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Analysis and Simplified Representations of Rectifier–Inverter Reluctance-Synchronous Motor Drives

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Abstract—**Methods of analyzing six-step and pulse-width-modulated (PWM) types of rectifier–inverter reluctance-synchronous motor drive systems are set forth. In this development the harmonic components due to the switching of the rectifier are neglected, and the operation of the inverter is expressed in a reference frame rotating in synchronism with the fundamental frequency of the inverter output voltages. In the case of the PWM inverter, the Fourier series expansion of a pulse train is used to express the operation of this type of inverter in the synchronously rotating reference frame. Simplified representations are obtained by neglecting the harmonics due to the inverter switching. These simplified representations are verified by comparing the results obtained from a computer study using these representations to those obtained using a detailed simulation of the system. The analysis set forth and the simplified representations that are developed can be used to determine small-displacement system stability, as well as provide a simple and direct technique of predicting the dynamic and steady-state performance of these involved systems. Moreover, the equations established for the PWM inverter can also be used in conjunction with an induction motor drive.**

Introduction

Analysis and design of rectifier–inverter drive systems have recently become an important area of research in electromechanics. The authors have reported a method of analyzing a six-step rectifier–inverter induction motor drive system wherein the harmonic components due to rectifier switching were neglected, and synchronously rotating reference frames were used to express the operation of the rectifier, inverter, and the induction motor [1]. Moreover, it was shown that this method of analysis yields a markedly simplified representation of this type of drive system if all harmonic components due to the switching of the inverter are also neglected. This simplified representation can be readily simulated on the analog computer for the purpose of performing feasibility studies of various control schemes. However, more important is the fact that this simplified representation permits the only convenient and accurate method of predicting the small-displacement stability of a rectifier–inverter induction motor drive system [2]. Also, in the analysis set forth in [1], equations of transformation of the inverter are developed. These relationships, which include the harmonics due to the switching of the inverter, permit new methods to analyze the steady-state behavior of the filter–inverter induction motor drive system when used in conjunction with the method of multiple reference frames or phasors [3], [4].

The work previously reported by the authors has dealt primarily with rectifier–inverter induction motor drive systems with six-step inverter output voltages [1]–[5]. There are other types of rectifier–inverter drive systems which are of current interest, for example, the six-step, rectifier–inverter reluctance-synchronous motor drive system and the rectifier–inverter drive system wherein pulse-width modulation (PWM) is employed to achieve reduced stator applied voltages during variable frequency operation [6]. Analysis and the development of simplified representations of these systems are the subjects of this paper.

The analysis of the six-step rectifier–inverter reluctance-synchronous drive system is similar to that of the six-step induction motor drive. The only difference is in the use of transformation equations between the inverter and the reference frame fixed in the rotor of the machine. The analysis of the PWM inverter is quite different in that a pulse train is employed to express analytically the operation of this type system in a reference frame rotating in synchronism with the frequency of the fundamental component of the inverter output voltages.

In this paper both the six-step and the PWM reluctance-synchronous drive systems are analyzed. Also, the simplified representations which are developed by neglecting all harmonic components are verified by comparing the results of an analog computer study of the complete system with the results of a computer study using the simplified representations. This verification establishes that these simplified representations for the six-step and PWM reluctance-synchronous drive systems may be used to predict small-displacement stability, and they offer valid simplified computer simulations of involved systems. Moreover, the equations of transformation for the PWM inverter are equally valid for an induction motor drive system. Hence, these relationships may be incorporated with those set forth in [1] to describe the operation of a PWM rectifier–inverter induction motor drive.

System Description and Basic Equations

A simplified diagram of the rectifier–inverter drive system considered in this paper is shown in Fig. 1. This system consists of a three-phase power source, a six-phase rectifier with filter, an inverter, and a three-phase reluctance-synchronous machine. The rectifier–inverter system is identical in configuration to that considered in previous papers wherein the inverter was switched so as to supply an induction motor with six-step or stepped phase voltages.

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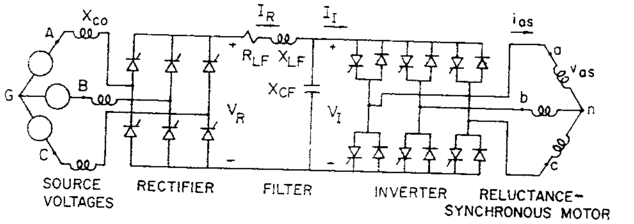


Fig. 1. System studied.

The purpose of this paper is to establish equivalent circuits of the rectifier–inverter reluctance-synchronous motor drive for both the six-step and the PWM modes of operation and to develop simplified representations from these equivalent circuits. Although the detailed analog computer simulation of this system is secondary to this purpose, it is important in that the validity of the simplified representations is established by comparing the system performance predicted by the computer simulation of the simplified representations with that predicted by the detailed analog computer simulation of the complete system. For this reason and for the convenience of one interested in simulating the complete system, the method of simulation will be discussed briefly and references cited while describing the system components.

Simulation of three-phase source voltages is readily accomplished by implementing a constant frequency oscillator. The simulation of the rectifier is set forth and verified in [7] and [8]. The voltage equations for the filter during continuous operation ($I_R > 0$) may be written

$$V_R = V_I + \left(\frac{p}{\omega_b} X_{LF} + R_{LF}\right) I_R \tag{1}$$

$$V_I = \frac{\omega_b}{p} X_{CF}(I_R - I_I). \tag{2}$$

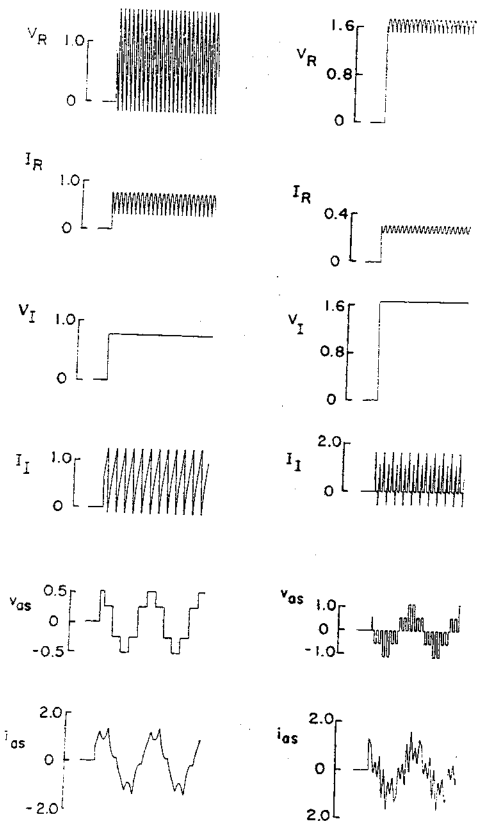
The rectifier current I_R can never be negative. When V_I exceeds V_R and I_R is forced to zero, V_R becomes equal to V_I.

