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STEADY-STATE ANALYSIS OF A CURRENT SOURCE INVERTER/RELUCTANCE MOTOR DRIVE PART I: ANALYSIS

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SUMMARY

The steady-state solution of a reluctance motor supplied from a current source inverter is obtained using state variable techniques. Closed-form solutions for the instantaneous motor phase voltages, rotor currents as well as the instantaneous and average component of the electromagnetic torque are derived. From the solution contained herein, all of the important performance characteristics of this type of drive can be calculated.

INTRODUCTION

AC adjustable speed drives incorporating voltage inverters or cycloconverters have been available for more than a decade. However, the current source inverter (CSI), in which the current rather than the voltage appears essentially as the independent variable, has only recently attracted attention. Most inverters in use today are essentially sources of adjustable voltage and frequency. Design of this type of inverter requires attention to numerous details such as gate pulse timing failure modes, fusing and thyristor specifications in order to realize a rugged and reliable device. On the other hand, the simplicity of design coupled with its low cost make the current source inverter an attractive alternative to voltage source inverter configurations.

Present interest in current source inverters was probably stimulated by the work of Ward [1] who cited earlier Russian investigators [2] in his paper. Perhaps the application area of most interest today is ac motor drives since a constant current characteristic is ideal for motor applications requiring a constant torque output. Associated with use in electrical machine drives, the current source inverter has the following properties:

- Simple circuit configuration.
- Thyristors requiring minimal protection.
- Better utilization of the current handling capacity of the thyristors.
- Commutation capability which is affected by switching frequency as well as load current thereby limiting the output frequency range.
- Ability to ride through commutation failure as well as to recover from momentary short-circuit faults.

Since the occurrence of short-circuit faults on electrical machine is by far more common than open-circuits, this advantage outweighs the hazards accompanying open-circuit faults. Although commutation capability is affected by load, this problem can be alleviated with auxilliary charging circuits. [3,4]

In present-day applications, both induction and reluctance (synchronous-reluctance) motors are widely used depending upon how precise a speed regulation is desired. In a recent paper, a detailed analysis of an induction motor fed from a current source inverter was presented. State variable techniques were employed to generate, in closed form, the time domain solution describing the system steady-state behavior. Such a solution is most important since it enables the designer to quickly evaluate the effects of parameter changes on the motor losses, efficiency, torque pulsations, etc. Also, this analysis clearly revealed the effects of various control strategies on steady-state performance. In particular, it was shown that operation under the most desirable strategy from the point of view of minimum torque pulsations necessitates the use of feedback control for stable operation.

Adjustable-speed reluctance motor drives are widely used in applications where speed and angular position synchronization are essential. Such applications form a substantial portion of the adjustable speed drive business. The simple and rugged rotor construction is of lower cost and greater reliability than that of a conventional wound-field machine. However, the salient rotor construction results in higher torque pulsations than an induction motor of equivalent size. This may be a critical problem in constant speed applications where the combined inertia of rotor and load assembly is insufficient to smooth out speed fluctuations. Although potential application areas are numerous, the operating characteristics of a synchronous-reluctance motor fed from a CSI have not been established. It is of immediate practical interest to investigate how such a source affects operation of this type of motor.

The solution of the reluctance machine equations under this mode of operation presents a formidable problem in motor analysis. Because of the 120° conduction time of the inverter thyristors, repetitive open circuits occur in the lines of the motor making conventional analysis techniques inapplicable. In such cases it has been shown that state variable techniques using a reference frame fixed in the stator can be employed. [5] However, this approach is not directly amenable to motors with rotor saliency such as a reluctance machine.

An extension of the state variable approach to the analysis of a synchronous-reluctance machine fed from a current source inverter is described in this paper. This study is a part of a detailed investigation carried out to achieve a thorough understanding of the CSI operating with a reluctance machine load. This investigation includes a steady-state analysis, equivalent circuit model, stability analysis and an analytical design method for the realization of a practical closed-loop adjustable speed drive. The theoretical results of these studies have been verified with experimental tests and with a detailed analog computer simulation. The steady-state analysis together with experimental correlation and discussion of results are the subjects of this two-part paper. The derivation of the equivalent circuit model, stability analysis and closed-loop design are subjects of companion papers. [6,7]

DESCRIPTION OF SYSTEM

System and Assumptions

The basic configuration of a controlled current inverter-reluctance motor drive system is shown in Fig. 1. The system consists of a thyristor dc bridge with associated control supplied from an ac voltage source, a dc link with a filter choke, a three-phase current inverter (CSI) and a three-phase reluctance (synchronous-reluctance) machine. Proper control of the thyristor bridge together with the operation of the choke serves to modify the ac voltage to appear as a current source to the three-phase CSI. The basic circuit arrangement of a CSI using the autosequential commutation described by Ward [1] is shown in Fig. 2. When the input current to the CSI is held constant, the ideal output waveform is the quasi-square wave current shown in Fig. 3.

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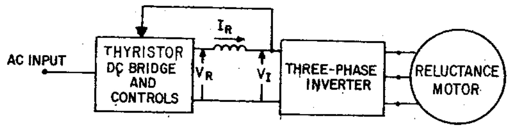

Fig. 1 Basic current source inverter-reluctance motor drive system.

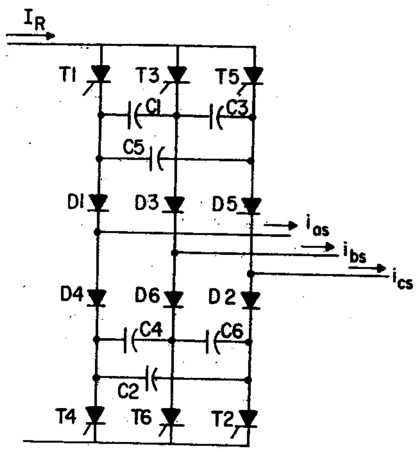

Fig. 2 Current source inverter with auto-sequential commutation.

