# Performance Calculations of a Reluctance Motor Drive by *dq* Harmonic Balance

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*Abstract*—A method of analyzing the constant speed steady-state performance of a reluctance motor fed from voltages of an arbitrary waveform is presented. The solution is obtained directly from Park's equations by a time-domain harmonic balancing approach related to the method of multiple reference frames. Both synchronous and asynchronous operation can be obtained from the same theory. The method is readily extended to incorporate any type of synchronous machine. Harmonic balancing is applied to the performance calculations of a reluctance motor supplied by a square wave voltage source inverter. Comparisons are made with more conventional approaches.

## INTRODUCTION

WITH THE development of static ac power supplies there has been an increasing demand for adjustable speed reluctance motor drives. Such drives find utility in a variety of industrial applications including fiber spinning, tape drives, machine tools, and nuclear reactor controls. In many of these applications the asynchronous or starting condition is an important performance aspect. Proper application of reluctance motors and sizing of the attendant inverter supply requires a sound knowledge of such aspects of asynchronous operation as starting torque, torque dips, pulsating torques, run-up times, inrush stator and rotor current, and synchronizing phenomena. The presence of inverter power sources typically having high harmonic content makes the analysis of these phenomena especially difficult.

Regardless of the problem to be solved, nearly any analysis of synchronous machinery begins with a transformation from physical phase variables to direct and quadrature ($d$, $q$, 0) variables expressed in a reference frame fixed on the rotor (Park's transformation) [1]. These $d$–$q$ components are universally accepted as the essential starting point in synchronous machine analysis and are nearly always used in transient simulation studies employing either analog or digital computation [2]. Park's $d$–$q$ axis approach has been extended to encompass induction machine analysis by Stanley [3] and Krause and Thomas [4]. In addition to simulation studies, $d$–$q$ techniques have been found ideal for stability analyses [5], [6], control analyses [7], and steady-state analyses involving complex waveforms [8], [9]. The $d$–$q$ axis approach is particularly useful since it is in gear with modern developments in control and circuit theory such as matrix analysis [6], state variable techniques [8], [9], transformation theory [10], and optimization techniques [11].

The notable exception to this nearly comprehensive approach, however, is unbalanced or asynchronous operation of synchronous machinery. Traditionally, either symmetrical components (0, 1, 2 or +, –, 0 components) or spin components ($f$, $b$, 0 components) have been used to investigate this condition. Although the details have been worked out for asynchronous operation [12], symmetrical components are generally used only for the study of voltage unbalances at synchronous speed [13]. When the machine rotates asynchronously with respect to the fundamental, the spin components have traditionally found acceptance [14], [15]. Although typically used for the sinusoidal case, the theory has also been worked out in each case for a general periodic voltage waveform [16], [17].

In general, solutions involving these variables involve transformations back and forth from one component system to another. The manipulation of variables between numerous component systems has traditionally been one of the distressing aspects of ac machine analysis. Since all analyses begin with $d$–$q$ components and in view of their widespread applicability, it would appear desirable to extend this approach to the problem of synchronous machine unbalances.

Recently, Krause [18] has shown that $d$–$q$ axis techniques can be used to analyze induction machines with voltages of arbitrary periodic waveform, providing in this case a viable alternative to symmetrical components. Although a $d$–$q$ approach to the synchronous machine equivalent has received some attention, a complete development has not appeared. Notable papers, however, include the work of Lauder [19], Widger and Adkins [20], Verma and Abo-Shady [21], and Lawrenson and Mathur [22]. In general, these authors employ a phasor form of the $d$–$q$ equations wherein the relevant impedances or admittances are calculated as a function of frequency. It appears that the only attempt to extend this method to the nonsinusoidal case is the work of Kankam and Slemon [23]. Unfortunately, only balanced waveforms were considered, and an expression for the electromagnetic torque, a key variable in any system study, was not given.

In this paper a comprehensive theory employing $d$–$q$ variables is set forth for the steady-state analysis of synchronous machines operating synchronously or nonsynchronously. Voltages of arbitrary periodic waveshape are considered, and harmonic balancing is used to generate the system equations. In particular, it is shown that these equations can be expressed in a form wherein all variables are constant. The approach is

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therefore the logical equivalent to the method of multiple reference frames. Being centered around matrix theory, the method of harmonic balance is ideally suited to machine computation and like multiple reference frames does not preclude phasor or complex impedance concepts in its solution.

In this paper particular reference is made to the reluctance (reluctance-synchronous) motor since operation from nonsinusoidal supplies is of primary importance. However, the extension to conventional wound-field synchronous machines is a straightforward process. Moreover, being a *d-q* technique, the method is ideally suited to implementing such important phenomena as saturation and deep bar effect and can directly incorporate rotor circuit models of any complexity [20], [22], [24].

## BASIC EQUATIONS AND ASSUMPTIONS

A simplified diagram of the system to be analyzed is given in Fig. 1. In general, three voltages of arbitrary but specified waveform having the same period are applied to the terminals of a three-phase three-wire reluctance machine. Although a four-wire system could have been considered in this analysis, the so-called zero-sequence circuit can be readily solved by conventional circuit theory and hence will not be considered in this paper.

The equations which describe the behavior of an idealized reluctance-synchronous machine have been developed in [5] wherein the following assumptions are made.

1) Each stator winding is distributed so as to produce a sinusoidal magnetomotive force (MMF) wave along the air gap.
2) The air gap is designed so as to produce a sinusoidal amplitude of flux density along the stator surface when excited by a single concentrated stator coil.
3) The effect of the rotor conductors can be modeled by two equivalent sinusoidally distributed windings.
4) Saturation of the magnetic circuit is neglected.
5) The rotor speed is constant.