Two modes of inverter operation are considered. The six-step (continuous-voltage) type inverter operation is simulated, and the simulation is verified in [5]. The PWM-type inverter operation is described in detail in [6]. Fig. 2 illustrates the form of the system variables during both modes of inverter operation. In particular V_R is the rectifier voltage, I_R the rectifier current, V_I the capacitor voltage, I_I the inverter current, v_{as} the phase voltage, and i_{as} the phase current. Six-step operation is shown in Fig. 2(a); PWM operation is shown in Fig. 2(b). In Fig. 2 the drive system is operating in the steady-state with 0.5 pu load torque applied to the reluctance-synchronous motor. The frequency of the source is 60 Hz, and the inverter is switched so as to produce output voltages with a frequency of 30 Hz. Hereafter, the frequency ratio f_R will be used to identify the frequency of inverter operation. This ratio is

$$f_R = \frac{\omega_e}{\omega_b} \tag{3}$$

where ω_e corresponds to the electrical angular velocity of the fundamental component of the output voltages of the inverter and ω_b is the base electrical angular velocity herein selected equal to the electrical angular velocity of the source voltages (60 Hz).

In the case of the six-step mode of inverter operation, Fig. 2(a), the delay angle of the rectifier is adjusted so as to reduce the output voltage of the rectifier according to some prescribed function of frequency. In this study, a constant volts-per-hertz ratio was employed.



(a) (b)

Fig. 2. Modes of inverter operation. (a) Six-step continuous voltage. (b) Pulse width modulated.

The PWM system, Fig. 2(b), also has a six-step envelope, but the applied voltage is switched periodically to zero. In this case, the method of reducing the terminal voltage with frequency differs from that used in the six-step (continuous-voltage) inverter. Generally, the rectifier is operated without phase delay, and the terminal voltage is reduced by increasing the time interval so that zero voltage is applied to the machine. In this paper the duration of each voltage pulse at the output of the inverter is fixed at 1/12 the period of the base frequency, that is

$$T_1 = \frac{\pi}{6\omega_b} \tag{4}$$

where T_1 is the duration of the voltage pulse in seconds. The period or interval between the leading edge of successive pulses is 1/12 the period of the fundamental component of the inverter output voltage. Thus,

$$T = \frac{\pi}{6 f_R \omega_b}. \tag{5}$$

Since T_1 is constant, if the capacitor voltage is maintained at a constant value, the relationship between the fundamental component of the inverter output voltage and frequency is obtained by dividing T_1 by T, which yields f_R. Hence, this PWM system also operates with a constant volts-per-hertz ratio. When the inverter is switched so as to establish rated frequency $f_R = 1.0$

$$T_1 = T \tag{6}$$

whereupon the PWM system develops continuous six-step voltages at the inverter terminals.

The zero-voltage mode is achieved in the PWM system by instantaneously switching all three phases of the machine to the same side of the capacitor. It is clear that when the inverter voltage is applied to the machine, two phases of the machine are connected to one terminal of the capacitor, while the third is connected to the other terminal. With all three phases connected to the same capacitor terminal, the stator phases of the machine are short-circuited. During this zero-voltage mode the inverter current I_I is zero, since the machine is effectively disconnected from the inverter.

The simulation of the PWM inverter is readily developed from the simulation of the continuous six-step inverter by incorporating additional logic signals to switch the comparator relays used in the simulation according to (4) and (5). Both the six-step- and the PWM-type inverters are simulated without regard to commutation or switching times. It is assumed that commutation occurs instantaneously in the inverter.

The reluctance-synchronous machine considered in this paper is identical in structure to the machine analyzed in [9]. This elementary two-pole three-phase machine has a short-circuited rotor winding in both the direct and quadrature axes. The equations that describe this machine in a reference frame fixed in the rotor (Park's equations) are

$$v_{qs}^r = \frac{p}{\omega_b}\psi_{qs}^r + \psi_{ds}^r\frac{p\theta_r}{\omega_b} + r_s i_{qs}^r \quad (7)$$

$$v_{ds}^r = \frac{p}{\omega_b}\psi_{ds}^r - \psi_{qs}^r\frac{p\theta_r}{\omega_b} + r_s i_{ds}^r \quad (8)$$

$$0 = \frac{p}{\omega_b}\psi_{qr}'^r + r_{qr}' i_{qr}'^r \quad (9)$$

$$0 = \frac{p}{\omega_b}\psi_{dr}'^r + r_{dr}' i_{dr}'^r \quad (10)$$

where

$$\psi_{qs}^r = X_{ls} i_{qs}^r + X_{aq}(i_{qs}^r + i_{qr}'^r) \quad (11)$$

$$\psi_{ds}^r = X_{ls} i_{ds}^r + X_{ad}(i_{ds}^r + i_{dr}'^r) \quad (12)$$

$$\psi_{qr}'^r = X_{lqr}' i_{qr}'^r + X_{aq}(i_{qs}^r + i_{qr}'^r) \quad (13)$$

$$\psi_{dr}'^r = X_{ldr}' i_{dr}'^r + X_{ad}(i_{ds}^r + i_{dr}'^r). \quad (14)$$

In the above equations the superscript r denotes variables in a reference frame fixed in the rotor, and the primes denote rotor quantities referred to the stator windings. Also, p is the operator d/dt, and with the appropriate subscripts, r and X_l denote resistances and leakage reactances, respectively. The angular position of the rotor is θ_r and $p\theta_r = \omega_r$. The direct- and quadrature-axis magnetizing reactances are X_{ad} and X_{aq}, respectively. The per-unit electromagnetic torque, positive for motor action, can be expressed

$$T = X_{ad} i_{dr}'^r i_{qs}^r - X_{aq} i_{qr}'^r i_{ds}^r + (X_{ad} - X_{aq}) i_{ds}^r i_{qs}^r. \quad (15)$$

