized inverter which supplies the induction machine with only a fundamental set of balanced three-phase variable-frequency voltages. Therefore, if the speed of the synchronously rotating reference frame appearing in the equations for the induction machine is changed to correspond to the frequency of this balanced three-phase set ($\omega = \omega_e$), the applied voltages in this synchronously rotating reference frame are related directly to the capacitor voltage.

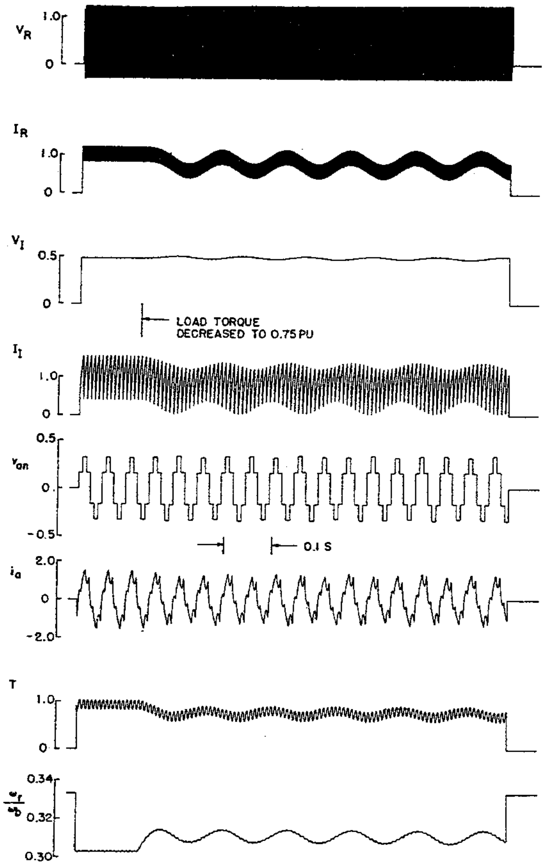


Fig. 5. Load torque switching from 0.925 to 0.75 pu; operation at 20 Hz. Detailed system representation.

It is clear, however, that in general both i_{qs}^e and i_{ds}^e will be made up of a constant and a series of harmonic components. Due to the selection of the time-zero position of the axes, the constant component of i_{ds}^e corresponds to the magnitude of the magnetizing or reactive component of the fundamental phase current. It is important to note that the current i_{ds}^e appears only as the coefficient of the 6th, 12th, ... harmonics in the expression for the inverter current (23). Thus the fundamental magnetizing current is supplied to the machine by the harmonic components (predominantly the 6th harmonic) of the inverter current. Therefore, if the harmonic components are neglected, as in (43)–(47), the effect of the magnetizing current flowing in the machine is not included in the inverter current. Consequently, neglecting all harmonic components may, at first, appear as an invalid means of approximating the performance of this static drive system. However, since the electromechanical performance of this system is determined primarily by the real power transferred through the inverter rather than the reactive power exchange, many of the dominant performance characteristics are preserved even though all harmonics are neglected.

COMPARISONS OF SYSTEM REPRESENTATIONS—COMPUTER STUDY

In order to investigate the validity of two of the simplified representations set forth in the previous sections, the results obtained from the analog computer simulations of these representations of the static drive system are compared to those obtained from the detailed analog computer simulation of the complete system. The simplified representations considered herein are 1) the representation where only the switching of the