These idealizing assumptions are commonly employed in analyzing transient and steady-state performance of salient-pole synchronous machines [1]. In general, assumptions 3) and 4) are made in the interest of simplicity, and the effect of saturation and multiple rotor circuits can be incorporated by established methods [2], [22]. In per unit, the resulting equations are expressed in a reference frame fixed in the rotor by

$$v_{qs}{}^r = \frac{p\psi_{qs}{}^r}{\omega_b} + \psi_{ds}{}^r \frac{\omega_r}{\omega_b} + r_s i_{qs}{}^r \tag{1}$$

$$v_{ds}{}^r = \frac{p\psi_{ds}{}^r}{\omega_b} - \psi_{qs}{}^r \frac{\omega_r}{\omega_b} + r_s i_{ds}{}^r \tag{2}$$

$$0 = \frac{p\psi_{qr}{}^r}{\omega_b} + r_{qr}{}' i_{qr}{}'^r \tag{3}$$

$$0 = \frac{p\psi_{dr}{}^r}{\omega_b} + r_{dr}{}' i_{dr}{}'^r \tag{4}$$

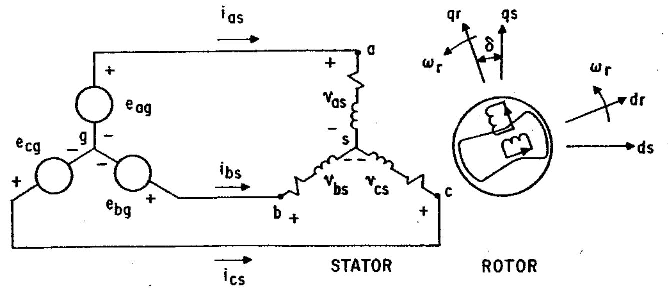


Fig. 1. Reluctance motor with arbitrary periodic source voltages.

where

$$\psi_{qs}{}^r = x_{ls} i_{qs}{}^r + x_{mq}(i_{qs}{}^r + i_{qr}{}'^r) \tag{5}$$

$$\psi_{ds}{}^r = x_{ls} i_{ds}{}^r + x_{md}(i_{ds}{}^r + i_{dr}{}'^r) \tag{6}$$

$$\psi_{qr}{}'^r = x_{lqr}{}' i_{qr}{}'^r + x_{mq}(i_{qs}{}^r + i_{qr}{}'^r) \tag{7}$$

$$\psi_{dr}{}'^r = x_{ldr}{}' i_{dr}{}'^r + x_{md}(i_{ds}{}^r + i_{dr}{}'^r) \tag{8}$$

and

$$T_e = \psi_{ds}{}^r i_{qs}{}^r - \psi_{qs}{}^r i_{ds}{}^r. \tag{9}$$

In these equations $p$ is the time derivative operator $d/dt$. The superscripts $r$ denotes a reference frame fixed in the rotor, $\omega_r$ is the equivalent rotor angular velocity; and $\omega_b$ is the base angular frequency used to define the per unit machine reactances $x_{ls}, x_{ldr}', x_{lqr}', x_{md}$, and $x_{mq}$.

## *d-q* REPRESENTATION OF SOURCE VOLTAGES

Since the three source voltages $e_{ag}$, $e_{bg}$, and $e_{cg}$ are periodic, they may be expressed by the following Fourier series expansions:

$$e_{ag} = \sum_{k=0}^{\infty} E_{ka\alpha} \cos k\omega_e t + \sum_{k=0}^{\infty} E_{ka\gamma} \sin k\omega_e t \tag{10}$$

$$e_{bg} = \sum_{k=0}^{\infty} E_{kb\alpha} \cos k\omega_e t + \sum_{k=0}^{\infty} E_{kb\gamma} \sin k\omega_e t \tag{11}$$

$$e_{cg} = \sum_{k=0}^{\infty} E_{kc\alpha} \cos k\omega_e t + \sum_{k=0}^{\infty} E_{kc\gamma} \sin k\omega_e t \tag{12}$$

where $\omega_e$ denotes the fundamental source angular frequency. Also, the $\alpha$ and $\gamma$ subscripts denote the coefficients of the cosine and sine terms of the expansions. In view of the previously mentioned resemblance to multiple reference frames, it should be noted that this notation is consistent with [18]. It is assumed that the dc term in the expansion is accounted for by the $\alpha$ term when $k = 0$. That is, by definition

$$E_{0a\gamma} = E_{0b\gamma} = E_{0c\gamma} = 0. \tag{13}$$

It can be noted that the source voltages are defined in the stationary reference frame whereas the motor equations are expressed in a reference frame rotating with the rotor, so that the effects of the source voltages must be transferred to the rotor frame. This can best be accomplished in three stages: first, solving for the three-phase voltages; second, transforming to a stationary $d$, $q$, 0 reference frame; and, finally, a subsequent transformation to a rotor $d$-$q$ frame.

The three load voltages are defined by

$$v_{as} = e_{ag} - v_{sg} \tag{14}$$

$$v_{bs} = e_{bg} - v_{sg} \tag{15}$$

$$v_{cs} = e_{cg} - v_{sg}. \tag{16}$$

If the sum of the three-phase currents is zero, it can be shown that the sum of the three-phase voltages will also be zero. Upon adding (14)-(16),

$$v_{sg} = \frac{1}{3}(e_{ag} + e_{bg} + e_{cg}) \tag{17}$$

$$v_{as} = \frac{2}{3}e_{ag} - \frac{1}{3}e_{bg} - \frac{1}{3}e_{cg} \tag{18}$$

$$v_{bs} = -\frac{1}{3}e_{ag} + \frac{2}{3}e_{bg} - \frac{1}{3}e_{cg} \tag{19}$$

$$v_{cs} = -\frac{1}{3}e_{ag} - \frac{1}{3}e_{bg} + \frac{2}{3}e_{cg}. \tag{20}$$

The equations relating phase variables to a $d$-$q$ variables expressed in a reference frame fixed in the stator, can be found from [4, Eqs. (28)-(30)], by aligning the $q$-axis of the arbitrary reference frame along the phase as axis ($\theta = 0$) and setting the speed of the reference frame to zero ($p\theta = 0$) whereupon

$$v_{qs}{}^{s} = \frac{2}{3}v_{as} - \frac{1}{3}v_{bs} - \frac{1}{3}v_{cs} \tag{21}$$

$$v_{ds}{}^{s} = \frac{1}{\sqrt{3}}(v_{cs} - v_{bs}) \tag{22}$$

$$v_{0s}{}^{s} = \frac{1}{3}(v_{as} + v_{bs} + v_{cs}). \tag{23}$$

The superscript $s$ is used to denote that the reference frame is fixed in the stator. Substituting (18)-(20),

$$v_{qs}{}^{s} = \frac{2}{3}e_{ag} - \frac{1}{3}e_{bg} - \frac{1}{3}e_{cg} \tag{24}$$

$$v_{ds}{}^{s} = \frac{1}{\sqrt{3}}(e_{cg} - e_{bg}) \tag{25}$$

$$v_{0s}{}^{s} = 0. \tag{26}$$

Note that the zero sequence voltage is identically zero, which will always be the case for a three-wire system. Hence this component will no longer be considered.