In the case of the symmetrical induction machine, the reference frame may be fixed in the stator, whereupon the phase voltages v_{as}, v_{bs}, and v_{cs} are arithmetically related to the d and q axes fixed in the stator [10]. This facility simplifies the simulation of a rectifier–inverter motor drive system [1]. However, since the rotor of a reluctance-synchronous machine is electrically asymmetrical, it is advantageous to fix the d and q axes in the rotor of the machine. Consequently, it is necessary to simulate the transformation of v_{as}, v_{bs}, and v_{cs} to v_{qs}^r and v_{ds}^r. By Park's equations

$$v_{qs}^r = \frac{2}{3}\left[v_{as}\cos\omega_r t + v_{bs}\cos\left(\omega_r t - \frac{2\pi}{3}\right) + v_{cs}\cos\left(\omega_r t + \frac{2\pi}{3}\right)\right] \quad (16)$$

$$v_{ds}^r = \frac{2}{3}\left[v_{as}\sin\omega_r t + v_{bs}\sin\left(\omega_r t - \frac{2\pi}{3}\right) + v_{cs}\sin\left(\omega_r t + \frac{2\pi}{3}\right)\right] \quad (17)$$

$$v_{0s} = \frac{1}{3}(v_{as} + v_{bs} + v_{cs}). \quad (18)$$

Since the stator windings are a symmetrical three-wire system, it can be shown that $v_{0s} = 0$ [11]. From (16) and (17) it is clear that a variable frequency oscillator must be implemented in order to generate $\cos\theta_r$ and $\sin\theta_r$, and multipliers are necessary to simulate the transformation of the phase voltages to the reference frame fixed in the rotor. Likewise, it is necessary to simulate the transformation of i_{qs}^r and i_{ds}^r to i_{as}, i_{bs}, and i_{cs}.

Equivalent Circuit with Rectifier and Inverter Represented in Synchronously Rotating Reference Frames

As a first step in the development of an equivalent circuit for both the six-step and PWM systems, a simplified or functional representation of the rectifier will be employed. If the harmonic components of the rectifier output voltage are neglected, the average output voltage of a six-phase converter can be expressed

$$V_R = \frac{3\sqrt{3}}{\pi} V_S \cos\alpha - \frac{3}{\pi} X_{co} I_R \quad (19)$$

where X_{co} is the commutating reactance, V_S is the magnitude of the line-to-neutral source voltage, and α the delay angle. Although (19) is the familiar expression for the average output voltage of a six-phase converter, this equation may also be interpreted as the average output of a six-phase converter expressed in a d–q axis rotating in synchronism with the source voltages [12]. Moreover, if (19) may be considered in a reference frame rotating in synchronism with the source voltages, the same interpretation may be applied to the filter equations (1) and (2). Thus, by neglecting the harmonic components in the output voltage of the rectifier, the operation of the rectifier and the filter may be compactly expressed in a synchronously rotating reference frame [1].

The six-step inverter voltages applied to the stator phases of the machine may be expressed in a Fourier series. For example, the line-to-neutral voltage for phase a is [1]

$$v_{as} = \frac{2V_I}{\pi}\left(\cos\omega_e t + \frac{1}{5}\cos 5\omega_e t - \frac{1}{7}\cos 7\omega_e t - \cdots\right). \quad (20)$$

It is clear that during normal operation the stator phase voltages may be considered as a series of balanced three-phase sets formed by the fundamental and the 5th, 7th, ... harmonic components. Thus, v_{bs} and v_{cs} may be expressed from (20) by replacing $\omega_e t$ with $\omega_e t - 2\pi/3$ and $\omega_e t + 2\pi/3$. Moreover, the amplitude of each of these balanced three-phase sets is determined by the instantaneous value of the capacitor voltage V_I. That is, the fundamental and the harmonic components of the phase voltages may be considered as sinusoidal functions modulated by the instantaneous value of V_I.

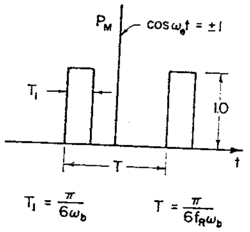


Fig. 3. Pulse train.

The operation of the PWM inverter may be conveniently analyzed by expressing the output voltages of the inverter as the product of the six-step phase voltages (20) and a pulse train with pulses of unit magnitude. Although it will be necessary to select the pulse train to correspond to the operation of a specific type of PWM inverter, the pulse train employed herein is shown in Fig. 3. Each six-step phase voltage is multiplied by the same pulse train, and in this study the period of the pulse train is always centered about the maximum value of the fundamental frequency component of phase a. This method of time reference is shown in Fig. 3. It is clear that $\cos \omega_e t = \pm 1$ always occurs at the center of the period of the pulse train $T/2$ and not the center of the period of the pulse. The pulse train may be expressed in a Fourier series as

$$P_M = f_R + \sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi}\left[\sin 12nf_R\omega_b t - \sin 12nf_R\omega_b\left(t - \frac{\pi}{6\omega_b}\right)\right]. \quad (21)$$

Thus, for the PWM inverter the line-to-neutral voltage of phase a may be expressed

$$v_{as} = \frac{2V_I}{\pi} P_M\left(\cos\omega_e t + \frac{1}{5}\cos 5\omega_e t - \frac{1}{7}\cos 7\omega_e t - \cdots\right). \quad (22)$$

It is clear that expressions for v_{bs} and v_{cs} may be obtained by multiplying P_M times the expression of the six-step voltage waveform.

The following equations of transformation will be used to transform v_{as}, v_{bs}, and v_{cs} to a reference frame rotating in synchronism with the fundamental frequency of these voltages:

$$v_{qs}^e = \frac{2}{3}\left[v_{as}\cos\omega_e t + v_{bs}\cos\left(\omega_e t - \frac{2\pi}{3}\right) + v_{cs}\cos\left(\omega_e t + \frac{2\pi}{3}\right)\right] \quad (23)$$

$$v_{ds}^e = \frac{2}{3}\left[v_{as}\sin\omega_e t + v_{bs}\sin\left(\omega_e t - \frac{2\pi}{3}\right) + v_{cs}\sin\left(\omega_e t + \frac{2\pi}{3}\right)\right] \quad (24)$$

$$v_{0s} = \frac{1}{3}(v_{as} + v_{bs} + v_{cs}). \quad (25)$$

Again, $v_{0s} = 0$. The voltages v_{qs}^e and v_{ds}^e become

$$v_{qs}^e = \frac{2V_I}{\pi} P_M\left(1 + \frac{2}{35}\cos 6\omega_e t - \frac{2}{143}\cos 12\omega_e t + \cdots\right) \quad (26)$$

$$v_{ds}^e = \frac{2V_I}{\pi} P_M\left(\frac{12}{35}\sin 6\omega_e t - \frac{24}{143}\sin 12\omega_e t + \cdots\right). \quad (27)$$