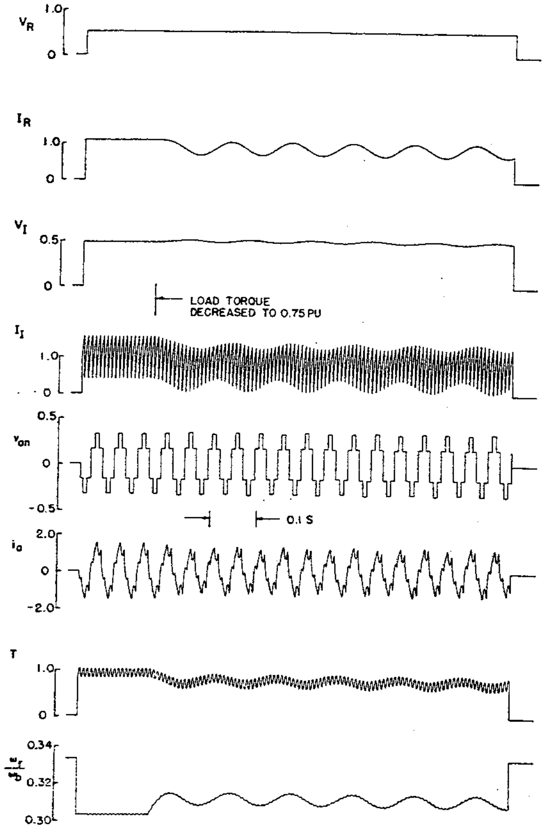


Fig. 6. Load torque switching from 0.925 to 0.75 pu; operation at 20 Hz. Functional representation of rectifier.

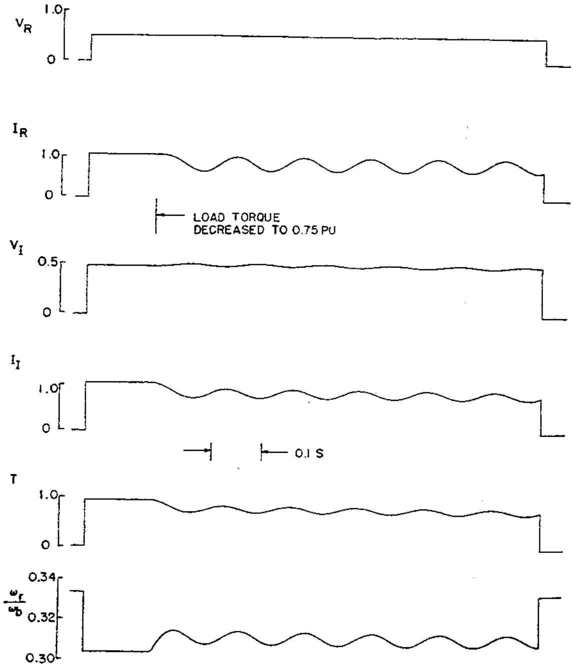


Fig. 7. Load torque switching from 0.925 to 0.75 pu; operation at 20 Hz. System represented in synchronously rotating reference frame with harmonics neglected.

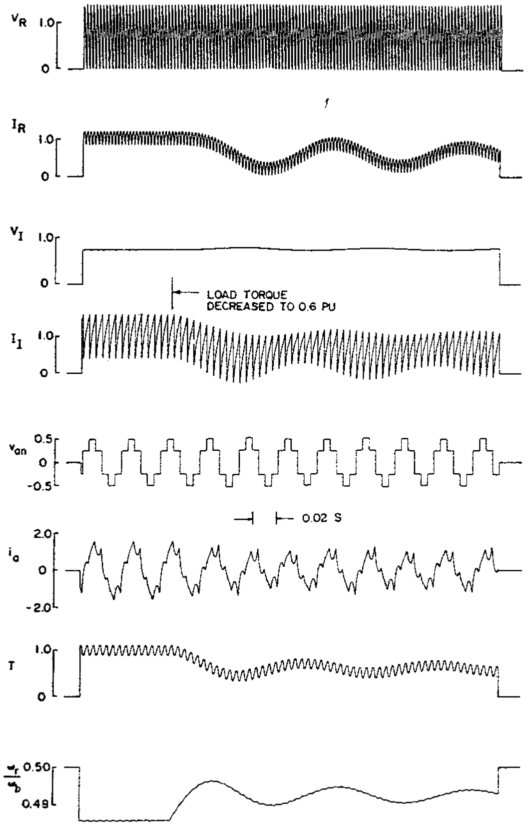


Fig. 8. Load torque switching from 1.0 to 0.6 pu; operation at 30 Hz. Detailed system representation.