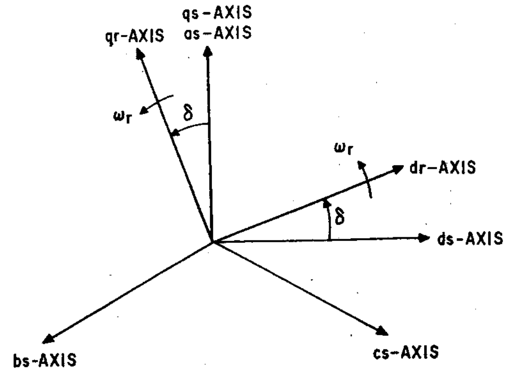

Fig. 2. Axes alignment at $t = 0$.

Since the $d$-$q$ load voltages are expressed in a stationary frame, the frequency components must be same as the source:

$$v_{qs}{}^{s} = \sum_{k=0}^{\infty} V_{kq\alpha} \cos k\omega_e t + \sum_{k=0}^{\infty} V_{kq\gamma} \sin k\omega_e t \tag{27}$$

$$v_{ds}{}^{s} = \sum_{k=0}^{\infty} V_{kd\alpha} \cos k\omega_e t + \sum_{k=0}^{\infty} V_{kd\gamma} \sin k\omega_e t. \tag{28}$$

Equations (10)-(12) and (24)-(26) imply that

$$V_{kq\alpha} = \frac{2}{3}E_{ka\alpha} - \frac{1}{3}E_{kb\alpha} - \frac{1}{3}E_{kc\alpha} \tag{29}$$

$$V_{kq\gamma} = \frac{2}{3}E_{ka\gamma} - \frac{1}{3}E_{kb\gamma} - \frac{1}{3}E_{kc\gamma} \tag{30}$$

$$V_{kd\alpha} = \frac{1}{\sqrt{3}}(E_{kc\alpha} - E_{kb\alpha}) \tag{31}$$

$$V_{kd\gamma} = \frac{1}{\sqrt{3}}(E_{kc\gamma} - E_{kb\gamma}) \tag{32}$$

for $k = 0, 1, \cdots, \infty$.

Because the inductances of a synchronous machine become time invariant only when expressed in the rotor reference frame, it is not practical to transform the stator voltages to synchronously rotating frames as employed in the method of multiple reference frames. It is useful to consider, however, the set of equations resulting when the $d$-$q$ voltages are transformed to a reference frame rotating at rotor speed. The equations of transformation from the $d$-$q$ stationary frame to the $d$-$q$ rotor frame illustrated by Fig. 2 are

$$v_{qs}{}^{r} = v_{qs}{}^{s} \cos(\omega_r t + \delta) - v_{ds}{}^{s} \sin(\omega_r t + \delta) \tag{33}$$

$$v_{ds}{}^{r} = v_{qs}{}^{s} \sin(\omega_r t + \delta) + v_{ds}{}^{s} \cos(\omega_r t + \delta) \tag{34}$$

where $\delta$ is the initial alignment of the rotating axis with respect to the stationary $q$ axis ($as$ axis) at $t = 0$. It is clear that when the rotor is rotating in synchronism with the fundamental voltage component ($\omega_r = \omega_e$) and the phase components are balanced and cosinusoidal, the angle $\delta$ corresponds to the conventional torque angle delta. Since the counterclockwise direction is considered positive, delta is positive for generator

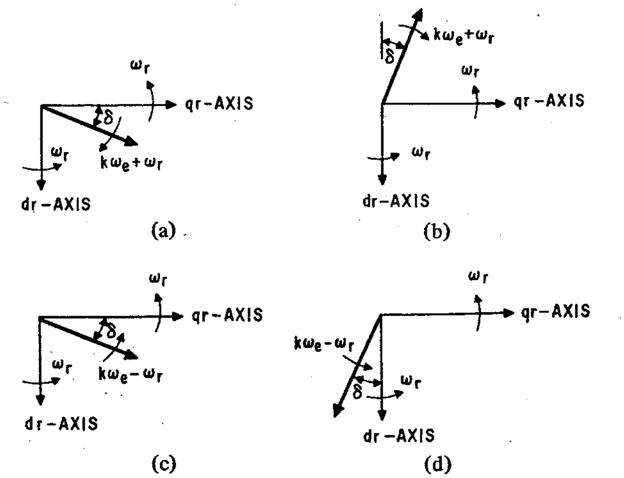


Fig. 3. Magnitude and direction of rotation at $t = 0$ of the four balanced voltage sets relative to a $d$-$q$ rotor reference frame. (a) First set: vector $\frac{1}{2}(V_{kq\alpha} + V_{kd\gamma})$. (b) Second set: vector $\frac{1}{2}(V_{kq\gamma} - V_{kd\alpha})$. (c) Third set: vector $\frac{1}{2}(V_{kq\alpha} - V_{kd\gamma})$. (d) Fourth set: vector $\frac{1}{2}(V_{kq\gamma} + V_{kd\alpha})$.

action. After some trigometric manipulation, the resulting equation can be expressed in the form

$$v_{qs}{}^r = \sum_{k=0}^{\infty} \frac{1}{2}(V_{kq\alpha} + V_{kd\gamma}) \cos[(k\omega_e + \omega_r)t + \delta] + \frac{1}{2}(V_{kq\gamma} - V_{kd\alpha}) \sin[(k\omega_e + \omega_r)t + \delta] + \frac{1}{2}(V_{kq\alpha} - V_{kd\gamma}) \cos[(k\omega_e - \omega_r)t - \delta] + \frac{1}{2}(V_{kq\gamma} + V_{kd\alpha}) \sin[(k\omega_e - \omega_r)t - \delta] \quad (35)$$

$$v_{ds}{}^r = \sum_{k=0}^{\infty} \frac{1}{2}(V_{kq\alpha} + V_{kd\gamma}) \sin[(k\omega_e + \omega_r)t + \delta] - \frac{1}{2}(V_{kq\gamma} - V_{kd\alpha}) \cos[(k\omega_e + \omega_r)t + \delta] - \frac{1}{2}(V_{kq\alpha} - V_{kd\gamma}) \sin[(k\omega_e - \omega_r)t - \delta] + \frac{1}{2}(V_{kq\gamma} + V_{kd\alpha}) \cos[(k\omega_e - \omega_r)t - \delta]. \quad (36)$$