It is clear that the speed of this synchronously rotating reference frame ω_e is determined by the frequency of the fundamental component of the output voltages of the inverter. In a previous paper the following substitution was introduced [1]:

$$g_{qs}^e = 1 + \frac{2}{35}\cos 6\omega_e t - \frac{2}{143}\cos 12\omega_e t + \cdots \quad (28)$$

$$g_{ds}^e = \frac{12}{35}\sin 6\omega_e t - \frac{24}{143}\sin 12\omega_e t + \cdots. \quad (29)$$

If it is assumed that no power is lost in the inverter

$$V_I I_I = \frac{3}{2}(v_{qs}^e i_{qs}^e + v_{ds}^e i_{ds}^e). \quad (30)$$

If (26) and (27) are substituted into (30), the inverter current becomes

$$I_I = \frac{3}{\pi} P_M (g_{qs}^e i_{qs}^e + g_{ds}^e i_{ds}^e). \quad (31)$$

For the purposes of constructing an equivalent circuit for the rectifier–inverter system, it is convenient to incorporate the following substitute quantities [1]:

$$V_{RO}' = \frac{6\sqrt{3}}{\pi^2} V_S \cos\alpha \quad (32)$$

$$X_{co}' = \frac{18}{\pi^3} X_{co} \quad (33)$$

$$I_R' = \frac{\pi}{3} I_R \quad (34)$$

$$V_R' = \frac{2}{\pi} V_R \quad (35)$$

$$R_{LF}' = \frac{6}{\pi^2} R_{LF} \quad (36)$$

$$X_{LF}' = \frac{6}{\pi^2} X_{LF} \quad (37)$$

$$I_I' = \frac{\pi}{3} I_I \quad (38)$$

$$V_I' = \frac{2}{\pi} V_I. \quad (39)$$

Hence

$$V_R' = V_{RO}' - X_{co}' I_R' \quad (40)$$

$$V_R' = V_I' + \left(\frac{p}{\omega_b} X_{LF}' + R_{LF}'\right) I_R' \quad (41)$$

$$V_I' = \frac{\omega_b}{p} X_{CF}'(I_R' - I_I'). \quad (42)$$

The above equations, which express the rectifier and inverter in their respective synchronously rotating reference frame, are valid for either an induction motor or a reluctance-synchronous motor drive system. By appropriate application of the above analysis, the equivalent circuit for the six-step inverter set forth in [1] may be modified to account for PWM inverter operation.

In the case of the reluctance-synchronous machine it is necessary to transform the voltage from the synchronously rotating reference frame of the inverter to the reference frame fixed in the

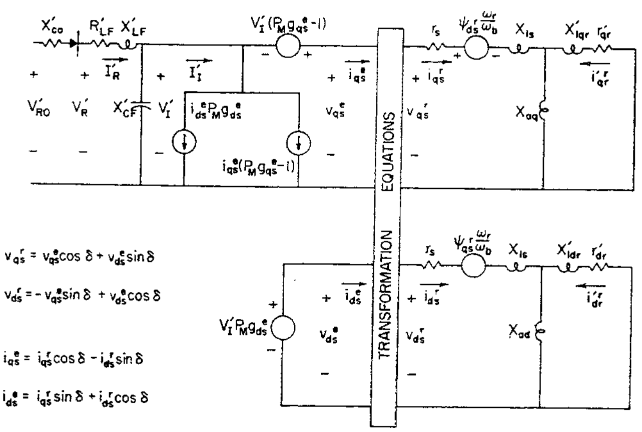


Fig. 4. Equivalent circuit in synchronously rotating reference frames. Harmonic components due to rectifier switching neglected.

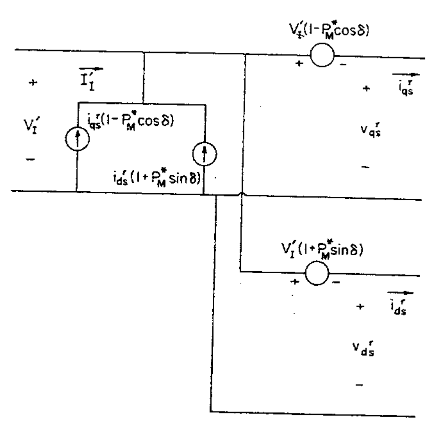


Fig. 5. Modifications of equivalent circuit. All harmonic components neglected and transformation to rotor incorporated.

rotor of the reluctance-synchronous machine. These equations of transformation may be written

$$v_{qs}^r = v_{qs}^e \cos\delta + v_{ds}^e \sin\delta \tag{43}$$

$$v_{ds}^r = -v_{qs}^e \sin\delta + v_{ds}^e \cos\delta \tag{44}$$

where

$$\delta = \frac{1}{p}(\omega_e - \omega_r). \tag{45}$$

Also,

$$i_{qs}^e = i_{qs}^r \cos\delta - i_{ds}^r \sin\delta \tag{46}$$

$$i_{ds}^e = i_{qs}^r \sin\delta + i_{ds}^r \cos\delta. \tag{47}$$

Incorporating the above equations of the rectifier–inverter system with the equations that describe the reluctance-synchronous machine, (7)–(14), yields the equivalent circuit shown in Fig. 4. This equivalent circuit can be used to describe the operation of both a six-step and PWM inverter. In the case of a six-step inverter, P_M is omitted or set equal to one. For PWM operation, P_M is defined by (21). Additional current and voltage generators may be incorporated in Fig. 4 for the purpose of portraying the equations of transformation from the synchronously rotating reference frame to the reference frame fixed in the rotor of the reluctance-synchronous machine. This refinement is not included, however, since it tends to distract from the fact that the equivalent circuit of the inverter can also be used to represent an inverter–induction motor drive wherein the transformation to the rotor is not used [1].

Simplified Representations and Results of Computer Study

The equivalent circuit shown in Fig. 4 offers a convenient method of describing this system with the harmonic components due to rectifier switching neglected. However, in its present form it does not yield the most convenient method of simulating the inverter on the analog computer. That is to say, if it is desirable to simulate the switching of the inverter in detail, the synchronously rotating reference frame representation is less convenient than the direct simulation employed in [5]. On the other hand, the representation in the synchronously rotating reference frame may be used to advantage if some of or all the harmonic components due to the inverter switching are neglected. In particular, it has been demonstrated that the synchronously rotating reference frame representation offers the only convenient means of 1) investigating stability, 2) developing simplified computer simulations, and 3) analyzing the steady-state performance of a rectifier–inverter induction motor drive with a six-step inverter output [1]–[4]. All the harmonic components due to inverter switching are neglected in [1] and [2], and only the salient harmonic components are included in the steady-state analyses presented in [3] and [4].