In the steady-state analysis to follow, it is assumed that:

(a) The dc link current I_R is maintained constant by the bridge rectifier and the series choke.

(b) The CSI is a zero impedance switching device. In particular, the duration of the commutation time can be assumed to be negligibly small so that at the output frequency of interest the inverter current can be assumed as the quasi-square waveshapes shown in Fig. 3.

(c) The reluctance machine can be represented as an idealized machine with uniformly distributed stator windings so as to produce a single sinusoidally distributed space MMF in the air gap.

(d) Effects of saturation can be accounted for by appropriate equivalent saturated values of mutual reactances which can be assumed constant for a particular operating point.

(e) All machine parameters are constant.

(f) Rotor and load assembly inertia is sufficiently large to minimize rotor speed fluctuation caused by harmonic components of torque so that the speed can be assumed constant.

(g) The stator windings of the machine is wye-connected. If need be, delta-connected windings can be represented as an equivalent wye connection.

Basic Machine Equations

The role of reference frame transformations in machine analysis is analogous to that of simple changes of reference axes in coordinate geometry. Apart from providing a unified approach to machine analysis, these transformations bring about simplifications in the equations describing the machine. The physical interpretation to a reference frame transformation is the replacement of the actual machine windings by equivalent ficticious windings, the axes of whose MMFs are aligned with the newly defined axes of reference.

It is common in machine analysis to refer machine variables to one or more of the following reference frames:

- The stationary reference frame.
- The synchronously rotating reference frame.
- The reference frame attached to the rotor.

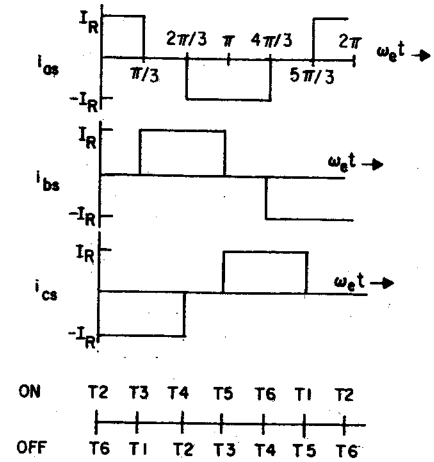

Fig. 3 Switching sequence and resulting phase currents.

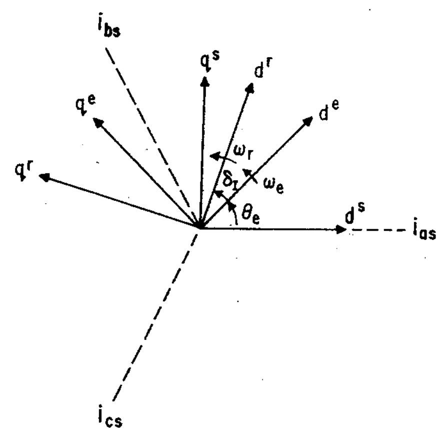

Fig. 4 Axes of reference.

For machines with asymmetrical rotor structure, simplification is gained by referring machine variables to the rotor reference frame. Following the usual convention, the d-axis of this reference frame is aligned with the path of minimum reluctance. For convenience, the axes of the above-mentioned reference frames relative to the axes of the stationary windings of a three phase machine have been sketched in Fig. 4.

The reluctance machine considered in this paper is a three-phase, three-wire machine. It has been shown that the rotor of the machine is adequately represented by one equivalent short-circuited rotor winding in both direct and quadrature axes. The voltage equations of the stator and rotor windings, which are obtained by analyzing the machine in a reference frame fixed on the rotor of the machine (Park's equations) are, in per unit [8]

$$v_{ds}^r = \frac{p}{\omega_b}\psi_{ds}^r - \frac{\omega_r}{\omega_b}\psi_{qs}^r + r_s i_{ds}^r \tag{1}$$

$$v_{qs}^r = \frac{p}{\omega_b}\psi_{qs}^r + \frac{\omega_r}{\omega_b}\psi_{ds}^r + r_s i_{qs}^r \tag{2}$$

$$0 = \frac{p}{\omega_b}\psi'^r_{dr} + r'_{dr} i'^r_{dr} \tag{3}$$

$$0 = \frac{p}{\omega_b}\psi'^r_{qr} + r'_{qr} i'^r_{qr} \tag{4}$$

where, in these equations the flux linkages

$$\psi_{ds}^r = x_{ds} i_{ds}^r + x_{ad} i'^r_{dr} \tag{5}$$

$$\psi_{qs}^{r} = x_{qs} i_{qs}^{r} + x_{aq} i_{qr}^{\prime r} \quad (6)$$

$$\psi_{dr}^{\prime r} = x_{dr}^{\prime} i_{dr}^{\prime r} + x_{ad} i_{ds}^{r} \quad (7)$$

$$\psi_{qr}^{\prime r} = x_{qr}^{\prime} i_{qr}^{\prime r} + x_{aq} i_{qs}^{r} \quad (8)$$

In the above equations the superscript r indicates variables in the rotor reference frame, the primed rotor quantities are referred to the stator windings by the appropriate turn-ratios and p denotes the operator d/dt. The additional subscripts s and r refer to stator and rotor quantities respectively. The electrical angular speed of the rotor is ω_r and ω_b is the base frequency. As a result of the assumptions cited, the zero-sequence stator and rotor quantities are absent, and hence, the equations relating these variables have been omitted.