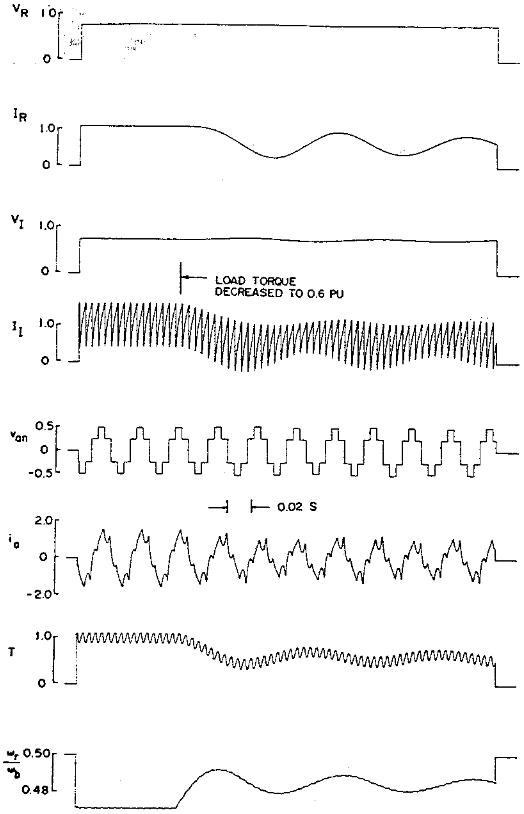


Fig. 9. Load torque switching from 1.0 to 0.6 pu; operation at 30 Hz. Functional representation of rectifier.

rectifier is neglected (18) while the inverter and the induction motor are represented in the stationary reference frame as in the detailed simulation, and 2) the representation of the static drive system in the synchronously rotating reference frame where the harmonic components due to the rectifier and the inverter are neglected (18) and (45)–(47).

The per unit parameters of the induction motor, filter, and commutating reactance are

$$X_{co} = 0.016 \quad R_{LF} = 0.025 \quad r_s = 0.025$$

$$X_{CF} = 0.0141 \quad X_{LF} = 0.5 \quad X_s = 2.075$$

$$r_r' = 0.020 \quad X_m = 2.0$$

$$X_r' = 2.075 \quad H = 0.2 \text{ s}.$$

The above parameters are based on a 7.5-hp induction motor having a base impedance of 9.45 ohms. The base frequency ω_b is assumed to be the rated frequency of the induction motor which is 60 Hz. In variable-frequency systems the amplitude of the applied stator voltages is decreased, in some manner, as frequency decreases. In the studies reported in this paper the amplitude of the fundamental component of the open-circuit inverter voltage is decreased linearly with frequency with 1.0-pu voltage occurring at 60 Hz [3]. In a previous paper it was shown that with the parameters and voltage–frequency relation given above, the static drive system will become unstable during operation at low frequencies [3]. For example, at 20 Hz the system is unstable if the load torque is equal to or less than 0.75 pu.

The computer traces given in Figs. 5, 6, and 7 show the system performance when the load torque is changed from a

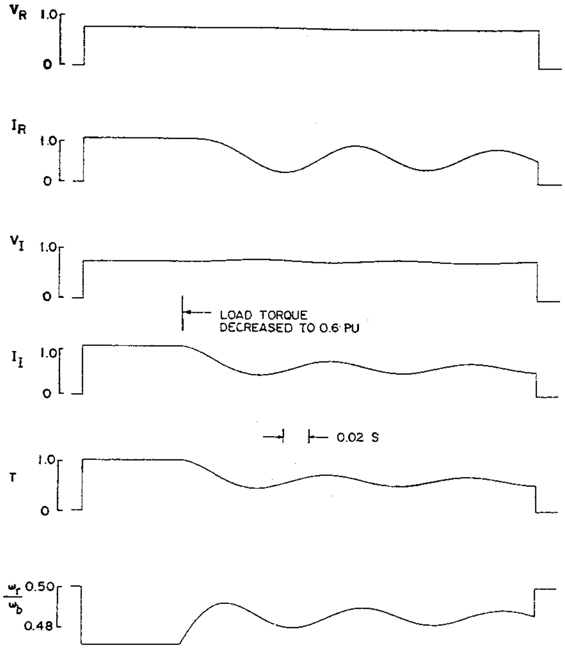
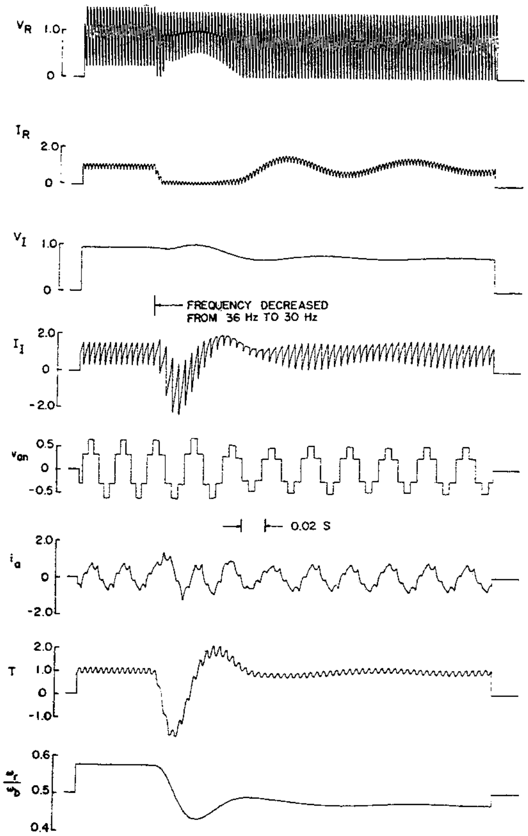