Although the expressions for the $d$-$q$ voltages appear even more complex in the rotor frame, note the balanced sets which appear as a result of this transformation. In particular, the first summation of (35) and (36) represents an infinite series of balanced sets of amplitude $(V_{kq\alpha} + V_{kd\gamma})/2$ rotating in the negative (clockwise) direction at an angular velocity $k\omega_e + \omega_r$. Similarly, the second summation represents another series of negatively rotating balanced sets. The third and fourth summations represent balanced sets having a positive rotation with respect to the $d$-$q$ axes and are rotating counterclosewise at an angular velocity $k\omega_e$-$\omega_r$. The alignment of these four sets of rotating vectors at $t = 0$ and their direction of rotation can be visualized by reference to Fig. 3.

## SOLUTION FOR MOTOR CURRENTS

In order to obtain the solution, these four infinite sets of balanced voltages must be "applied" to Park's equations, (1)-(4). To simplify the procedure only one term $k = n$ of the positively rotating sets will be considered, and these terms will be designated as $v_{qs}{}^r(+n)$ and $v_{ds}{}^r(+n)$. In particular,

$$v_{qs(+n)}{}^r = \frac{1}{2}(V_{nq\alpha} - V_{nd\gamma}) \cos[(n\omega_e - \omega_r)t - \delta] + \frac{1}{2}(V_{nq\gamma} + V_{nd\alpha}) \sin[(n\omega_e - \omega_r)t - \delta] \quad (37)$$

$$v_{ds(+n)}{}^r = \frac{1}{2}(V_{nq\gamma} + V_{nd\alpha}) \cos[(n\omega_e - \omega_r)t - \delta] - \frac{1}{2}(V_{nq\alpha} - V_{nd\gamma}) \sin[(n\omega_e - \omega_r)t - \delta]. \quad (38)$$

It can be recalled that the rotor speed has been assumed constant so that Park's equations become linear. Hence, in the steady state, these voltages will excite only currents of the same frequency. That is

$$i_{qs(+n)}{}^r = I_{qs\alpha(+n)} \cos(n\omega_e - \omega_r)t + I_{qs\gamma(+n)} \sin(n\omega_e - \omega_r)t \quad (39)$$

$$i_{ds(+n)}{}^r = I_{ds\alpha(+n)} \cos(n\omega_e - \omega_r)t + I_{ds\gamma(+n)} \sin(n\omega_e - \omega_r)t \quad (40)$$

with similar definitions for $i_{dr}{}'^r(+n)$, and $i_{qr}{}'^r(+n)$. Substituting the explicit voltage and current expressions into Park's equations, (1)-(4), and algebraically eliminating the flux linkage variables by means of (5)-(8) results in the following voltage equation for the $qs$ circuit:

$$\frac{1}{2}(V_{nq\alpha} - V_{nd\gamma}) \cos[(n\omega_e - \omega_r)t - \delta] + \frac{1}{2}(V_{nq\gamma} + V_{nd\alpha}) \sin[(n\omega_e - \omega_r)t - \delta]$$
$$= r_s I_{qs\alpha(+n)} \cos(n\omega_e - \omega_r)t + r_s I_{qs\gamma(+n)} \sin(n\omega_e - \omega_r)t + \frac{n\omega_e - \omega_r}{\omega_b} x_{qs}[-I_{qs\alpha(+n)} \sin(n\omega_e - \omega_r)t + I_{qs\gamma(+n)} \cos(n\omega_e - \omega_r)t] + \frac{n\omega_e - \omega_r}{\omega_b} x_{mq} \cdot [-I_{qr\alpha(+n)} \sin(n\omega_e - \omega_r)t + I_{qr\gamma(+n)} \cos(n\omega_e - \omega_r)t] + \frac{\omega_r}{\omega_b} x_{ds}[I_{ds\alpha(+n)} \cos(n\omega_e - \omega_r)t + I_{ds\gamma(+n)} \sin(n\omega_e - \omega_r)t] + \frac{\omega_r}{\omega_b} x_{md}[I_{dr\alpha(+n)} \cdot \cos(n\omega_e - \omega_r)t + I_{dr\gamma(+n)} \sin(n\omega_e - \omega_r)t]. \quad (41)$$

In order that 41 be valid, the cosine and sine components on the left and right half sides of (41) must be equal, which implies that the following two equations must be valid:

$$\frac{1}{2}[(V_{nq\alpha} - V_{nd\gamma})\cos\delta - (V_{nq\gamma} + V_{nd\alpha})\sin\delta] = r_s I_{qs\alpha(+n)} + \frac{(n\omega_e - \omega_r)}{\omega_b} x_{qs} I_{qs\gamma(+n)} + \frac{(n\omega_e - \omega_r)}{\omega_b} x_{mq} I_{qr\gamma(+n)} + \frac{\omega_r}{\omega_b} x_{ds} I_{ds\alpha(+n)} + \frac{\omega_r}{\omega_b} x_{md} I_{dr\alpha(+n)} \quad (42)$$

$$\frac{1}{2}[(V_{nq\gamma} + V_{nd\alpha})\cos\delta + (V_{nq\alpha} - V_{nd\gamma})\sin\delta] = r_s I_{qs\gamma(+n)} - \frac{(n\omega_e - \omega_r)}{\omega_b} x_{qs} I_{qs(+n)} - \frac{(n\omega_e - \omega_r)}{\omega_b} x_{mq} I_{qr\alpha(+n)} + \frac{\omega_r}{\omega_b} x_{ds} I_{ds\gamma(+n)} + \frac{\omega_r}{\omega_b} x_{md} I_{dr\gamma(+n)} \quad (43)$$

