SIMULATION OF A SYNCHRONOUS MACHINE WITH AN OPEN PHASE

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ABSTRACT

Techniques are described for simulating an open circuit of a three phase synchronous machine using Park's equations. Set forth in detail is an exact analytical technique that results in an all-dq0 simulation eliminating entirely the need for matrix inversion inherent to the commonly used phase variable approach. In addition, it is shown that this method of simulation markedly reduces the computation time. This work sets the stage for an all dq0 simulation of unbalanced operation of power systems wherein variable step size integration may be employed to full advantage. Also reported is the application of this technique to the simulation of a single phase synchronous machine.

INTRODUCTION

Transient analyses of power system networks during system faults has traditionally been a key tool for the evaluation of power system performance. Because of the ease in its implementation, three phase faults are generally studied since the network remains balanced. Because of the balanced conditions conventional dq0 variables can be used to represent the network as well as the synchronous machine during all stages of the fault sequence. Since dq0 variables become constants during the steady-state, the full power of the digital computer is brought to bear by utilizing variable step size integration whereby the computational step size is automatically increased as the new steady-state condition is approached. Although less severe, unbalanced faults such as line-to-ground faults occur much more frequently in a power system. For this reason, unbalanced faults are being increasingly used as the stability criterion for a power system. In this case, three phase faults are handled by isolating part of the network should instability occur.

Conventional techniques for analyzing unbalanced faults utilize symmetrical component theory. In this approach three sequence networks are identified and interconnected according to the fault condition. The resulting circuit is solved as one large equivalent network. Whereas the machine time constants might be modeled accurately, network currents are processed as if they were simple algebraic quantities. Although the machine equations are nonlinear, superposition theory is inherently assumed. Although adequate for electromechanical swings, numerous important effects cannot be accurately calculated. These effects include pulsating torques due to negative sequence currents which may result in shaft resonances, increased losses and damping resulting from negative sequence rotor currents, and recovery voltages across line breaker and arrestors which are critical for sizing such equipment.

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One straightforward alternative to the use of sequence networks which has been extensively studied is to simulate the synchronous machine as well as the network directly in phase variables (abc variables).[1-4] Unfortunately, this type of simulation is inherently slow since it requires updating and inverting the machine inductance matrix at each time increment.

Other "hybrid" representations have been developed which can also accommodate unbalanced fault conditions.[5-8] In this case dq0 variables are utilized for the machine and phase variables used to model the network. Axes transformation equations are then required to interface each machine to the network. When time rate of change of voltages and currents are included, solution of the network by phase variables is again slow, since 60 Hertz line frequencies are involved during both transient and steady-state operation. Alternatively, multiple switching between balanced and unbalanced conditions requires storing two models of the network to minimize computing time which consequently taxes the storage capacity of the computer.

Another logical alternative is to extend the approach for three phase faults and to simulate both the machines and the network directly in dq0 variables. Conventional representations of both machine and network could then be used and all relevant transient effects could be studied. Appreciable savings in computation time would again be possible by use of variable-step size integration. The potential of this approach was recognized in (9). However, the numerical methods required as a result of the formulation imposed serious restrictions since numerical differentiation was needed. Also, arbitrary approximations were assigned to quantities in order to avoid a divergent solution.

As evidenced by (9) a major obstacle in implementing an all dq0 approach is the development of suitable models for each faulted condition which does not result in numerical instability. Such models have recently been reported for both line-to-line and line-to-ground faults.[10,11] A related problem which has not yet been addressed is proper modeling of the open circuit condition which occurs after the fault is cleared. Whereas a three phase open circuit at the point of fault is balanced and hence straightforward, single phase open circuits which typically occur during single-pole breaker operation present a formidable problem.[12] In this paper it is shown that the dq0 simulation of an open circuit can be accomplished by applying, in the simulation, the appropriate voltage to the dq0 equivalent circuit so as to maintain a current zero in the open phase. The approach totally eliminates computational stability problems and matrix inversion is not required. Also, this paper describes how this technique can be extended to simulate a single phase synchronous machine. The development of this open circuit model sets the stage for an all dq0 representation of unbalanced operation of a power system.