Fig. 10. Load torque switching from 1.0 to 0.6 pu; operation at 30 Hz. System represented in synchronously rotating reference frame with harmonics neglected.



g. 11. Load torque held at 1.0 pu; frequency stepped from 36 to 30 Hz. Detailed system representation.

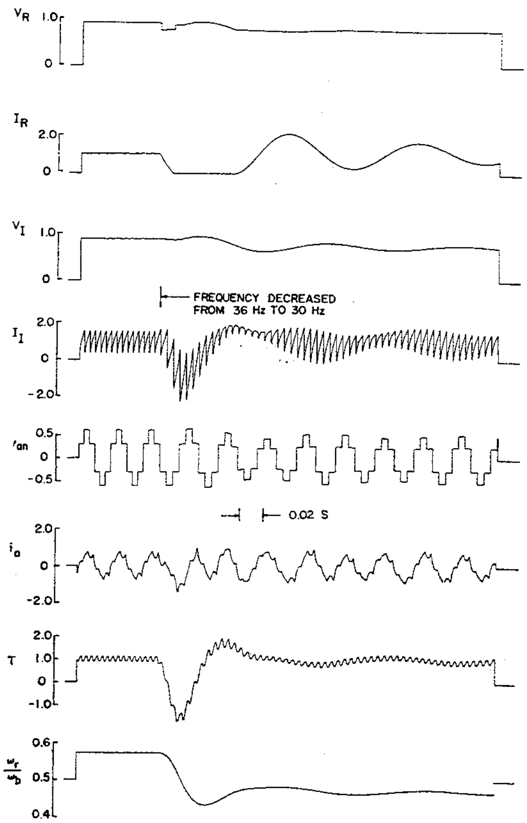


Fig. 12. Load torque held at 1.0 pu; frequency stepped from 36 to 30 Hz. Functional representation of rectifier.

able to an unstable load point at an operating frequency of) Hz. In particular, with the machine operating at 0.925-pu rque, a stable operating point, the load torque is switched to 75 pu, an unstable operating point. The computer recordings own in Fig. 5 were obtained using the detailed simulation of e complete system. Computer tracings shown in Fig. 6 were tained using the representation of the system wherein a nctional (average value) representation of the rectifier is nployed (18). The traces given in Fig. 7 were obtained using e representation in the synchronously rotating reference frame th all harmonic components neglected (18) and (45)–(47). Figs. 5 and 6 the following system variables are recorded:

V_R	rectifier output voltage
I_R	rectifier current
V_I	capacitor voltage
v_{an}	line-to-neutral stator voltage
i_a	phase current
T	electromagnetic torque
ω_r/ω_b	per unit electrical angular velocity of the rotor.