The process of matching coefficients can be continued for the *ds*, *qr*, and *dr* circuits. This process will generate six additional equations. All eight equations can be compactly described by a single matrix equation:

$$\bar{V}_{(+n)} = \bar{Z}_{(+n)} \bar{I}_{(+n)} \quad (44)$$

where

$$\bar{V}_{(+n)} = \frac{1}{2} \begin{bmatrix} (V_{nq\alpha} - V_{nd\gamma})\cos\delta - (V_{nq\gamma} + V_{nd\alpha})\sin\delta \\ (V_{nq\gamma} + V_{nd\alpha})\cos\delta + (V_{nq\alpha} - V_{nd\gamma})\sin\delta \\ (V_{nq\gamma} + V_{nd\alpha})\cos\delta + (V_{nq\alpha} - V_{nd\gamma})\sin\delta \\ -(V_{nq\alpha} - V_{nd\gamma})\cos\delta + (V_{nq\gamma} + V_{nd\alpha})\sin\delta \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (45)$$

$$\bar{I}_{(+n)} = [I_{qs\alpha(+n)}, I_{qs\gamma(+n)}, I_{ds\alpha(+n)}, I_{ds\gamma(+n)}, I_{qr\alpha(+n)}, I_{qr\gamma(+n)}, I_{dr\alpha(+n)}, I_{dr\gamma(+n)}]^t \quad (46)$$

and $t$ denotes the transpose. The matrix $\bar{Z}_{(+n)}$ is defined by the 8 X 8 matrix

$$\bar{z}_{(+n)} = \begin{bmatrix} r_s & \frac{n\omega_e - \omega_r}{\omega_b} x_{qs} & \frac{\omega_r}{\omega_b} x_{ds} & 0 & 0 & \frac{n\omega_e - \omega_r}{\omega_b} x_{mq} & \frac{\omega_r}{\omega_b} x_{md} & 0 \\ -\frac{(n\omega_e - \omega_r)}{\omega_b} x_{qs} & r_s & 0 & \frac{\omega_r}{\omega_b} x_{ds} & -\frac{(n\omega_e - \omega_r)}{\omega_b} x_{mq} & 0 & 0 & \frac{\omega_r}{\omega_b} x_{md} \\ -\frac{\omega_r}{\omega_b} x_{qs} & 0 & r_s & \frac{n\omega_e - \omega_r}{\omega_b} x_{ds} & -\frac{\omega_r}{\omega_b} x_{mq} & 0 & 0 & \frac{n\omega_e - \omega_r}{\omega_b} x_{md} \\ 0 & -\frac{\omega_r}{\omega_b} x_{qs} & -\frac{(n\omega_e - \omega_r)}{\omega_b} x_{ds} & r_s & 0 & -\frac{\omega_r}{\omega_b} x_{mq} & -\frac{(n\omega_e - \omega_r)}{\omega_b} x_{md} & 0 \\ 0 & \frac{n\omega_e - \omega_r}{\omega_b} x_{mq} & 0 & 0 & r_{qr}' & \frac{n\omega_e - \omega_r}{\omega_b} x_{qr}' & 0 & 0 \\ -\frac{(n\omega_e - \omega_r)}{\omega_b} x_{mq} & 0 & 0 & 0 & -\frac{(n\omega_e - \omega_r)}{\omega_b} x_{qr}' & r_{qr}' & 0 & 0 \\ 0 & 0 & 0 & \frac{n\omega_e - \omega_r}{\omega_b} x_{md} & 0 & 0 & r_{dr}' & \frac{n\omega_e - \omega_r}{\omega_b} x_{dr}' \\ 0 & 0 & -\frac{(n\omega_e - \omega_r)}{\omega_b} x_{md} & 0 & 0 & 0 & -\frac{(n\omega_e - \omega_r)}{\omega_b} x_{dr}' & r_{dr}' \end{bmatrix}. \quad (47)$$

In an entirely similar manner the solution for the negatively rotating sets can be found for a particular $k = n$. The relevant equations can be expressed in the matrix form

$$\bar{V}_{(-n)} = \bar{Z}_{(-n)} \bar{I}_{(-n)}. \quad (48)$$

The vector $\bar{I}_{(-n)}$ is defined similarly to $\bar{I}_{(+n)}$, and the matrix $\bar{Z}_{(-n)}$ is the same as $Z_{(+n)}$ with the quantity $(n\omega_e - \omega_r)$ replaced by $(n\omega_e + \omega_r)$. The vector $V_{(-n)}$ that expresses the negatively rotating voltage sets is

$$\bar{V}_{(-n)} = \frac{1}{2} \begin{bmatrix} (V_{nq\alpha} + V_{nd\gamma})\cos\delta + (V_{nq\gamma} - V_{nd\alpha})\sin\delta \\ (V_{nq\gamma} - V_{nd\alpha})\cos\delta - (V_{nq\alpha} + V_{nd\gamma})\sin\delta \\ -(V_{nq\gamma} - V_{nd\alpha})\cos\delta + (V_{nq\alpha} + V_{nd\gamma})\sin\delta \\ (V_{nq\alpha} + V_{nd\gamma})\cos\delta + (V_{nq\gamma} - V_{nd\alpha})\sin\delta \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (49)$$

The solution for the two current vectors $\overline{I}_{(+n)}$ and $\overline{I}_{(-n)}$ is readily obtained by inverting the two impedance matrices $\overline{Z}_{(+n)}$ and $\overline{Z}_{(-n)}$.

It should be mentioned that the real 8 dimensional voltage equations (44) and (48) can be manipulated into complex four-dimensional equations by appropriate multiplication of the rows by the operator $j$. However, the savings in computing time involved in inverting two 4 × 4 complex matrices rather than one 8 × 8 real matrix is not substantial so that little is gained from such additional manipulation. Note than when $n = 1$ and $\omega_r = \omega_e$ (synchronous speed) the alpha components are automatically zero since $\sin(\omega_e - \omega_r)t = 0$, so that the solution reduces to the usual result for balanced synchronous operation. The total solution is found by repeating the process for all $k = n$, $k = 0, 1, 2, \cdots, \infty$. In the rotor reference frame, the solution is given by