SYNCHRONOUS MACHINE REPRESENTATION

The equations which describe the behavior of a synchronous machine (Park's equations) may be expressed[13,14]

$$v_q = \frac{p}{\omega_b}\psi_q + \psi_d\frac{\omega_r}{\omega_b} + r_a i_q \tag{1}$$

$$v_d = \frac{p}{\omega_b}\psi_d - \psi_q\frac{\omega_r}{\omega_b} + r_a i_d \tag{2}$$

$$v_0 = \frac{p}{\omega_b}\psi_0 + r_a i_0 \tag{3}$$

$$0 = \frac{p}{\omega_b} \psi_{kq} + r_{kq} i_{kq} \tag{4}$$

$$0 = \frac{p}{\omega_b} \psi_{kd} + r_{kd} i_{kd} \tag{5}$$

$$E_x = \frac{X_{ad}}{r_{fd}} \left(\frac{p}{\omega_b} \psi_{fd} + r_{fd} i_{fd} \right) \tag{6}$$

In the above equations, ω_b is the base electrical angular velocity, ω_r is the electrical angular velocity of the rotor, and p is the operator d/dt, also

$$\psi_q = X_{\ell a} i_q + \psi_{aq} \tag{7}$$

$$\psi_d = X_{\ell a} i_d + \psi_{ad} \tag{8}$$

$$\psi_0 = X_{\ell a} i_0 \tag{9}$$

$$\psi_{kq} = X_{\ell kq} i_{kq} + \psi_{aq} \tag{10}$$

$$\psi_{kd} = X_{\ell kd} i_{kd} + \psi_{ad} \tag{11}$$

$$\psi_{fd} = X_{\ell fd} i_{fd} + \psi_{ad} \tag{12}$$

where by definition

$$\psi_{aq} = X_{aq}(i_q + i_{kq}) \tag{13}$$

$$\psi_{ad} = X_{ad}(i_d + i_{fd} + i_{kd}) \tag{14}$$

The parameters used in the above equations are defined as

r_a = armature resistance

$X_{\ell a}$ = armature leakage reactance

r_{fd} = field resistance

$X_{\ell fd}$ = field leakage reactance

r_{kd} = d-axis damper resistance

$X_{\ell kd}$ = d-axis damper leakage reactance

r_{kq} = q-axis damper resistance

$X_{\ell kq}$ = q-axis damper leakage reactance

X_{ad} = d-axis magnetizing reactance

X_{aq} = q-axis magnetizing reactance

If Eqs. 7-12 are solved for the currents and the results substituted into Eqs. 1-6, the equations may be arranged in the following form, convenient for computer implementation [14],

$$\psi_q = \frac{\omega_b}{p} \left[v_q - \psi_d \frac{\omega_r}{\omega_b} + \frac{r_a}{X_{\ell a}} (\psi_{aq} - \psi_q) \right] \tag{15}$$

$$\psi_d = \frac{\omega_b}{p} \left[v_d + \psi_q \frac{\omega_r}{\omega_b} + \frac{r_a}{X_{\ell a}} (\psi_{ad} - \psi_d) \right] \tag{16}$$

$$\psi_0 = \frac{\omega_b}{p} \left(v_0 - \frac{r_a}{X_{\ell a}} \psi_0 \right) \tag{17}$$

$$\psi_{kq} = \frac{\omega_b}{p} \frac{r_{kq}}{X_{\ell kq}} (\psi_{aq} - \psi_{kq}) \tag{18}$$

$$\psi_{kd} = \frac{\omega_b}{p} \frac{r_{kd}}{X_{\ell kd}} (\psi_{ad} - \psi_{kd}) \tag{19}$$

$$\psi_{fd} = \frac{\omega_b}{p} \left[\frac{r_{fd}}{X_{ad}} E_x + \frac{r_{fd}}{X_{\ell fd}} (\psi_{ad} - \psi_{fd}) \right] \tag{20}$$

where

$$\psi_{aq} = X_{mq} \left(\frac{\psi_q}{X_{\ell a}} + \frac{\psi_{kq}}{X_{\ell kq}} \right) \tag{21}$$

$$\psi_{ad} = X_{md} \left(\frac{\psi_d}{X_{\ell a}} + \frac{\psi_{kd}}{X_{\ell kd}} + \frac{\psi_{fd}}{X_{\ell fd}} \right) \tag{22}$$

in which

$$X_{mq} = \left[\frac{1}{X_{\ell a}} + \frac{1}{X_{aq}} + \frac{1}{X_{\ell kq}} \right]^{-1} \tag{23}$$

$$X_{md} = \left[\frac{1}{X_{\ell a}} + \frac{1}{X_{ad}} + \frac{1}{X_{\ell kd}} + \frac{1}{X_{\ell fd}} \right]^{-1} \tag{24}$$

The above integral equations are in terms of voltages and flux linkages, however, the currents can be expressed from Eqs. 7-12 and Eqs. 21 and 22. Hence, the equations give rise to a voltage input-current output simulation. Saturation may be taken into account by a method set forth by Thomas [14].