Since ac motor drives involve both ac and dc quantities, it is advantageous to adopt a per unit system with peak rated line-to-neutral voltage and peak rated line current of the machine as base values. [8-10] The per unit instantaneous electromagnetic torque developed by a three-phase synchronous-reluctance machine can then be expressed as

$$T_e = (x_{ad} - x_{aq})\, i_{ds}^{r} i_{qs}^{r} + x_{ad} i_{dr}^{\prime r} i_{qs}^{r} - x_{aq} i_{qr}^{\prime r} i_{ds}^{r} \quad (9)$$

The torque as defined by Eq. 9 has a positive value for motor action.

METHOD OF SOLUTION

Transformation of the Input Currents

With stator currents as input variables and stator voltages and torque as outputs of the machine, it is convenient to rewrite the voltage equations, Eqs. 1-4

$$\frac{p}{\omega_b} i_{dr}^{\prime r} = \frac{-r_{dr}^{\prime}}{x_{dr}^{\prime}} i_{dr}^{\prime r} - \frac{p}{\omega_b}\left(\frac{x_{ad}}{x_{dr}^{\prime}} i_{ds}^{r}\right) \quad (10)$$

$$\frac{p}{\omega_b} i_{qr}^{\prime r} = \frac{-r_{qr}^{\prime}}{x_{qr}^{\prime}} i_{qr}^{\prime r} - \frac{p}{\omega_b}\left(\frac{x_{aq}}{x_{qr}^{\prime}} i_{qs}^{r}\right) \quad (11)$$

$$v_{ds}^{r} = r_s i_{ds}^{r} + \frac{p}{\omega_b}(x_{ds} i_{ds}^{r} + x_{ad} i_{dr}^{\prime r}) - \frac{\omega_r}{\omega_b}(x_{qs} i_{qs}^{r} + x_{aq} i_{qr}^{\prime r}) \quad (12)$$

$$v_{qs}^{r} = r_s i_{qs}^{r} + \frac{p}{\omega_b}(x_{qs} i_{qs}^{r} + x_{aq} i_{qr}^{\prime r}) + \frac{\omega_r}{\omega_b}(x_{ds} i_{ds}^{r} + x_{ad} i_{dr}^{\prime r}) \quad (13)$$

In order to completely characterize the system differential equations it is necessary to properly define the input variables in Eqs. 10-13. This is accomplished by transforming the stator phase currents shown in Fig. 3 to d-q stator currents i_{ds}^{r} and i_{qs}^{r} in the reference frame fixed on the rotor. For clarity, it is convenient to perform this transformation in three distinct stages. The three stages of variable transformation can be visualized geometrically by referring to Fig. 4. In the first stage the phase currents as shown in Fig. 3 are transformed to d-q components in the stationary reference frame using the following equations of transformation

$$f_{ds}^{s} = f_{as} \quad (14)$$

$$f_{qs}^{s} = \frac{1}{\sqrt{3}}(f_{bs} - f_{cs}) \quad (15)$$

where 'f' denotes either the symbol 'i' for current or 'v' for voltage. In the above equations of transformation, it is assumed that the d^s-axis is aligned with the stator current vector at t = 0. Since a three-wire system has been assumed the zero sequence quantities are again identically zero.

As a second step the d-q variables f^s in the stationary reference frame are transformed to d-q variables f^e in the synchronously rotating reference frame. In this case the desired transformation is of the form

$$[f_{ds}^{e}, f_{qs}^{e}]^t = \underline{T}(\theta_e)\, [f_{ds}^{s}, f_{qs}^{s}]^t \quad [16]$$

where

$$\underline{T}(\theta_e) = \begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix} \quad (17)$$

and t denotes the transpose. The angle θ_e is defined by Fig. 4 and, when the switching intervals are specified as in Fig. 3, $\theta_e = \omega_e t - \pi/6$ so that the d^s and d^e axes are aligned at $\omega_e t = \pi/6$.

The final step consists of transforming the d-q variables f^e in the synchronously rotating frame to d-q variables f^r in a reference frame fixed on the rotor using the transformation

$$[f_{ds}^{r}, f_{qs}^{r}]^t = \underline{T}(\delta_I)\, [f_{ds}^{e}, f_{qs}^{e}]^t \quad (18)$$

Here, $\underline{T}(\delta_I)$ is the same as $\underline{T}(\theta_e)$ with the argument θ_e replaced by δ_I. The angle δ_I is the rotor angle measured by the location of the d-axis of the machine relative to the d-axis of the synchronously rotating reference frame and is positive for generator action. It should be noted that the synchronously rotating reference frame is, in this case, defined by the rotation of a current rather than a voltage vector and hence δ_I is not the same as the conventional torque angle δ.

Input Currents

The thyristor gating sequence of the CSI together with the resulting output currents has been shown in Fig. 3. It can be noted that this mode of operation has six distinct states. In Fig. 5 the current paths through the machine stator windings for each of these six states are indicated. By applying the equations of transformation given by Eqs. 14-15, the d-q currents i_{ds}^{s} and i_{qs}^{s} in the stationary reference frame can be obtained and are given in Fig. 5 adjacent to the corresponding diagram for each state. It is apparent from Fig. 5 that when the inverter switching is ideal, stator currents i_{ds}^{s} and i_{qs}^{s} can be expressed as

$$[i_{ds}^{s}, i_{qs}^{s}]^t = I_R^{\prime}\, [g_{ds}^{s}, g_{qs}^{s}]^t \quad (19)$$

where $I_R^{\prime} = 2\sqrt{3} I_R / \pi$ and g_{ds}^{s} and g_{qs}^{s} are switching functions plotted in Fig. 6. Similarly the stator currents i_{ds}^{e} and i_{qs}^{e} in the synchronously rotating reference frame using the transformation given in Eqs. 16-17 can be written in terms of switching functions as

$$[i_{ds}^{e}, i_{qs}^{e}]^t = I_R^{\prime}\, [g_{ds}^{e}, g_{qs}^{e}]^t \quad (20)$$

where

$$[g_{ds}^{e}, g_{qs}^{e}]^t = \underline{T}(\theta_e)\, [g_{ds}^{s}, g_{qs}^{s}]^t \quad (21)$$