In this section the simplified representations of the six-step and PWM rectifier–inverter reluctance-synchronous drive systems are set forth and verified. It is clear that once this verification is established, these simplified representations may be used to predict system stability and steady-state performance. Moreover, the analog computer simulation of the simplified representation may be used to advantage in determining the feasibility of control schemes.

If the harmonic components due to inverter switching are neglected, the equations that describe the six-step and PWM drive systems are markedly simplified. In particular, if all harmonic components are neglected $g_{qs}^e = 1$ and $g_{ds}^e = 0$. For PWM operation $P_M = f_R$, whereas six-step operation may be represented by setting P_M equal to unity. For the sake of brevity, the quantity P_M^* will be introduced and only one set of equations will be given. It is clear that both six-step or PWM operation, with all harmonic components neglected, may be described from this set of equations by setting $P_M^* = 1.0$ or f_R, respectively.

With $g_{qs}^e = 1$ and $g_{ds}^e = 0$ the following equations may be written:

$$v_{qs}^e = P_M^* V_I' \tag{48}$$

$$v_{ds}^e = 0 \tag{49}$$

$$v_{qs}^r = P_M^* V_I' \cos\delta \tag{50}$$

$$v_{ds}^r = -P_M^* V_I' \sin\delta \tag{51}$$

$$I_I' = P_M^* i_{qs}^e \tag{52}$$

where i_{qs}^e is expressed by (46). The above equations suggest the equivalent circuit shown in Fig. 5. In Fig. 5 the equations of transformation to the rotor are incorporated. The modifications necessary to represent an induction motor rather than a reluctance-synchronous machine are straightforward [1].

It is important to note that with all harmonics neglected, the inverter current I_I is independent of i_{ds}^e. That fact is apparent since $g_{ds}^e = 0$, and hence the current generator equal to i_{ds}^e-$P_M g_{ds}^e$ in Fig. 4 is zero. With the time zero reference stipulated by the equations of transformation of the inverter, the nonsinusoidal component of i_{ds}^e corresponds to the amplitude of the out-of-phase component of the fundamental inverter current, which is the magnetizing or reactive component of the fundamental motor current. As pointed out in [1], the current i_{ds}^e appears only as the coefficient of the 6th, 12th, ... harmonics in the expression of the inverter current [(29) and (31)]. Thus, the fundamental magnetizing current is supplied to the machine by the harmonic components (predominantly the 6th harmonic) of the inverter current. Therefore, if the harmonic components are neglected, the effect of the magnetizing current is not included in the inverter current. Consequently, neglecting all harmonic components may, at first, appear as an invalid means of approximating the performance of this static drive system. However, since the electromechanical performance of this system is determined primarily by the real power transferred through the inverter rather than the reactive power exchange, many of the dominant performance characteristics are preserved even though all harmonics are neglected.

In order to establish verification of the simplified representations of the six-step and PWM systems wherein all harmonics are neglected, the results of an analog computer study using these simplified representations are compared to the detailed analog computer simulation of the complete system. The results of this computer study are shown in Figs. 6–13. The per-unit parameters of the reluctance-synchronous motor, filter, and commutating reactance used in this study are

$$r_s = 0.045 \quad r_{qr}' = 0.015 \quad r_{dr}' = 0.030$$

$$X_{ls} = 0.100 \quad X_{lqr}' = 0.100 \quad X_{ldr}' = 0.100$$

$$H = 0.4 \text{ s} \quad X_{aq} = 0.5 \quad X_{ad} = 2.0$$

$$X_{co} = 0.160 \quad R_{LF} = 0.025$$

$$X_{CF} = 0.00705 \quad X_{LF} = 0.500.$$

The computer recordings from the detailed simulation, Figs. 6,8,10, and 12, show the following system variables: V_R is the rectifier output voltage, I_R the rectifier current, V_I the capacitor voltage, I_I the inverter current, v_{as} the line-to-neutral stator voltage, i_{as} the phase current, T the electromagnetic torque, and ω_r/ω_b the per-unit electrical angular velocity of the rotor. The computer recordings obtained using the simplified representations, Figs. 7, 9, 11, and 13, show only V_R, I_R, V_I, I_I, T, and ω_r/ω_b. However, $V_I = (\pi/2) P_M{}^* v_{qs}^e$ and $I_I = (3/\pi) P_M{}^* i_{qs}^e$.

A comparison of system performance obtained from a detailed computer simulation of the complete system and the system performance predicted by the simplified representations for a six-step inverter are shown in Figs. 6–9. In the case of the six-step inverter the amplitude of the fundamental component of the open-circuit inverter voltage is decreased linearly with frequency with 1.0-pu voltage at 60 Hz.

In Figs. 6–9 the system is initially operating at $f_R = 0.5$ (30 Hz) with a load torque of 0.5 pu. The computer tracings shown in Figs. 6 and 8 were obtained from the detailed computer simulation of the system. Figs. 7 and 9 show recordings obtained from an analog computer study of the representation; $P_M{}^*$ is set equal to one.

The performance of the system during a load-torque disturbance is shown in Fig. 6 (detailed representation) and Fig. 7 (simplified representation). With the system initially operating at 0.5-pu load torque, the load torque is switched to 0.3 pu. The one mode of operation that is not accurately predicted by the simplified representation is illustrated by this comparison. In the development of the simplified representations of the rectifier, the harmonic components of the output voltage are neglected. Consequently, the simplified representations do not properly account for all modes of discontinuous rectifier operation. The recording of V_I in Fig. 6 shows a slow increase in the capacitor voltage during the sustained, periodic, discontinuous operation of the rectifier at the light load. This relatively slow increase in the capacitor voltage during the settling out of the system occurs due to the harmonic components of the rectifier voltage (predominantly the 6th harmonic). Without automatic control of V_I, this periodic discontinuous operation will persist at the 6th harmonic frequency, and the capacitor voltage will increase to a value determined by a combination of the torque load, the amplitude of the source voltage, and the system parameters. The harmonic components of the rectifier voltage are neglected in the simplified representations. Therefore, continuous rectifier operation will be established in the steady state for all values of load torque greater than zero but less than the breakdown value. Although the functional (average value) representation of the rectifier will exhibit discontinuous operation when the average value of the capacitor voltage is larger than the average value of the rectifier voltage, the "harmonic charging" of the capacitor due to periodic discontinuous operation of the rectifier will not occur (Fig. 7). However, in practical systems where the amplitude of the inverter voltage is controlled automatically, the control will correct for the 6th harmonic charging of the capacitor, and the inverter voltage will be maintained at the desired value.