The per unit torque, positive for motor action, is expressed

$$T_e = \psi_d i_q - \psi_q i_d \tag{25}$$

The rotor speed is expressed

$$\omega_r = \frac{\omega_b}{2H\,p}(T_e - T_m) \tag{26}$$

where H is the inertia constant in seconds, and T_m is positive for load torque.

Assuming that a phase current ceases to flow after a normal current zero, the phase current can be maintained at zero by replacing, in the simulation, the system voltage with the appropriate open circuit voltage. If, for example, i_a in Fig. 1 is to be held at zero after a normal current zero, then the applied voltages which will force i_a to remain at zero are

$$v_a = \frac{p}{\omega_b} \psi_a \tag{27}$$

$$v_b = e_{gb} - v_{ng} \tag{28}$$

$$v_c = e_{gc} - v_{ng} \tag{29}$$

It is, however, necessary to express ψ_a in terms of the appropriate d and q variables to maintain i_a at zero. Using Park's transformation for the three wire system considered, ψ_a may be expressed

$$\psi_a = \psi_q \cos\theta_r + \psi_d \sin\theta_r \tag{30}$$

where θ_r is the rotor electrical angular displacement. The restriction of zero i_a yields

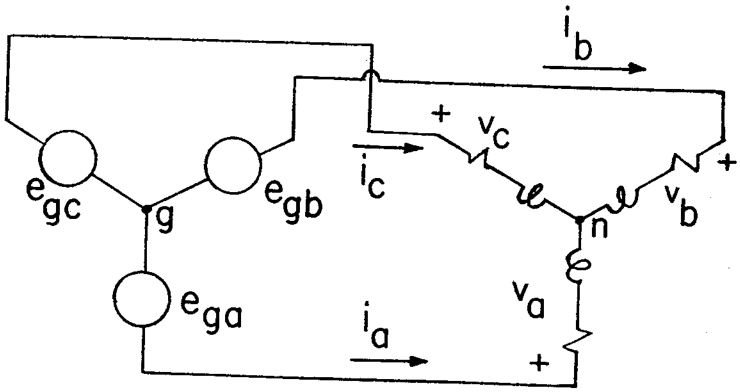

Fig. 1. System studied.

$$0 = i_q \cos\theta_r + i_d \sin\theta_r \tag{31}$$

If Eqs. 7 and 8 are solved for i_q and i_d, respectively, and the results substituted into Eq. 31, Eq. 31 may be written

$$0 = \frac{1}{X_{\ell a}} \left[(\psi_q - \psi_{aq}) \cos\theta_r + (\psi_d - \psi_{ad}) \sin\theta_r\right] \tag{32}$$

Equation 32 may be put in the form

$$\psi_q \cos\theta_r + \psi_d \sin\theta_r = \psi_{aq} \cos\theta_r + \psi_{ad} \sin\theta_r \tag{33}$$

Comparing Eqs. 30 and 33, it is clear that for zero i_a

$$\psi_a = \psi_{aq} \cos\theta_r + \psi_{ad} \sin\theta_r \tag{34}$$

Equation 34 may be written in terms of state variables by substituting Eqs. 21 and 22 for ψ_{aq} and ψ_{ad}, respectively. Hence, v_a during $i_a = 0$ becomes

$$v_a = \frac{p}{\omega_b} \left[X_{mq} \left(\frac{\psi_q}{X_{\ell a}} + \frac{\psi_{kq}}{X_{\ell kq}}\right) \cos\theta_r + X_{md} \left(\frac{\psi_d}{X_{\ell a}} + \frac{\psi_{fd}}{X_{\ell fd}} + \frac{\psi_{kd}}{X_{\ell kd}}\right) \sin\theta_r\right] \tag{35}$$

In the case of a three wire machine it can be shown that

$$v_a + v_b + v_c = 0 \tag{36}$$

Thus, v_{ng} may be expressed by adding Eqs. 28 and 29 and utilizing Eq. 36,

$$v_{ng} = \tfrac{1}{2}\left(e_{gb} + e_{gc} + v_a\right) \tag{37}$$

If Eq. 37 is substituted into Eqs. 28 and 29 and then Park's equations are used to transform v_a, v_b and v_c to the reference frame fixed in the rotor, v_q and v_d become

$$v_q = v_a \cos\theta_r + \frac{1}{\sqrt{3}} (e_{gb} - e_{gc}) \sin\theta_r \tag{38}$$

$$v_d = v_a \sin\theta_r - \frac{1}{\sqrt{3}} (e_{gb} - e_{gc}) \cos\theta_r \tag{39}$$

It should be noted that Eqs. 38 and 39 are valid for the three wire system shown in Fig. 1 for both zero and nonzero values of i_a. For normal operations $v_a = e_{ga}$, for open circuit operation ($i_a = 0$) v_a in Eqs. 38 and 39 is replaced by Eq. 35 at the instant of a normal current zero.