In a similar fashion in the rotor reference frame

$$[i_{ds}^{r}, i_{qs}^{r}]^t = I_R^{\prime}\, [g_{ds}^{r}, g_{qs}^{r}]^t \quad (22)$$

where

$$[g_{ds}^{r}, g_{qs}^{r}]^t = \underline{T}(\delta_I)\, \underline{T}(\theta_e)\, [g_{ds}^{s}, g_{qs}^{s}]^t \quad (23)$$

Over the basic interval defined by $0 < \omega_e t < \pi/3$, it can be shown that the functions g_{ds}^{r} and g_{qs}^{r} are defined by

$$g_{ds}^{r} = \frac{\pi}{3} \cos(\omega_e t - \pi/6 + \delta_I) \quad (24)$$

$$g_{qs}^{r} = \frac{-\pi}{3} \sin(\omega_e t - \pi/6 + \delta_I) \quad (25)$$

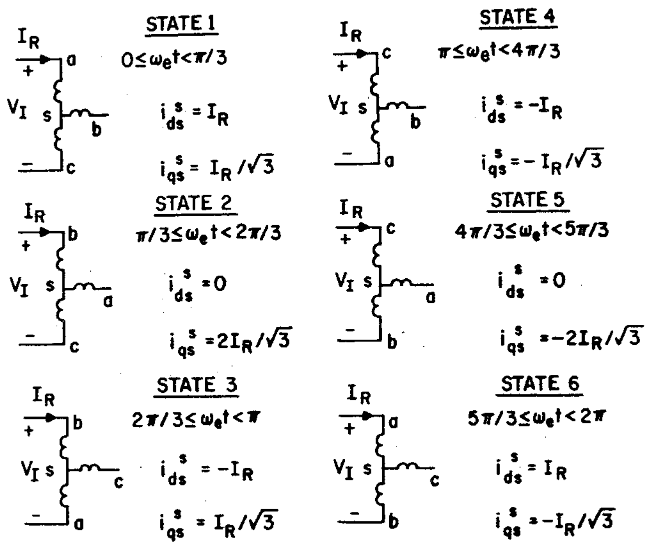

Fig. 5 The six inverter states and the resulting d-q currents.

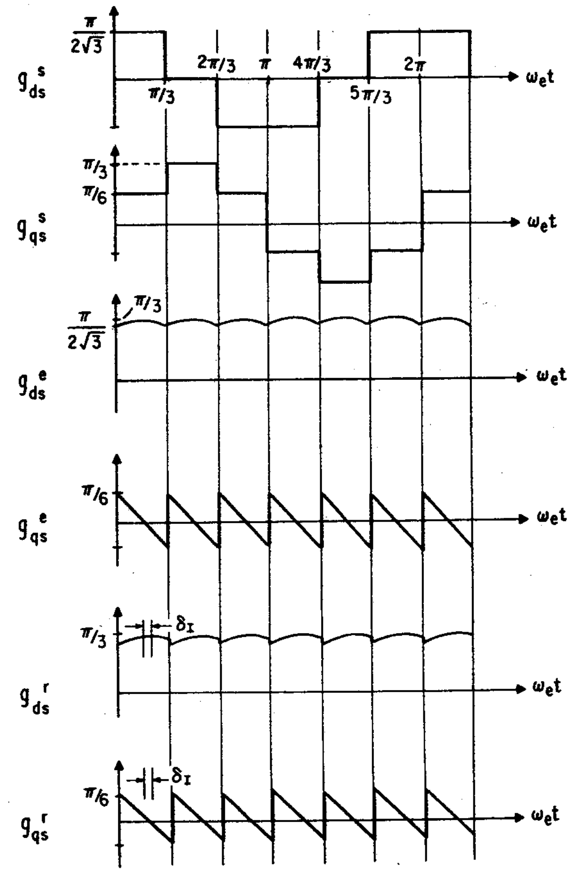

Fig. 6 The g functions.

The functions g_{ds}^e, g_{qs}^e and g_{ds}^r, g_{qs}^r are also plotted in Fig. 6, for negative δ_I (motor action). Since I_R is assumed constant and since $i_{ds}^s = g_{ds}^s I'_R$ etc., it is apparent that Fig. 6 also effectively defines the d-q stator current waveforms in the three reference frames.