It is important to note that the amplitude of the 6th harmonic component of the rectifier current is determined primarily by the phase delay of the rectifier and the filter inductance X_{LF}. Therefore, the range of light loads where the 6th harmonic discontinuous operation will occur, and thus the range of light loads where the simplified representation will not exactly predict the performance of the system, will be determined by the phase delay and the value of X_{LF}. It is interesting to note that the 6th harmonic discontinuity appears to add damping to the system; this feature is apparent from the recording of speed in Figs. 6 and 7. Analysis of intermittent discontinuous operation of the rectifier due to the 6th harmonic component warrants investigation.

Before proceeding, it should be pointed out that the source voltages at the rectifier used in the study shown in Fig. 6 were set at a value somewhat larger than 1.0 pu. However, since the phase angle was adjusted to give the proper value of the inverter voltage, this larger value of V_S has little or no effect upon the performance of the system. In the remaining studies V_S was set at 1.0 pu.

The computer tracings given in Figs. 8 and 9 show a comparison of the detailed simulation and the simplified representation for a step change in the frequency of the fundamental component of the six-step inverter voltages. Initially, this system is operating at $f_R = 0.5$ with a load torque of 0.5 pu. The frequency is then switched to 36 Hz ($f_R = 0.6$) and simultaneously, the delay angle of the rectifier is decreased so that the open-circuit inverter voltage is increased linearly with frequency. Other than the slight discrepancy which occurs due to the brief period of discontinuous rectifier operation, the response of the system variables predicted by the simplified representa-

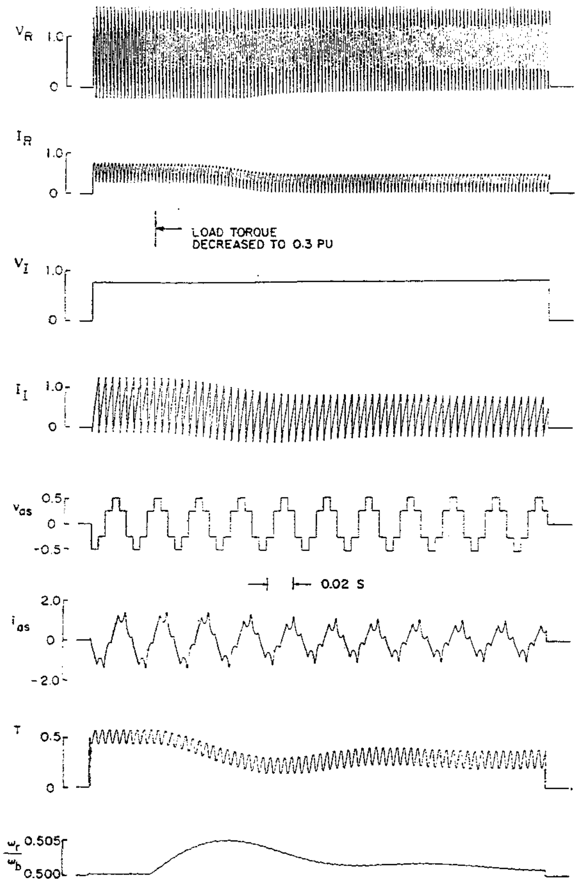


Fig. 6. Load-torque switching from 0.5 to 0.3 pu, operation at 30 Hz. Detailed representation of six-step inverter system.

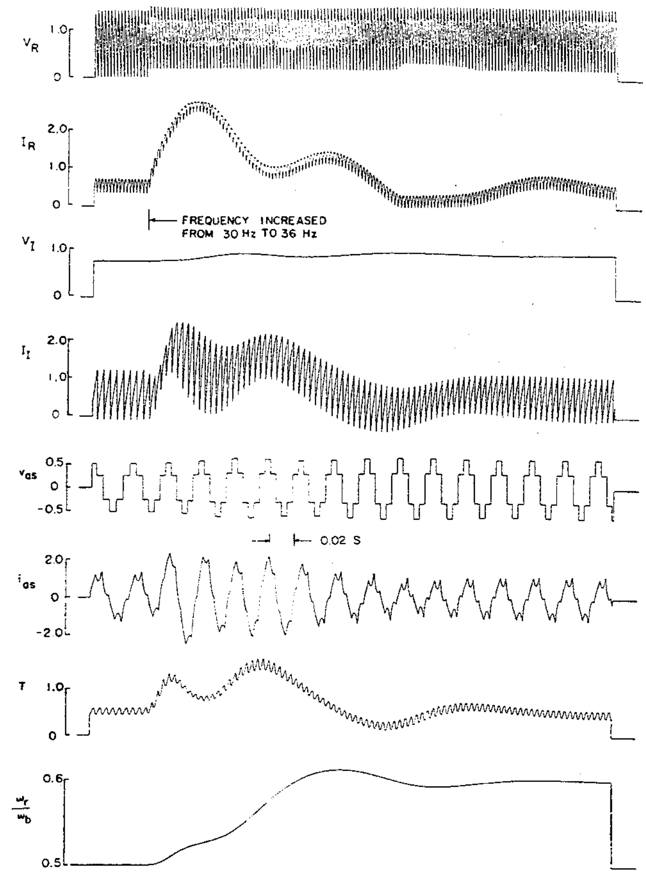


Fig. 8. Load torque held at 0.5 pu, frequency stepped from 30 to 36 Hz. Detailed representation of six-step inverter system.