Fig. 7, only V_R, I_R, V_I, T, and ω_r/ω_b are recorded. However, is clear that in this case

$$V_I = (\pi/2)v_{qs}^e, \quad I_I = (3/\pi)i_{qs}^e.$$

comparison of the computer traces given in Figs. 5–7 reveals at the average response of the system variables is identical. In rticular, the computer recordings of the system variables tained using the representation in the synchronously rotating erence frame (Fig. 7) are the average of the corresponding stem variables recorded from the detailed simulation (Fig. 5)

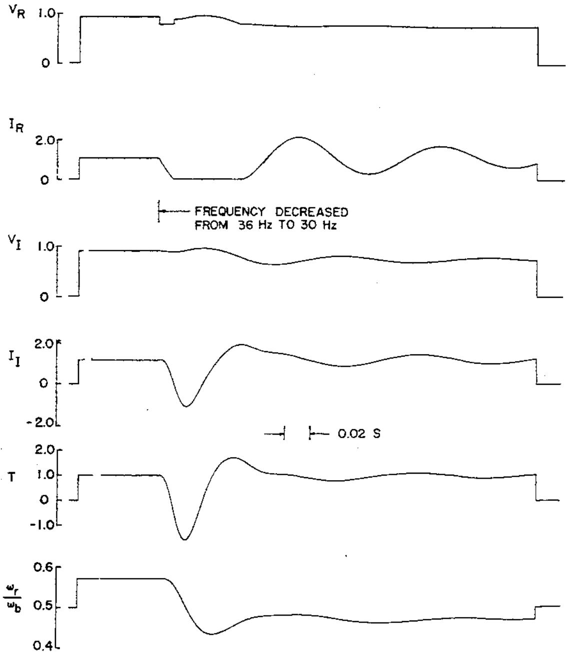


Fig. 13. Load torque held at 1.0 pu; frequency stepped from 36 to 30 Hz. System represented in synchronously rotating reference frame with harmonics neglected.

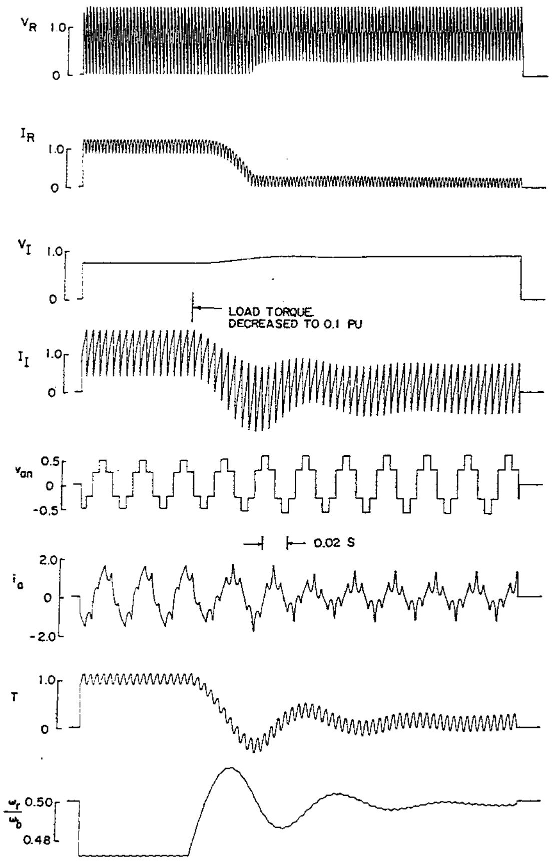


Fig. 14. Load torque switching from 1.0 to 0.1 pu; operation at 30 Hz. Detailed system representation.