From the plots of g_{ds}^r and g_{qs}^r and Eq. 22 it can be seen that both the input current i_{ds}^r and i_{qs}^r have discontinuous time derivatives for finite values of load angle δ_I. To avoid dealing with impulse functions arising from these inputs it is useful to define a new pair of pseudo-current variables i_D^r and i_Q^r.

$$i_D^r = i'^{\,r}_{dr} + \frac{x_{ad}}{x'_{dr}} i_{ds}^r \tag{26}$$

$$i_Q^r = i'^{\,r}_{qr} + \frac{x_{aq}}{x'_{qr}} i_{qs}^r \tag{27}$$

The motive behind this choice of variables is apparent from Eqs. 10-11. To lend a physical interpretation, it might be added that these variables are equivalent to rotor flux linkages, which by virtue of the theorem of constant flux linkages have continuous derivative over the switching instant. In terms of the new variables Eqs. 10-11 can now be written in matrix notation as

$$\frac{p}{\omega_b}\begin{bmatrix} i_D^r \\ i_Q^r \end{bmatrix} = \begin{bmatrix} \frac{-r'_{dr}}{x'_{dr}} & 0 \\ 0 & \frac{-r'_{qr}}{x'_{qr}} \end{bmatrix}\begin{bmatrix} i_D^r \\ i_Q^r \end{bmatrix} + \begin{bmatrix} \frac{r'_{dr} x_{ad}}{x'^{\,2}_{dr}} & 0 \\ 0 & \frac{r_{qr} x_{aq}}{x'^{\,2}_{qr}} \end{bmatrix}\begin{bmatrix} i_{ds}^r \\ i_{qs}^r \end{bmatrix} \tag{28}$$

Eq. 28 is in the conventional state variable form

$$\frac{p}{\omega_b} \underline{i}_{DQ}^r = \underline{A}\, \underline{i}_{DQ}^r + \underline{B}\, \underline{i}_{dqs}^r \tag{29}$$

In Eq. 29 the vectors are

$$\underline{i}_{DQ}^r = [i_D^r, i_Q^r]^t \tag{30}$$

$$\underline{i}_{dqs}^r = [i_{ds}^r, i_{qs}^r]^t \tag{31}$$

The matrices $\underline{A}$ and $\underline{B}$ are the system coefficient matrices.

If the initial condition and the input for Eq. 29 are known, the solution over the specified interval is defined in terms of a convolution integral. However, it is possible to include the input variables as state variables and thus eliminate the convolution integral term if we observe that

$$\frac{p}{\omega_e} i_{ds}^r = i_{qs}^r \tag{32}$$

$$\frac{p}{\omega_e} i_{qs}^r = -i_{ds}^r \qquad \text{for } 0 \leq t < \pi/3\omega_e \tag{33}$$

From the definitions of i_{ds}^r and i_{qs}^r in terms of the 'g' functions and from Eqs. 24 and 25, these currents have initial conditions

$$i_{ds}^r(0) = \frac{\pi}{3} I'_R \cos(\pi/6 - \delta_I) \tag{34}$$

$$i_{qs}^r(0) = \frac{\pi}{3} I'_R \sin(\pi/6 - \delta_I) \tag{35}$$

Defining the per unit frequency ratio

$$f_R = \omega_e/\omega_b \tag{36}$$

and the augmented current vector

$$\underline{I} = [i_D^r, i_Q^r, i_{ds}^r, i_{qs}^r]^t \tag{37}$$

Eq. 29 can then be written as

$$\frac{p}{\omega_b}\underline{I} = \hat{\underline{A}}\,\underline{I} \tag{38}$$

where

$$\hat{\underline{A}} = \begin{bmatrix} \frac{-r'_{dr}}{x'_{dr}} & 0 & \frac{r'_{dr}x_{ad}}{x'^{2}_{dr}} & 0 \\ 0 & \frac{-r'_{qr}}{x'_{qr}} & 0 & \frac{r'_{qr}x_{aq}}{x'^{2}_{qr}} \\ 0 & 0 & 0 & f_R \\ 0 & 0 & -f_R & 0 \end{bmatrix} \tag{39}$$

The solution of Eq. 38 is given by

$$\underline{I}(\omega_e t) = e^{\hat{\underline{A}}\omega_e t/f_R}\underline{I}(0) \tag{40}$$

where the matrix exponential term is the state transition matrix for the unforced four variable system. The system state transition matrix is given in Appendix I.

Periodicity Relations

Because of the symmetrical nature of the inverter switching it can be shown that it is not generally necessary to solve for the solution explicitly over a complete cycle. [5] If the solution over one basic switching interval is known, it can be used to generate the solution for the remaining intervals of an entire cycle of inverter operation. From the waveform properties of the balanced set of stator excitation currents it can be shown that the following periodic relations are valid for the stator d-q currents.

$$\begin{bmatrix} i_{ds}^r(\omega_e t + \pi/3) \\ i_{qs}^r(\omega_e t + \pi/3) \end{bmatrix} = \begin{bmatrix} i_{ds}^r(\omega_e t) \\ i_{qs}^r(\omega_e t) \end{bmatrix} \tag{41}$$

This periodicity of the d-q stator currents is also apparent from Fig. 6.

Since the system forcing functions, namely i_{ds}^r and i_{qs}^r, are periodic and since the differential equation describing the system is time-invariant, the system response currents i_D^r and i_Q^r have similar periodic relations

$$\begin{bmatrix} i_D^r(\omega_e t + \pi/3) \\ i_Q^r(\omega_e t + \pi/3) \end{bmatrix} = \begin{bmatrix} i_D^r(\omega_e t) \\ i_Q^r(\omega_e t) \end{bmatrix} \tag{42}$$

From Eqs. 26 and 27 the rotor currents i'^r_{dr} and i'^r_{qr} are written as a linear combination of the stator d-q currents and the pseudo-currents i_D^r and i_Q^r. Therefore, it is apparent that the same periodic symmetry applies for the d-q rotor currents.