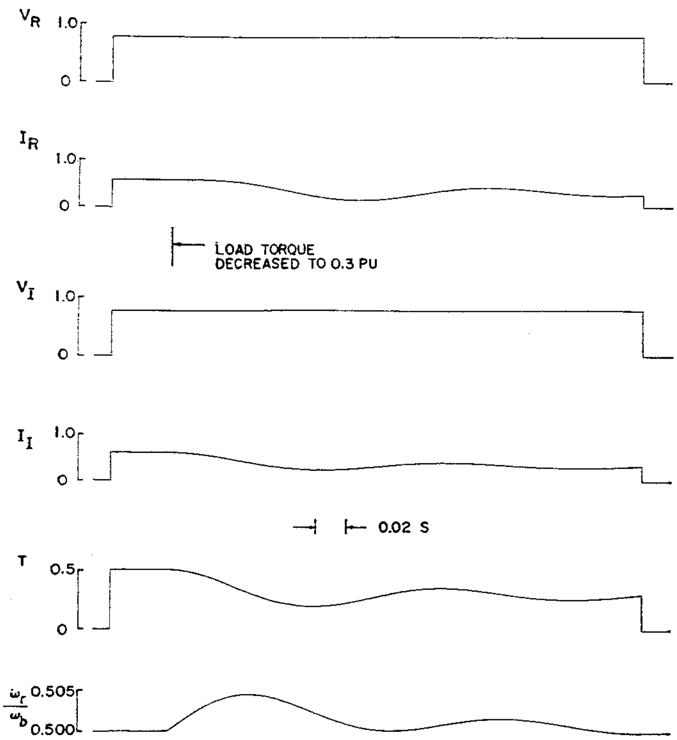


Fig. 7. Load-torque switching from 0.5 to 0.3 pu, operation at 30 Hz. Simplified representation of six-step inverter system.

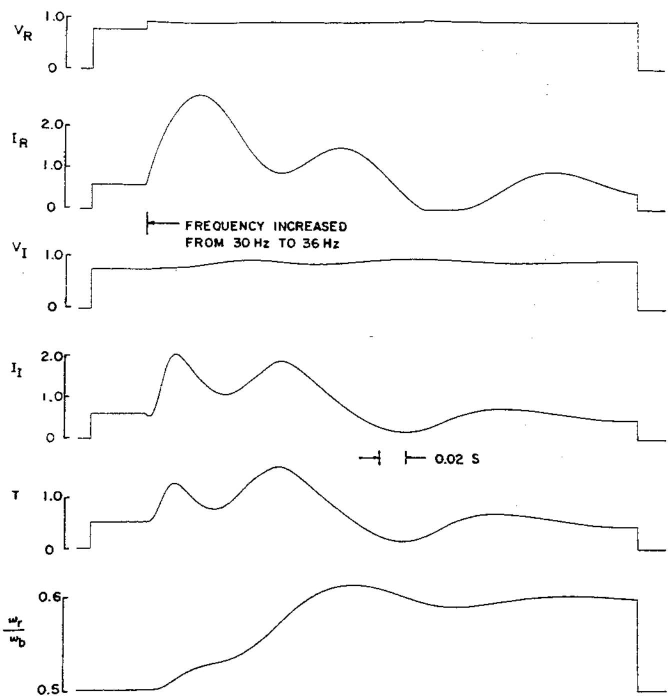


Fig. 9. Load torque held at 0.5 pu, frequency stepped from 30 to 36 Hz. Simplified representation of six-step inverter system.

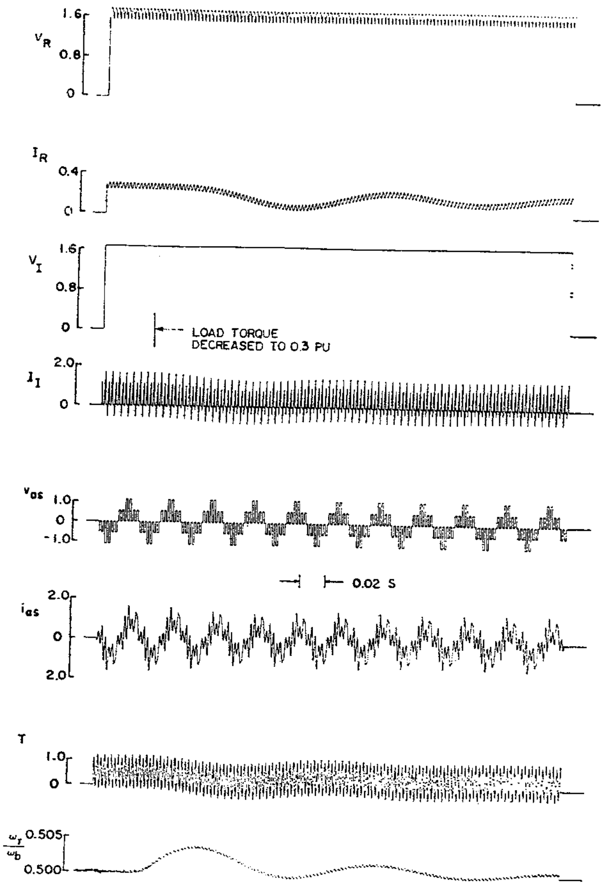


Fig. 10. Load-torque switching from 0.5 to 0.3 pu, operation at 30 Hz. Detailed representation of PWM system.

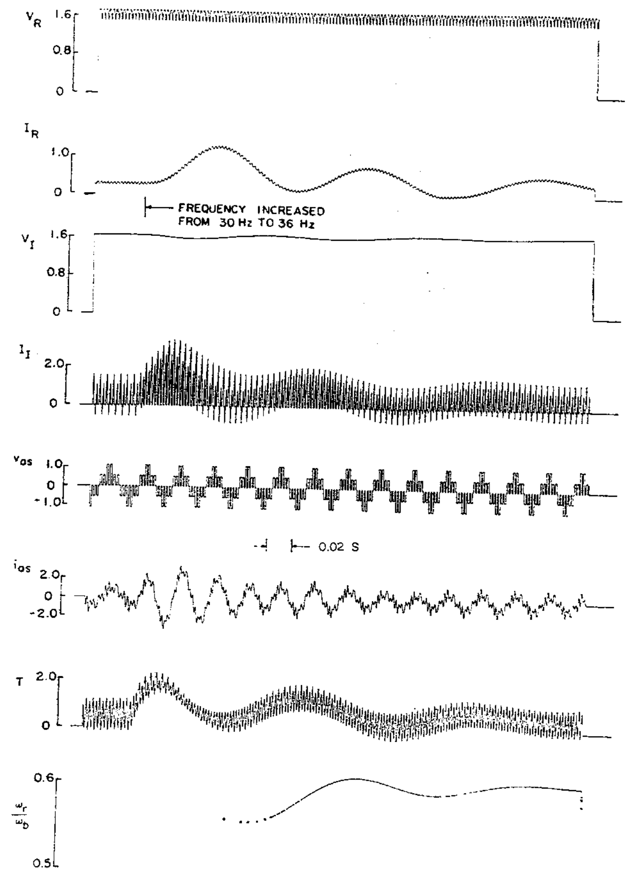


Fig. 12. Load torque held at 0.5 pu, frequency stepped from 30 to 36 Hz. Detailed representation of PWM system.