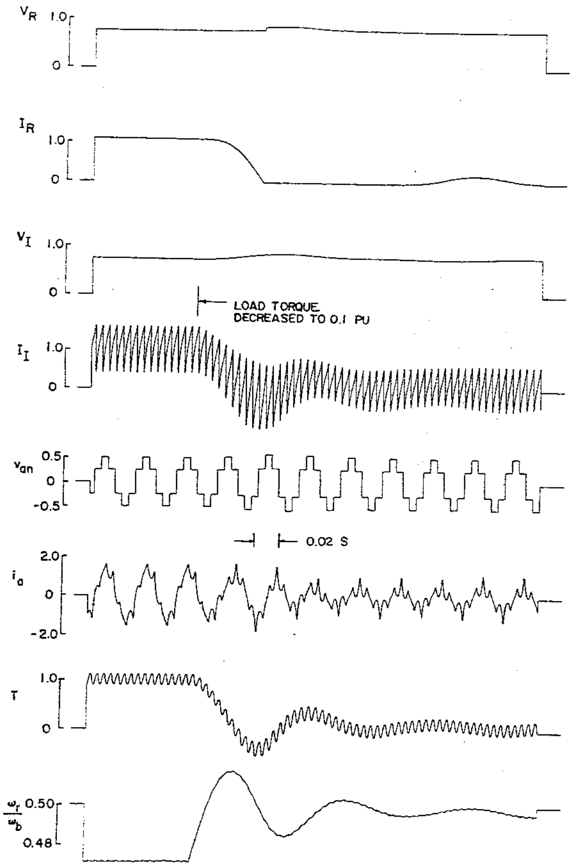


Fig. 15. Load torque switching from 1.0 to 0.1 pu; operation at 30 Hz. Functional representation of rectifier.

and those obtained using the functional representation of the rectifier (Fig. 6).

The computer traces given in Figs. 8–10 permit a comparison of the representations during load-torque switching at an operating frequency of 30 Hz. With the system operating initially at 1.0-pu load torque, the load torque is decreased (stepped) to 0.6 pu. Although the rotor speed is lightly damped at small load torques with the system parameters used, the system is stable for all load torques less than the breakdown value [3]. The average response of the system variables given in Figs. 8–10 is identical for the three types of representations.

Figs. 11–13 show the system response using the three representations during a step change in the frequency of the fundamental component of the voltages applied to the induction machine. In particular, the system is initially operating at 36 Hz with 1.0-pu load torque. The frequency is then stepped to 30 Hz, and simultaneously the delay angle of the rectifier is increased so that the open-circuit inverter voltage is decreased linearly with frequency. A comparison of Figs. 11–13 reveals an excellent correlation between the system performance obtained using the three methods of representation.

Although the simplified representations enable one to predict accurately the performance of the static drive system for many operating conditions, these representations do not properly account for all modes of discontinuous rectifier operation. This feature is demonstrated in Figs. 14–16, where the system is operating at 30 Hz and the load torque is switched from 1.0 to 0.1 pu. The recording of V_I, in Fig. 14, shows an increase in the capacitor voltage during the sustained, periodic, discontinuous operation of the rectifier at the light load. This relatively slow

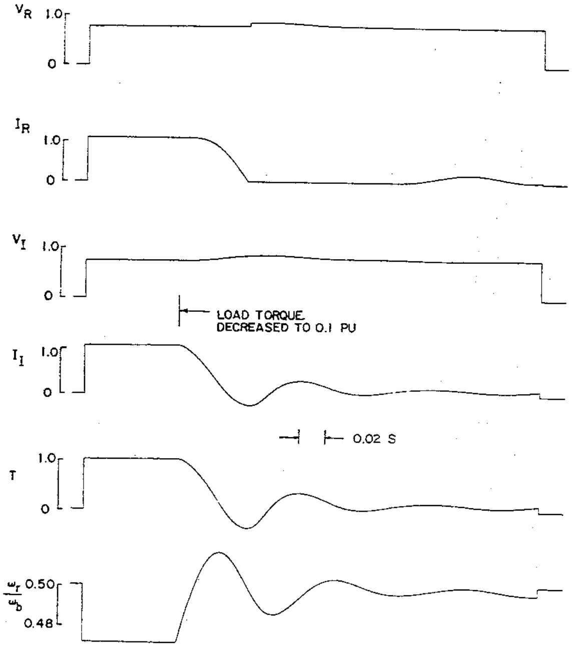


Fig. 16. Load torque switching from 1.0 to 0.1 pu; operation at 30 Hz. System represented in synchronously rotating reference frame with harmonics neglected.

increase in the capacitor voltage during the settling out of the system occurs due to the harmonic components of the rectifier voltage (predominately the 6th harmonic). This periodic discontinuous operation will persist at the 6th harmonic frequency, and the capacitor voltage will increase to a value determined by a combination of the torque load, the amplitude of the source voltage, and the system parameters. The harmonic components of the rectifier voltage are neglected in the simplified representations. Therefore, continuous rectifier operation will be established in the steady state for all values of load torque greater than zero but less than the breakdown value. Although the functional (average value) representation of the rectifier will exhibit discontinuous operation when the average value of the capacitor voltage is larger than the average value of the rectifier voltage, the harmonic charging of the capacitor due to periodic discontinuous operation of the rectifier will not occur (Figs. 15 and 16).