Because of the nature of digital computation it is advantageous to express the voltage equations in a slightly different form. This becomes apparent once Eqs. 38 and 39, with v_a replaced by Eq. 35, are substituted into Eqs. 15 and 16 to simulate an open circuit. In this case ψ_q and ψ_d are expressed in terms of the derivatives of ψ_q and ψ_d making it necessary to perform numerical integration of the derivative of the state variables being computed. This problem can be overcome by rearranging Eqs. 15 and 16. If Eq. 35 is substituted into Eqs. 38 and 39 and the resulting expressions for v_q and v_d substituted into Eqs. 15 and 16, ψ_q and ψ_d may be expressed in matrix form as follows

$$\begin{bmatrix} \psi_q \\ \psi_d \end{bmatrix} = \frac{\omega_b}{p} B^{-1} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \qquad (40)$$

where

$$B = \begin{bmatrix} 1 - \frac{X_{mq}}{X_{\ell a}} \cos^2 e_r & -\frac{X_{md}}{X_{\ell a}} \tfrac{1}{2} \sin 2\theta_r \\ -\frac{X_{mq}}{X_{\ell a}} \tfrac{1}{2} \sin 2\theta_r & 1 - \frac{X_{md}}{X_{\ell a}} \sin^2\theta_r \end{bmatrix} \qquad (41)$$

The variables ϕ_1 and ϕ_2 may be written

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots\alpha_{17} \\ \alpha_{21} & \alpha_{22} & \cdots\alpha_{27} \end{bmatrix} \begin{bmatrix} \psi_q \\ \psi_d \\ \psi_{kq} \\ \psi_{kd} \\ \psi_{fd} \\ \psi_{aq} \\ \psi_{ad} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \end{bmatrix} \begin{bmatrix} E_x \\ e_{gb} \\ e_{gc} \end{bmatrix} \qquad (42)$$

where

$$\alpha_{11} = -\frac{\omega_r X_{mq}}{\omega_b X_{\ell a}} \tfrac{1}{2} \sin 2\theta_r - \frac{r_a}{X_{\ell a}} \qquad (43)$$

$$\alpha_{12} = \frac{\omega_r X_{md}}{\omega_b X_{\ell a}} \cos^2\theta_r - \frac{\omega_r}{\omega_b} \qquad (44)$$

$$\alpha_{13} = \frac{-X_{mq} r_{kq}}{X_{\ell kq}^2} \cos^2\theta_r - \frac{\omega_r X_{mq}}{\omega_b X_{\ell kq}} \tfrac{1}{2} \sin 2\theta_r \qquad (45)$$

$$\alpha_{14} = \frac{-X_{md} r_{kd}}{X_{\ell kd}^2} \tfrac{1}{2} \sin 2\theta_r + \frac{\omega_r X_{md}}{\omega_b X_{\ell kd}} \cos^2\theta_r \qquad (46)$$

$$\alpha_{15} = \frac{-X_{md} r_{fd}}{X_{\ell fd}^2} \tfrac{1}{2} \sin 2\theta_r + \frac{\omega_r X_{md}}{\omega_b X_{\ell fd}} \cos^2\theta_r \qquad (47)$$

$$\alpha_{16} = \frac{X_{mq} r_{kq}}{X_{\ell kq}^2} \cos^2\theta_r + \frac{r_a}{X_{\ell a}} \qquad (48)$$

$$\alpha_{17} = \frac{X_{md}r_{fd}}{X_{\ell fd}^2}\tfrac{1}{2}\sin 2\theta_r + \frac{X_{md}r_{kd}}{X_{\ell kd}^2}\tfrac{1}{2}\sin 2\theta_r \tag{49}$$

$$\alpha_{21} = \frac{-\omega_r X_{mq}}{\omega_b X_{\ell a}}\sin^2\theta_r + \frac{\omega_r}{\omega_b} \tag{50}$$