Initial Values and Solution

As yet the initial values of i_D^r and i_Q^r are unknown. However, these currents can be obtained by using the periodic relations in Eq. 42 at $t = 0$ and $t = \pi/3\omega_e$, or

$$\begin{bmatrix} i_D^r(\pi/3) \\ i_Q^r(\pi/3) \end{bmatrix} = \begin{bmatrix} i_D^r(0) \\ i_Q^r(0) \end{bmatrix} \tag{43}$$

Substituting the above values of i_D^r and I_Q^r, with $t = \pi/3\omega_e$ into Eq. 40 and rearranging

$$[\underline{U} - e^{\frac{\pi}{3}\frac{\hat{A}}{f_R}}] \begin{bmatrix} i_D^r(0) \\ i_Q^r(0) \\ i_{ds}^r(0) \\ i_{qs}^r(0) \end{bmatrix} = \underline{0} \tag{44}$$

where $\underline{U}$ is a 4x 4 identity matrix. The quantities $i_D^r(0)$ and $i_Q^r(0)$ can now be solved in terms of the initial values of the stator currents given in Eqs. 34 and 35 and the elements of the $\hat{\underline{A}}$ matrix

$$i_D^r(0) = \frac{(\pi/3)\, r'_{dr}x_{ad}I'_R}{x'_{dr}\sqrt{r'^{2}_{dr} + f_R^2 x'^{2}_{dr}}\,[1 - \exp[-(\pi/3)(r'_{dr}/f_R x'_{dr})]]},$$

$$[\exp[-(\pi/3)(r'_{dr}/f_R x'_{dr})]\cos(\phi_d + \delta_I - \pi/6) - \cos(\phi_d + \delta_I + \pi/6)] \tag{45}$$

$$i_Q^r(0) = \frac{(\pi/3)\, r'_{qr}x_{aq}I'_R}{x'_{qr}\sqrt{r'^{2}_{qr} + f_R^2 x'^{2}_{qr}}\,(1 - \exp[-(\pi/3)(r'_{qr}/f_R x'_{qr})])},$$

$$[\exp[-(\pi/3)(r'_{qr}/f_R x'_{qr})]\sin(\phi_q + \delta_I - \pi/6) + \sin(\phi_q + \delta_I + \pi/6)]$$

where the angles (46)

$$\phi_d = \tan^{-1}(-f_R x'_{dr}/r'_{dr}) \tag{47}$$

$$\phi_q = \tan^{-1}(-f_R x'_{qr}/r'_{qr}) \tag{48}$$

Having obtained the initial value of the augmented current vector $\underline{I}(0)$ the solution for the system is then known over the interval $0 < \theta_e < \pi/3$ from Eq. 40. The complete closed form expressions for i_D^r and i_Q^r are given in Appendix II. Substitution for the known values of stator currents i_{ds}^r and i_{qs}^r and pseudo-currents i_D^r and i_Q^r in Eqs. 26 and 27 then gives the rotor currents. The values of rotor currents i'^r_{dr} and i'^r_{qr} and the pseudo-currents i_D^r and i_Q^r over the remaining five intervals of the cycle are known by virtue of the periodicity relations described by Eq. 42.

The solution for the rotor d-q currents, stator d-q voltages and the electromagnetic torque can be obtained from Eqs. 12-13 and Eq. 9 respectively. The stator phase voltages are obtained by transforming the stator d-q voltages in the rotor reference frame fixed on the rotor back to stator d-q voltages in the stationary reference frame using Eqs. 16 and 18, then to phase voltages using Eqs. 14-15.

Due to the presence of finite jumps in the stator current, the stator voltages contain impulses at these instances. Equations of solution for the rotor currents, phase voltages, electromagnetic torque are presented in Appendix II. Also included in Appendix II are expressions for the strength of the voltage impulses which ride on the stator voltages and the corresponding changes in magnetic energy which occur in the d- and q-axis magnetic field at instants of current switching.

CONCLUSION

In this paper the complete steady-state solution of a reluctance motor supplied from a current-source inverter has been obtained. Since the solution is expressed as an explicit function of time, the effects of all motor parameters on any variable can be readily observed. In the accompanying paper, Part II, the significant results of this analysis will be discussed and correlated with experimental results.

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APPENDIX I
State Transition Matrix of Augmented System

Taking the Laplace transform of both sides with respect to $\omega_b t$, Eq. 38 can be written in the form

$$\underline{I}(s) = [s\underline{U} - \underline{\hat{A}}]^{-1}\underline{I}(0) \tag{49}$$

where $\underline{U}$ is a 4× 4 identity matrix.

Expressed explicitly in term of the elements of $\underline{\hat{A}}$ the determinant of $[s\underline{U}-\underline{\hat{A}}]$ is

$$\det[s\underline{U}-\underline{\hat{A}}] = \left(s + \frac{r'_{dr}}{x'_{dr}}\right)\left(s + \frac{r'_{qr}}{x'_{qr}}\right)(s^2 + f_R{}^2) \tag{50}$$

The resolvent matrix $[s\underline{U} - \underline{\hat{A}}]^{-1}$ is

$$[s\underline{U} - \underline{\hat{A}}]^{-1} = \begin{bmatrix} \frac{1}{s + r'_{dr}/x'_{dr}} & 0 & \frac{r'_{dr}x_{ad}s}{x'^2_{dr}(s + r'_{dr}/x'_{dr})(s^2 + f_R{}^2)} & \frac{r'_{dr}x_{ad}f_R}{x'^2_{dr}(s + r'_{dr}/x'_{dr})(s^2 + f_R{}^2)} \\ 0 & \frac{1}{s + r'_{qr}/x'_{qr}} & \frac{-r'_{qr}x_{aq}f_R}{x'^2_{qr}(s + r'_{qr}/x'_{qr})(s^2 + f_R{}^2)} & \frac{r'_{qr}x_{aq}s}{x'^2_{qr}(s + r'_{qr}/x'_{qr})(s^2 + f_R{}^2)} \\ 0 & 0 & \frac{s}{s^2 + f_R{}^2} & \frac{f_R}{s^2 + f_R{}^2} \\ 0 & 0 & \frac{-f_R}{s^2 + f_R{}^2} & \frac{s}{s^2 + f_R{}^2} \end{bmatrix} \tag{51}$$