The computer traces show that at the load torque of 0.1 pu, the capacitor voltage increases to approximately 0.9 pu in the actual system (detailed simulation, Fig. 14), whereas the simplified representations give a capacitor voltage of about 0.78 pu (Figs. 15 and 16). Although discontinuous rectifier operation occurred briefly during the change in frequency shown in Figs. 11–13, the time interval during discontinuous operation was small, and the capacitor voltage (Fig. 11) increases only slightly above that predicted by functional representation of the rectifier (Figs. 12 and 13).

Since the simplified representations may be readily simulated and conveniently used, these representations may be employed to advantage in conducting feasibility and preliminary control studies. The simplicity of the simulation in the synchronously rotating reference frame is indeed a desirable feature, and in many cases the results obtained using this representation will predict the system performance with sufficient accuracy. However, in modes of operation where the exact system performance during sustained discontinuous operation is of importance the detailed simulation must be used in order to portray accurately the operation of the system. Perhaps in some cases it may be desirable to incorporate a detailed representation of the rectifier with the synchronously rotating reference frame representation of the inverter and induction motor wherein the harmonics due to inverter switching are neglected.

CONCLUSIONS

Simplified representations of the rectifier-inverter induction motor drive system have been developed. A computer study has been conducted wherein the system performances predicted by two of these simplified representations are compared to that obtained from a detailed computer simulation of the complete static drive system. The modes of operation of the static drive system which are portrayed accurately by the simplified representations as well as those operating conditions which can not be duplicated exactly are clearly established.

Since the simplified representations yield computer simulations which can be readily implemented and easily handled, these representations should be invaluable when conducting preliminary control studies. Moreover, the equations of transformation which express the operation of the inverter in the synchronously rotating reference frame can be employed to describe conveniently the effects of the induction motor upon the filter. Although the harmonic components in these equations of transformation are neglected when establishing the most simple representation of the static drive system, the complete equations, wherein the harmonics are included, offer a new method of analyzing the steady-state behavior of filter-inverter-induction motor combination.

ACKNOWLEDGMENT

The computer studies were performed at the University of Wisconsin Hybrid Computer Laboratory, using equipment provided in part by an NSF grant.

REFERENCES

[1] P. C. Krause and L. T. Woloszyk, "Comparison of computer and test results of a static ac drive system," *IEEE Trans. Industry and General Applications*, vol. IGA-4, pp. 583–588, November/December 1968.

[2] L. T. Woloszyk, "An analog computer study of a static ac drive system," M. S. thesis, University of Wisconsin, Madison, 1967.

[3] T. A. Lipo and P. C. Krause, "Stability analysis of a rectifier-inverter induction motor drive," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-88, pp. 55–66, January 1969.

[4] T. A. Lipo, P. C. Krause, and H. E. Jordan, "Harmonic torque and speed pulsations in a rectifier-inverter induction motor drive," this issue, pp. 579–587.

[5] P. C. Krause and C. H. Thomas, "Simulation of symmetrical induction machinery," *IEEE Trans Power Apparatus and Systems*, vol. PAS-84, pp. 1038–1053, November 1965.

[6] P. C. Krause, "Method of multiple reference frames applied to the analysis of symmetrical induction machinery," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-87, pp. 218–227, January 1968.

[7] P. C. Krause and C. H. Thomas, "The arbitrary reference frame applied to the analysis of symmetrical induction machinery," presented at the IEEE Winter Power Meeting, New York, N. Y., January 28–February 2, 1968.

[8] P. C. Krause, Jr., discussion of H. E. Jordan, "Analysis of induction machines in dynamic systems," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-84, pp. 1085–1088, November 1965.

[9] H. A. Peterson and P. C. Krause, Jr., "A direct- and quadrature-axis representation of a parallel ac and dc power system," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-85, pp. 210–225, March 1966.