$$\alpha_{22} = \frac{\omega_r X_{md}}{\omega_b X_{\ell a}}\tfrac{1}{2}\sin 2\theta_r - \frac{r_a}{X_{\ell a}} \tag{51}$$

$$\alpha_{23} = \frac{-X_{mq}r_{kq}}{X_{\ell kq}^2}\tfrac{1}{2}\sin 2\theta_r - \frac{\omega_r X_{mq}}{\omega_b X_{\ell kq}}\sin^2\theta_r \tag{52}$$

$$\alpha_{24} = \frac{-X_{md}r_{kd}}{X_{\ell kd}^2}\sin^2\theta_r + \frac{\omega_r X_{md}}{\omega_b X_{\ell kd}}\tfrac{1}{2}\sin 2\theta_r \tag{53}$$

$$\alpha_{25} = \frac{-X_{md}r_{fd}}{X_{\ell fd}^2}\sin^2\theta_r + \frac{\omega_r X_{md}}{\omega_b X_{\ell fd}}\tfrac{1}{2}\sin 2\theta_r \tag{54}$$

$$\alpha_{26} = \frac{X_{mq}r_{kq}}{X_{\ell kq}^2}\tfrac{1}{2}\sin 2\theta_r \tag{55}$$

$$\alpha_{27} = \frac{X_{md}r_{fd}}{X_{\ell fd}^2}\sin^2\theta_r + \frac{X_{md}r_{kd}}{X_{\ell kd}^2}\sin^2\theta_r + \frac{r_a}{X_{\ell a}} \tag{56}$$

$$\beta_{11} = \frac{X_{md}r_{fd}}{X_{\ell fd}X_{ad}}\tfrac{1}{2}\sin 2\theta_r \tag{57}$$

$$\beta_{12} = \frac{1}{\sqrt{3}}\sin\theta_r \tag{58}$$

$$\beta_{13} = -\frac{1}{\sqrt{3}}\sin\theta_r \tag{59}$$

$$\beta_{21} = \frac{X_{md}r_{fd}}{X_{\ell fd}X_{ad}}\sin^2\theta_r \tag{60}$$

$$\beta_{22} = \frac{-1}{\sqrt{3}}\cos\theta_r \tag{61}$$

$$\beta_{23} = \frac{1}{\sqrt{3}}\cos\theta_r \tag{62}$$

It should be noted that the quantity ($\tfrac{1}{2}\sin 2\theta_r$) can be determined from the identity $\sin 2\theta_r = 2\cos\theta_r \sin\theta_r$ so that only $\sin\theta_r$ and $\cos\theta_r$ need be computed at each time step. This formulation yields a set of simultaneous differential equations without numerical differentiation or repetitive matrix inversion.

RESULTS OF COMPUTER STUDIES

Figure 2 shows the results of a computer study of opening and reclosing of phase a of the system shown in Fig. 1 using the method of representation