$$i_{qs}^r = \sum_{k=0}^{\infty} [i_{qs(+k)}^r + i_{qs(-k)}^r] \tag{50}$$

$$i_{ds}^r = \sum_{k=0}^{\infty} [i_{ds(+k)}^r + i_{ds(-k)}^r] \tag{51}$$

$$i_{qr}'^r = \sum_{k=0}^{\infty} [i_{qr(+k)}'^r + i_{dr(-k)}'^r] \tag{52}$$

$$i_{dr}'^r = \sum_{k=0}^{\infty} [i_{dr(+k)}'^r + i_{dr(-k)}'^r] \tag{53}$$

where

$$i_{qs(+k)}^r = I_{qs\alpha(+k)} \cos(k\omega_e - \omega_r)t + I_{qs\gamma(+k)} \sin(k\omega_e - \omega_r)t \tag{54}$$

$$i_{qs(-k)}^r = I_{qs\alpha(-k)} \cos(k\omega_e + \omega_r)t + I_{qs\gamma(-k)} \sin(k\omega_e + \omega_r)t \tag{55}$$

and so forth for

$$i_{ds(+k)}^r,\ i_{ds(-k)}^r,\ i_{qr(+k)}'^r,\ i_{qr(-k)}'^r,\ i_{dr(+k)}'^r,\ i_{dr(-k)}'^r.$$

Equations (52) and (53) represent currents flowing in the rotor circuits of the reluctance machine, and since the reference frame has been fixed in the rotor, they correspond to the physical currents. However, the stator currents actually flow in stationary circuits, so that the $d$-$q$ stator currents defined by (50) and (51) must be transformed to the stator. The equations of transformation are

$$i_{qs}^s = i_{qs}^r \cos(\omega_r t + \delta) + i_{ds}^r \sin(\omega_r t + \delta) \tag{56}$$

and

$$i_{ds}^s = -i_{qs}^r \sin(\omega_r t + \delta) + i_{ds}^r \cos(\omega_r t + \delta). \tag{57}$$

Substituting (50) and (51) into (56) and (57) results in the following equations for stator currents:

$$\begin{aligned} i_{qs}^s = \sum_{k=0}^{\infty} \frac{1}{2}\{&[I_{qs\alpha(+k)} - I_{ds\gamma(+k)}] \cos(k\omega_e t + \delta) \\ &+ [I_{qs\gamma(+k)} + I_{ds\alpha(+k)}] \sin(k\omega_e t + \delta) \\ &+ [I_{qs\alpha(-k)} + I_{ds\gamma(-k)}] \cos(k\omega_e t - \delta) \\ &+ [I_{qs\gamma(-k)} - I_{ds\alpha(-k)}] \sin(k\omega_e t - \delta) \\ &+ [I_{qs\alpha(+k)} + I_{ds\gamma(+k)}] \cos[(k\omega_e - 2\omega_r)t - \delta] \\ &+ [I_{qs\gamma(+k)} - I_{ds\alpha(+k)}] \sin[(k\omega_e - 2\omega_r)t - \delta] \\ &+ [I_{qs\alpha(-k)} - I_{ds\gamma(-k)}] \cos[(k\omega_e + 2\omega_r)t + \delta] \\ &+ [I_{qs\gamma(-k)} + I_{ds\alpha(-k)}] \sin[(k\omega_e + 2\omega_r)t + \delta]\} \end{aligned} \tag{58}$$

$$\begin{aligned} i_{ds}^s = \sum_{k=0}^{\infty} \frac{1}{2}\{&[I_{qs\gamma(+k)} + I_{ds\alpha(+k)}] \cos(k\omega_e t + \delta) \\ &+ [-I_{qs\alpha(+k)} + I_{ds\gamma(+k)}] \sin(k\omega_e t + \delta) \\ &+ [-I_{qs\gamma(-k)} + I_{ds\alpha(-k)}] \cos(k\omega_e t - \delta) \\ &+ [I_{qs\alpha(-k)} + I_{ds\gamma(-k)}] \sin(k\omega_e t - \delta) \\ &+ [-I_{qs\gamma(+k)} + I_{ds\alpha(+k)}] \cos[(k\omega_e - 2\omega_r)t - \delta] \\ &+ [I_{qs\alpha(+k)} + I_{ds\gamma(+k)}] \sin[(k\omega_e - 2\omega_r)t - \delta] \\ &+ [I_{qs\gamma(-k)} + I_{ds\alpha(-k)}] \cos[(k\omega_e + 2\omega_r)t + \delta] \\ &+ [-I_{qs\alpha(-k)} + I_{ds\gamma(-k)}] \sin[(k\omega_e + 2\omega_r)t + \delta]\}. \end{aligned} \tag{59}$$

Although the equations that have been solved are linear, it is interesting to note the additional frequencies $k\omega_e - 2\omega_r$ and $k\omega_e + 2\omega_r$ that appear in the expressions for stator current. These terms are not necessarily part of the source voltage frequency spectrum and arise due to the periodic variation in machine inductances.

## SOLUTION FOR ELECTROMAGNETIC TORQUE

The per unit expression for electromagnetic torque is given by (9). For simplicity the flux linkages involving stator leakage reactance can be eliminated from (9) so that a more suitable form is given by

$$T_e = \psi_{md}^r i_{qs}^r - \psi_{mq}^r i_{ds}^r \tag{60}$$

$$\psi_{md}^r = x_{md}(i_{ds}^r + i_{dr}'^r) \tag{61}$$

$$\psi_{mq}^r = x_{mq}(i_{qs}^r + i_{qr}'^r). \tag{62}$$

Substituting (50)–(53) into (60), the torque is expressed in terms of harmonic components as

$$\begin{aligned} T_e = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} &[i_{qs(+k)}^r + i_{qs(-k)}^r][\psi_{md(+l)}^r \\ &+ \psi_{md(-l)}^r] - [i_{ds(+k)}^r + i_{ds(-k)}^r][\psi_{mq(+l)}^r \\ &+ \psi_{mq(-l)}^r] \end{aligned} \tag{63}$$