The state transition matrix of the system is obtained by taking the inverse Laplace transform of the resolvent matrix. The result is

$$e^{\underline{\hat{A}}\omega_e t/f_R} = \begin{bmatrix} e^{-r'_{dr}\omega_b t/x'_{dr}} & 0 & \frac{r'_{dr}x_{ad}}{x'_{dr}(r'^2_{dr} + f_R^2 x'^2_{dr})}[-r'_{dr}(e^{-r'_{dr}\omega_b t/x'_{dr}} - \cos\omega_e t) + f_R x'_{dr}\sin\omega_e t] & \frac{r'_{dr}x_{ad}}{x'_{dr}(r'^2_{dr} + f_R^2 x'^2_{dr})}[f_R x'_{dr}(e^{-r'_{dr}\omega_b t/x'_{dr}} - \cos\omega_e t) + r'_{dr}\sin\omega_e t] \\ 0 & e^{-r'_{qr}\omega_b t/x'_{qr}} & \frac{r'_{qr}x_{aq}}{x'_{qr}(r'^2_{qr} + f_R^2 x'^2_{qr})}[-f_R x'_{qr}(e^{-r'_{qr}\omega_b t/x'_{qr}} - \cos\omega_e t) - r'_{qr}\sin\omega_e t] & \frac{r'_{qr}x_{aq}}{x'_{qr}(r'^2_{qr} + f_R^2 x'^2_{qr})}[-r'_{qr}(e^{-r'_{qr}\omega_b t/x'_{qr}} - \cos\omega_e t) + f_R x'_{qr}\sin\omega_e t] \\ 0 & 0 & \cos\omega_e t & \sin\omega_e t \\ 0 & 0 & -\sin\omega_e t & \cos\omega_e t \end{bmatrix} \tag{52}$$

APPENDIX II
Closed Form Solution

Pseudo-Currents i_D^r and i_Q^r

From the expression for the solution given by Eq. 40 and the initial values of the pseudo-currents i_D^r and i_Q^r in Eqs. 47-48, the values of the pseudo-currents within any interval $(k-1)\,\pi/3 \leq \omega_e t < k\pi/3$, $k = 1, 2, \ldots, 6$ are

$$i_D^r(\omega_e t) = \frac{(\pi/3)\,r'_{dr}x_{ad}I'_R}{x'_{dr}\sqrt{r'^2_{dr} + f_R{}^2 x'^2_{dr}}}\Big[-\cos(\omega_e t + \delta_I + \phi_d - \pi/6) - \frac{\exp(-r'_{dr}\omega_e t/f_R x'_{dr})}{1 - \exp(-\pi r'_{dr}/3 f_R x'_{dr})}\,[\cos(\delta_I + \phi_d + \pi/6) - \cos(\delta_I + \phi_d - \pi/6)]\Big] \tag{53}$$

$$i_Q^r(\omega_e t) = \frac{(\pi/3)\, r'_{qr} x_{aq} I'_R}{x'_{qr}\sqrt{r'^2_{qr} + f_R^2 x'^2_{qr}}} [\sin(\omega_e t + \delta_I + \phi_q - \pi/6)$$

$$+ \frac{\exp(-r'_{qr}\omega_e t/f_R x'_{qr})}{1-\exp(-\pi r'_{qr}/3 f_R x'_{qr})} [\sin(\delta_I + \phi_q + \pi/6) - \sin(\delta_I + \phi_q - \pi/6)]] \quad (54)$$

Rotor d-q Currents

$$i'^r_{dr}(\omega_e t) = \frac{-(\pi/3)\, f_R x_{ad} I'_R}{\sqrt{r'^2_{dr} + f_R^2 x'^2_{dr}}} [\sin(\omega_e t + \delta_I + \phi_d - \pi/6)$$

$$- \frac{r'_{dr}\exp(-r'_{dr}\omega_e t/f_R x'_{dr})}{f_R x'_{dr}[1-\exp(-\pi r'_{dr}/3 f_R x'_{dr})]} [\cos(\delta_I + \phi_d + \pi/6) - \cos(\delta_I + \phi_d - \pi/6)]] \quad (55)$$

$$i'^r_{qr}(\omega_e t) = \frac{-(\pi/3)\, f_R x_{aq} I'_R}{\sqrt{r'^2_{qr} + f_R^2 x'^2_{qr}}} [\cos(\omega_e t + \delta_I + \phi_q - \pi/6)$$

$$- \frac{r'_{qr}\exp(-r'_{qr}\omega_e t/f_R x'_{qr})}{f_R x'_{qr}[1-\exp(-\pi r'_{qr}/3 f_R x'_{qr})]} [\sin(\delta_I + \phi_q + \pi/6) - \sin(\delta_I + \phi_q - \pi/6)]] \quad (56)$$