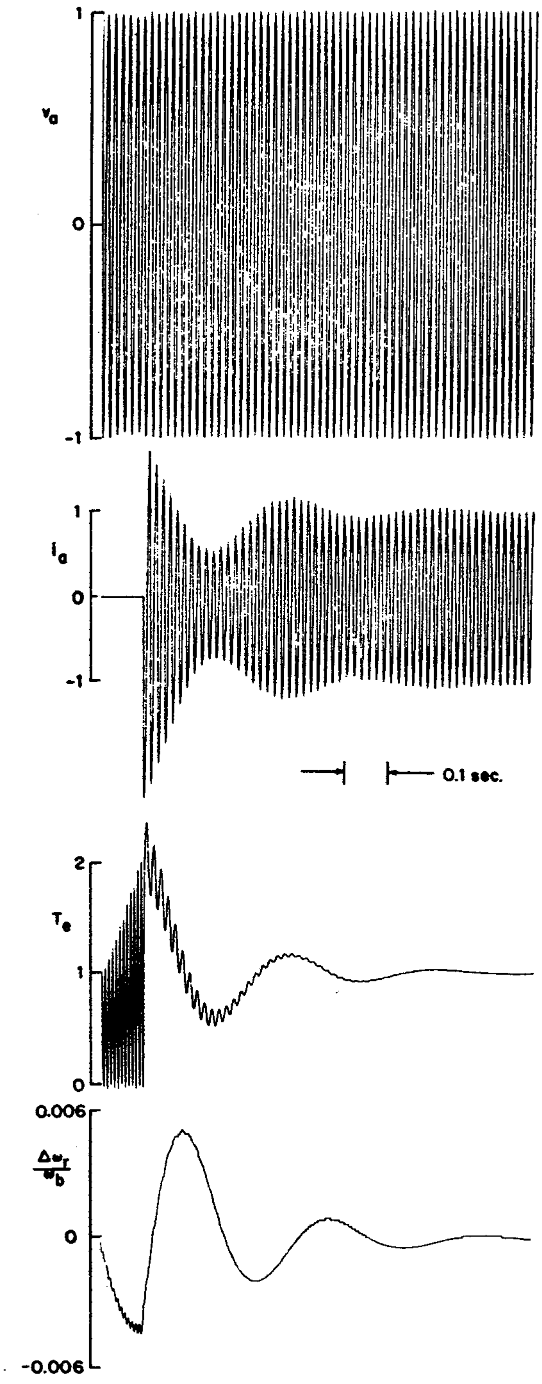

Fig. 2. Opening and reclosing phase a of a synchronous machine - digital computer study.

established in the previous section. The following variables are plotted; v_a - phase a voltage, i_a - phase a current, T_e - electromagnetic torque, and $\Delta\omega_r/\omega_b$ change in rotor speed. The per unit parameters of the machine are given in the Appendix. Initially the machine is operating normally as a motor with 1.0 per unit load torque and 1.0 per unit source voltage. The machine is overexcited with the excitation set to 2.0 per unit. Phase a is opened at a normal current zero and help open for 0.1 sec. whereupon the source voltage is reapplied.

In order to compare the core requirements and execution time of the new method using direct and quadrature axis flux linkages with the standard approach which uses machine currents as state variables, the algorithm given in [1] was used to simulate the same problem. Both digital simulations were programmed on a CDC 1700 digital computer using a time increment of 0.001 sec. with a fourth order Runge-Kutta numerical integration routine. Using the direct and quadrature axis flux linkages as state variables the core requirements were 3826 computer words with an execution time per increment of 0.14 sec. Using phase variables the core requirements were 5092 computer words and an execution time of 0.62 sec. per increment was required. Clearly, the direct and quadrature axis approach is much faster. Moreover, the direct and quadrature axis flux linkages are similar in waveform to the torque and become constant during steady state operation and pulsating only during the time the phase is open circuited. Hence, the execution time can be markedly reduced by using a variable step size integration routine. This advantage is not possible in the simulation employing phase variables as state variables since in the steady state the phase variables are sinusoidal in waveform.

The techniques used to simulate an open phase of a three phase machine may also be used to simulate a single phase synchronous machine. This can be accomplished by first assuming the single phase machine to be a two phase machine (two identical stator windings), whereupon, Park's equations for a two phase machine can be used to represent the machine in the direct and quadrature axis. The necessary voltage is then applied to one of the stator phases to maintain a zero phase current in a manner similar to the three phase case. This method of simulating a single phase machine employing Park's equations has also proven to be more convenient than a phase variable simulation. Moreover, the technique is not only of value from the simulation standpoint but it also permits analysis of a single phase synchronous machine using Park's equations. The advantage of this feature appears to be significant.

CONCLUSIONS

An open phase of a synchronous machine has been simulated using dq0 variables to represent the machine rather than phase (abc) variables which has been the accepted approach in the past. It has been shown that a digital simulation is possible using the machine representation dq0 variables by applying, in the simulation, the voltage to the terminals of the machine which maintains a current zero in one phase. This key feature yields a simulation which eliminates numerical differentiation and repetitive matrix inversion making it much faster than other digital methods.

Application of the method to the simulation of a single phase synchronous machine is also discussed. This approach offers the advantage of simulating and analyzing a single phase synchronous machine using Park's equations.

The general concept described in this paper should serve as a guide to dq0 representations of open circuits anywhere in a power system. Important in this type of simulation is that the 60 Hz variation is not present in the dq0 representation; hence, variable step size integration can be used once the unbalanced condition subsides. This method of decreasing computation time is not possible in a phase variable simulation. This paper together with Ref. 10 provides the analytical tools for all dq0 representation of a power system. The potential of this type of representation warrants consideration.

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APPENDIX

The per unit machine parameters are

$r_a = 0.003$	$X_{aq} = 1.120$	$r_{kd} = 0.0078$
$X_{\ell a} = 0.093$	$X_{ad} = 1.44$	$X_{\ell kd} = 0.025$
$r_{kq} = 0.0075$	$H = 2.95$ sec.	$r_{fd} = 0.0008$
$X_{\ell kq} = 0.0523$		$X_{\ell fd} = 0.1075$

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