where

$$\psi_{md(+l)}{}^r = x_{md}[i_{ds(+l)}{}^r + i_{dr(+l)}{}'^r] \quad (64)$$

$$\psi_{md(-l)}{}^r = x_{md}[i_{ds(-l)}{}^r + i_{dr(-l)}{}'^r] \quad (65)$$

$$\psi_{mq(+l)}{}^r = x_{mq}[i_{qs(+l)}{}^r + i_{qr(+l)}{}'^r] \quad (66)$$

$$\psi_{mq(-l)}{}^r = x_{mq}[i_{qs(-l)}{}^r + i_{qr(-l)}{}'^r]\,. \quad (67)$$

Upon making the substitution for the explicit time harmonic form of the positive and negative rotating sets ((54), (55), etc.) in (63) yields an explicit time function form for the electromagnetic torque. In general, time harmonics can be identified having harmonics $(k \pm l)\omega_e$ and $(k \pm l)\omega_e \pm \omega_r$. Again, the relevant equations can be assembled into a convenient matrix form. If $I_k$ is a column vector of stator current components,

$$\bar{I}_k = [I_{qs\alpha(+k)}, I_{qs\gamma(+k)}, I_{ds\alpha(+k)}, I_{ds\gamma(+k)}, I_{qs\alpha(-k)}, I_{qs\gamma(-k)}, I_{ds\alpha(-k)}, I_{ds\gamma(-k)}]^t, \quad (68)$$

and $\psi_k$ is a column vector of air gap flux components:

$$\bar{\psi}_k = [\psi_{qs\alpha(+k)}, \psi_{qs\gamma(+k)}, \psi_{ds\alpha(+k)}, \psi_{ds\gamma(+k)}, \psi_{qs\alpha(-k)}, \psi_{qs\gamma(-k)}, \psi_{ds\alpha(-k)}, \psi_{ds\gamma(-k)}]^t. \quad (69)$$

Then the electromagnetic torque can be described by

$$T_e = \sum_{j=1}^{6} T_{ej\alpha} + T_{ej\gamma} \quad (70)$$

where $\alpha$ and $\gamma$ again pertain to the cosine and sine terms of the six frequency components. The definitions of $T_{e1\alpha}, \cdots, T_{e6\gamma}$ is given in the Appendix.

The results of this section are completely general in that synchronous or asynchronous operation with arbitrary periodic voltage waveforms can be investigated. Although a machine with only a single $d$- and $q$-axis rotor winding has been analyzed, it is clear that rotor windings of any complexity can be incorporated into the analysis by appropriately increasing the dimension of the matrices $\bar{Z}_{(+n)}$ and $\bar{Z}_{(-n)}$ by two for each additional winding. In particular, the conventional wound-field synchronous machine can be analyzed. However, in this case, the presence of field excitation introduces an additional term to the stator phase current having frequency $\omega_r$. These terms are readily calculated with the same theory.

## APPLICATION TO RELUCTANCE MOTOR DRIVES

A problem of interest which is particularly well suited to the harmonic balance method is the steady-state behavior of reluctance motors supplied by a conventional square wave voltage source inverter. A simplified diagram of this sytem is given in Fig. 4. The power source is comprised of a controlled ac/dc rectifier, a dc filter circuit, and a force-commutated dc/ac voltage inverter. In general, the dc smoothing filter tends to attenuate the harmonics inherent in the rectifier output and to maintain a constant dc inverter voltage $V_I$. When the inverter frequency is not too low, $V_I$ may be assumed constant. In this case, a typical line-to-ground, line-to-line, and line-to-neutral voltage is shown in Fig. 5.

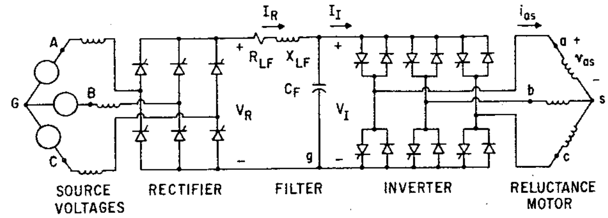


Fig. 4. Rectfier–inverter, reluctance motor system.

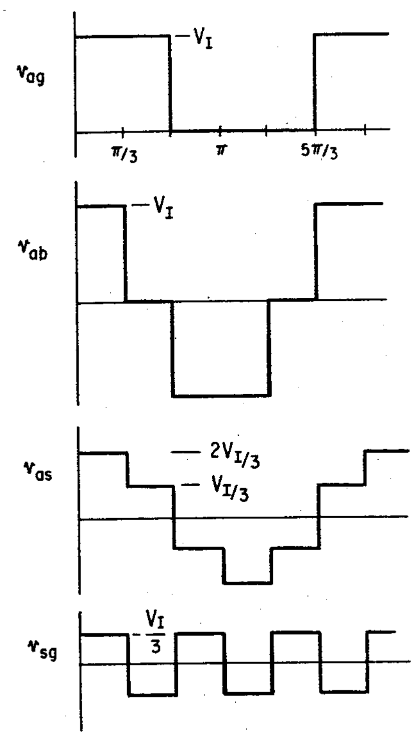


Fig. 5. System voltages with stiff dc bus.

The reluctance motor chosen for purposes of this study was a 10-hp 220-V four-pole, 60-Hz delta-connected machine. Using nameplate horsepower, voltage, and frequency as base quantities, the measured per unit parameters of this machine are $r_s = 0.01212$, $x_{ls} = 0.0306$, $r_{dr}' = 0.00955$, $x_{ldr}' = 0.00685$, $r_{qr}' = 0.02783$, $x_{lqr}' = 0.1235$, $x_{md} = 0.7791$, $x_{mq} = 0.28935$.

Figs. 6 and 7 show typical results for synchronous and asynchronous operation when this reluctance motor is fed from an inverter supply. When a Fourier analysis is made of the voltage waveshapes of Fig. 5, it can be shown that

$$e_{ag} = \frac{V_I}{2} + \frac{2V_I}{\pi}\left(\cos\omega_e t - \frac{1}{3}\cos 3\omega_e t + \frac{1}{5}\cos 5\omega_e t - \frac{1}{7}\cos 7\omega_e t + \cdots\right). \quad (71)$$

Voltages $e_{bg}$ and $e_{cg}$ are phase displaced from $e_{ag}$ by 120°. Using (18) it can be shown that

$$v_{as} = \frac{2V_I}{\pi}\left(\cos\omega_e t + \frac{1}{5}\cos 5\omega_e t - \frac{1}{7}\cos 7\omega_e t - \frac{1}{11}\cos 11\omega_e t + \cdots\right), \quad (72)$$