Stator d-q Voltages

The d-q stator voltages expressed in terms of the d-q pseudo-currents and d-q stator input currents in a general interval $(k-1)\pi/3 < \omega_e t < k\pi/3$ are

$$v_{ds}^r(\omega_e t) = (\pi/3\omega_b) I'_R (x_{ds} - x_{ad}^2/x'_{dr}) \sin\delta_I U_1 [(k-1)\pi/3)]$$

$$+ (r_s + r'_{dr} x_{ad}^2/x'^2_{dr})\, i_{ds}^r + f_R [(x_{ds} - x_{qs}) - (x_{ad}^2/x'_{dr} - x_{aq}^2/x'_{qr})]\, i_{qs}^r$$

$$- x_{ad} r'_{dr} i_D^r/x'_{dr} - f_R x_{aq} i_Q^r \quad (57)$$

$$v_{qs}^r(\omega_e t) = (\pi/3\omega_b) I'_R (x_{qs} - x_{aq}^2/x'_{qr}) \cos\delta_I U_1 [(k-1)\pi/3)]$$

$$+ (r_s + r'_{qr} x_{aq}^2/x'^2_{qr})\, i_{qs}^r + f_R [(x_{ds} - x_{qs}) - (x_{ad}^2/x'_{dr} - x_{aq}^2/x'_{qr})]\, i_{ds}^r$$

$$- x_{aq} r'_{qr} i_Q^r/x'_{qr} + f_R x_{ad} i_D^r \quad (58)$$

In Eqs. 57 and 58 the symbol U_1 (T) denotes the unit impulse function at t = T. Note that the first terms in these equations correspond to the impulse 'strengths' at the switching instants. The effective reactances opposing these step changes of currents are the d- and q-axes transient reactances.

The stator phase voltages are readily obtained by transforming the d-q stator voltages in the rotor reference frame back to a-b-c quantities. That is

$$v_{as} = v_{ds} \cos\theta_r - v_{qs}^r \sin\theta_r \quad (59)$$

$$v_{bs} = v_{ds}^r \cos(\theta_r - 2\pi/3) - v_{qs}^r \sin(\theta_r - 2\pi/3) \quad (60)$$

$$v_{cs} = v_{ds}^r \cos(\theta_r + 2\pi/3) - v_{qs}^r \sin(\theta_r + 2\pi/3) \quad (61)$$

Change in Magnetic Field Energy

Accompanying the step change in stator currents a sudden change in the motor d- and q-axes magnetic field energy occurs given by the following expressions

$$\Delta W_d = (\pi^2 I'^2_R/18\omega_b)(x_{ds} - x_{ad}^2/x'_{dr}) \sin^2\delta_I \quad (62)$$

$$\Delta W_q = (\pi^2 I'^2_R/18\omega_b)(x_{qs} - x_{aq}^2/x'_{qr}) \cos^2\delta_I \quad (63)$$

Electromagnetic Torque

The instantaneous torque can be obtained from the expressions for the instantaneous stator and rotor currents. Substituting Eq. 22, 55 and 56 in Eq. 9, the instantaneous torque over any 60° interval may be expressed as

$$T_e = -(\pi^2 I'^2_R/18)(x_{ad} - x_{aq}) \sin 2(\omega_e t + \delta_I - \pi/6)$$

$$+ (\pi^2 I'^2_R/18)(x_{ad}^2/x'_{dr} - x_{aq}^2/x'_{qr}) \sin 2(\omega_e t + \delta_I - \pi/6)$$

$$- C_d \sin\phi_d - C_q \sin\phi_q + C_d \sin 2(\omega_e t + \delta_I + \phi_d/2 - \pi/6)$$

$$- C_q \sin 2(\omega_e t + \delta_I + \phi_q/2 - \pi/6)$$

$$- \frac{2C_d \exp(-r'_{dr}\omega_e t/f_R x'_{dr})}{1-\exp(-\pi r'_{dr}/3 f_R x'_{dr})} \sin(\delta_I + \phi_d) \sin(\omega_e t + \delta_I - \pi/6)$$

$$- \frac{2C_q \exp(-r'_{qr}\omega_e t/f_R x'_{qr})}{1-\exp(-\pi r'_{qr}/3 f_R x'_{qr})} \cos(\delta_I + \phi_q) \cos(\omega_e t + \delta_I - \pi/6) \quad (64)$$

where

$$C_d = \frac{(\pi^2 I'^2_R/18)\, r'_{dr} x_{ad}^2}{x'_{dr}\sqrt{r'^2_{dr} + f_R^2 x'^2_{dr}}} \quad (65)$$

$$C_q = \frac{(\pi^2 I'^2_R/18)\, r'_{qr} x_{aq}^2}{x'_{qr}\sqrt{r'^2_{qr} + f_R^2 x'^2_{qr}}} \quad (66)$$

The average torque developed by the machine calculated from the mean-value of the instantaneous torque is

$$T_e(\text{avg}) = -(\pi\sqrt{3} I'^2_R/12)\, [(x_{ad} - x_{aq}) \sin 2\delta_I$$

$$+ (x_{ad}^2/x'_{dr} - x_{aq}^2/x'_{qr}) \sin 2\delta_I] - C_d \sin\phi_d$$

$$- C_q \sin\phi_q + (3\sqrt{3}/2\pi)\, [C_d \sin(2\delta_I + \phi_d) - C_q \sin(2\delta_I + \phi_q)]$$

$$- \frac{(6/\pi)\, C_d f_R x'_{dr}}{\sqrt{r'^2_{dr} + f_R^2 x'^2_{dr}}} \sin(\delta_I + \phi_d)\, [\exp(-\pi r'_{dr}/3 f_R x'_{dr}) \cdot$$

$$\sin(\delta_I - \phi_d + \pi/6) - \sin(\delta_I - \phi_d - \pi/6)]$$

$$- \frac{(6/\pi)\, C_q f_R x'_{qr}}{\sqrt{r'^2_{qr} + f_R^2 x'^2_{qr}}} \cos(\delta_I + \phi_q)\, [\exp(-\pi r'_{qr}/3 f_R x'_{qr}) \cdot$$

$$\cos(\delta_I - \phi_q + \pi/6) - \cos(\delta_I - \phi_q - \pi/6)] \quad (67)$$