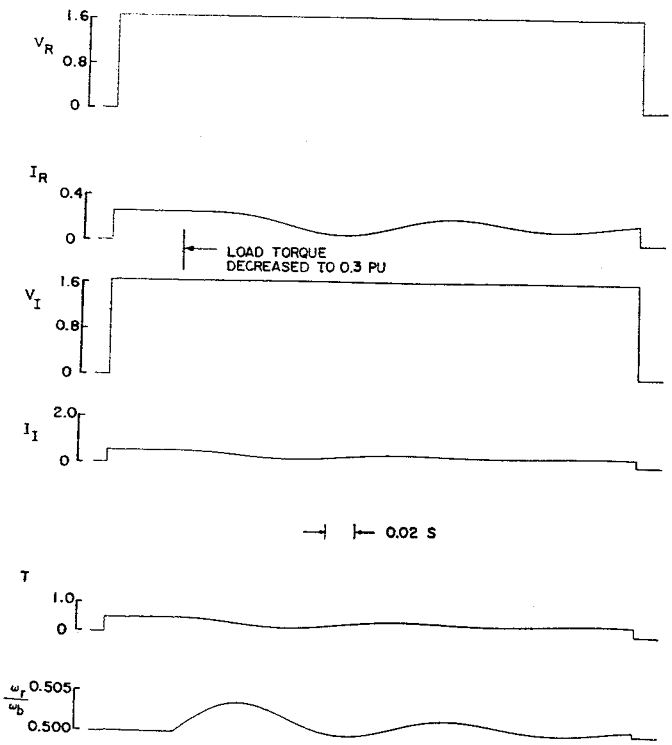


Fig. 11. Load-torque switching from 0.5 to 0.3 pu, operation at 30 Hz. Simplified representation of PWM system.

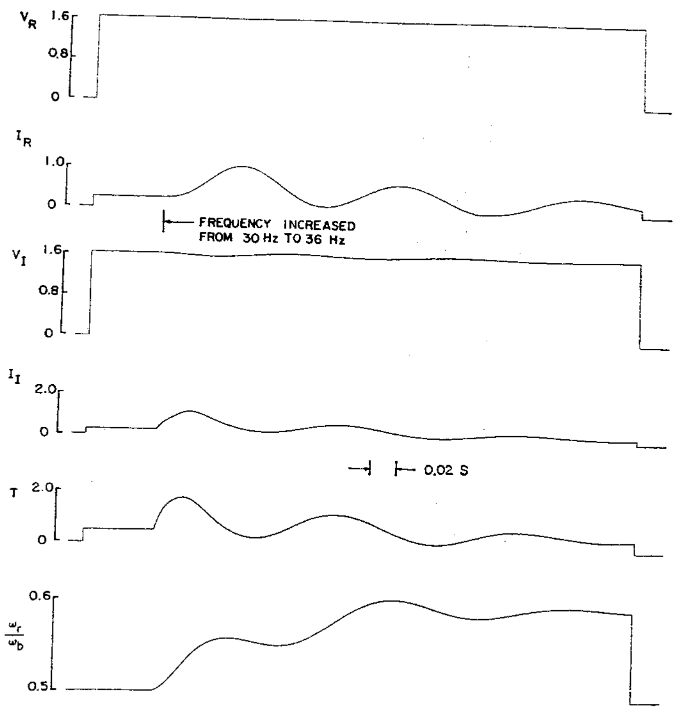


Fig. 13. Load torque held at 0.5 pu, frequency stepped from 30 to 36 Hz. Simplified representation of PWM system.

tion, Fig. 9, is identical to the average value of the variables of the actual system, Fig. 8.

Figs. 10–13 show a comparison of system performance obtained from a detailed computer simulation and the system performance predicted by the simplified representations for a PWM inverter. In this system the rectifier was fired without phase delay, and the period of the pulse train was varied so as to produce a linear relation between frequency and the amplitude of the fundamental component of the open-circuit inverter voltage with 1.0 pu voltage at 60 Hz ($T_1 = T$). In the simplified representation $P_M^* = f_R$ for a constant volts-per-hertz ratio.

Comparison of the detailed and simplified representations during a load disturbance is given by Figs. 10 and 11. The load-torque switching from 0.5 to 0.3 pu is identical to that used for the six-step inverter (Figs. 6 and 7). Figs. 12 and 13 show a comparison of the system response obtained from the detailed and simplified representations during a step change in frequency. As in Figs. 8 and 9, the frequency was switched from $f_R = 0.5$ to $f_R = 0.6$, with the period of the pulse train increased appropriately at the instant the switching frequency of the inverter was changed. Due to the high value of rectifier voltage, and thus a small 6th harmonic component in the rectifier voltage, discontinuous operation of the rectifier did not occur during the load switching or for the step changes in frequency. During the modes of operation considered, the simplified representation of the PWM system predicts exactly the response of the detailed representation of the system.

If it is desirable to investigate modes of operation of the six-step or PWM inverter wherein sustained discontinuous rectifier operation is of importance, the detailed simulation of these systems must be used. Perhaps in some cases the discontinuous operation due to the 6th harmonic content of the rectifier output may be accurately portrayed by simulating only the switching of the rectifier in detail while retaining the simplified representation of the inverter switching. It is clear that the simplified representations of the six-step and PWM inverter reluctance-synchronous drive systems may be used to accurately predict the small-displacement stability of these systems. Moreover, the computer simulations of these simplified representations may be employed to advantage in conducting feasibility and preliminary studies for many proposed control schemes.

Conclusions

An analysis of six-step and PWM rectifier–inverter reluctance-synchronous drive systems has been set forth. In this analysis, the harmonic components of the rectifier output voltage are neglected, while the harmonic components due to the inverter switching are represented in a reference frame rotating in synchronism with the fundamental frequency of the inverter output. Simplified representations were established by neglecting the harmonic components in the six-step or PWM inverter systems. These simplified representations have been verified for continuous rectifier operation, and the modes of operation that may not be accurately predicted by these simplified representations have been pointed out. The method of analysis set forth, as well as the resulting simplified representations, should serve to establish a new and convenient means of predicting the dynamic and steady-state performance of six-step and PWM inverter drive systems.

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