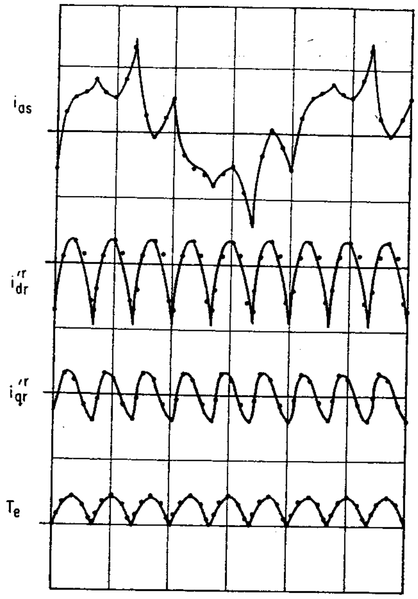


Fig. 6. Steady-state solution at synchronous speed. $V_I = 0.4/\pi$, $\omega_e = 2\pi\cdot 12$ rad/s, $\delta = -30°$. Solid line is solution using state variable techniques. Points denote solution using $d$-$q$ harmonic balance. Scale: $i_{as} = 2.5$ $A$/div, $i_{dr}'^r = 2$ $A$/div, $i_{qr}'^r = A$/div, $T_e = 2.5$ $N\cdot m$/div. Time scale 90°/div.

which can also be verified by a Fourier analysis of $v_{as}$ in Fig. 5. Voltages $v_{bs}$ and $v_{cs}$ are also phase displaced from $v_{as}$ by 120°. Finally, from (24) and (25),

$$v_{qs}^s = \frac{2}{\pi} V_I \left( \cos \omega_e t + \frac{1}{5} \cos 5\omega_e t - \frac{1}{7} \cos 7\omega_e t - \cdots \right) \tag{73}$$

$$v_{ds}^s = -\frac{2}{\pi} V_I \left( \sin \omega_e t + \frac{1}{5} \sin 5\omega_e t - \frac{1}{7} \sin 7\omega_e t - \cdots \right). \tag{74}$$

Hence

$$V_{1q\alpha} = \frac{2V_I}{\pi} \qquad V_{1d\gamma} = -\frac{2V_I}{\pi}$$

$$V_{5q\alpha} = \frac{2V_I}{5\pi} \qquad V_{5d\gamma} = -\frac{2V_I}{5\pi}$$

$$V_{7q\alpha} = -\frac{2V_I}{7\pi} \qquad V_{7d\gamma} = \frac{2V_I}{7\pi}$$

and so forth. This result can also be obtained from (29)–(32) so that only the frequency spectrum of the source voltages need be defined and the $d$-$q$ voltages efficiently calculated by a computer.

Fig. 6 shows the steady-state behavior of the reluctance machine during synchronous speed motoring at a torque angle $\delta = -30°$. The fundamental line frequency is 12 Hz, and the dc bus voltage is adjusted to maintain constant volts/hertz,

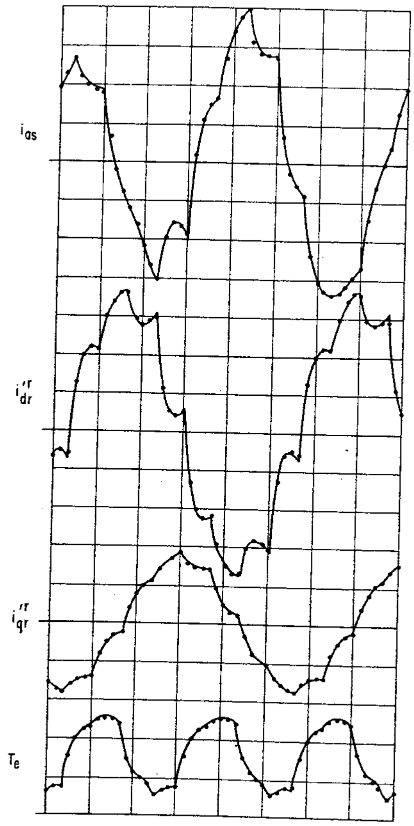


Fig. 7. Steady-state solution during asynchronous operation. $V_I = 0.4/\pi$, $\omega_e = 2\pi\cdot 12$ rad/s, $\omega_r = 2\pi\cdot 3$ rad/s, $\delta = -30°$. Solid line is output of digital computer simulation. Points denote solution using $d$-$q$ harmonic balance. Scale: $i_{as} = 2.5$ $A$/div, $i_{dr}'^r = 2.5$ $A$/div, $i_{qr}'^r = 0.625$ $A$/div, $T_e = 2.5$ $N\cdot m$/div. Time scale = 90°/div (20.8 $ms$/div).

that is $V_I = 0.4/\pi$. The solid line shows the exact solution obtained by state variable techniques [9]. The points indicate the approximate solution using harmonic balancing, and including all relevant harmonics up to the 23rd. Note the characteristic sixth harmonic pulsation in the torque as well as the $d$-$q$ rotor currents. Very good correlation is evident.

In Fig. 7 the operating frequency and dc bus voltage has been maintained the same; however, the machine is now operated subsynchronously. Because state variable techniques become impractical for asynchronous operation, the analytical results are now compared to the output of a digital computer simulation. In general, the harmonic balancing method has no such restriction and is valid for arbitrary rotor speeds. Unfortunately, an arbitrary rotor speed can result in nonperiodic waveshapes which would make correlation difficult. In Fig. 7 the rotor speed is fixed at 0.25 per unit of synchronous speed (0.05 per unit of base speed). Under this condition it can be shown that the stator current begins to repeat after two fundamental cycles, the rotor currents begins to repeat after 4/3 cycles, and the torque begins to repeat after 2/3 cycles. Note again the close correlation between the two methods. The time required to compute one operating point such as given in Fig. 6 or 7 is 1.5 s on a GE 600 Series digital computer. A summary of the first several harmonics corresponding to $i_{as}$ and $T_e$ in Fig. 7 is given in Table I.

TABLE I
STATOR CURRENT AND ELECTROMAGNETIC TORQUE COMPONENTS CORRESPONDING TO FIG. 7

| $i_{as}$ | | | $T_e$ | | |
|---|---|---|---|---|---|
| Amplitude | Ang. Freq. | Phase (Deg.) | Amplitude | Ang. Freq. | Phase (Deg.) |
| 1.554 | $0.5\omega_e$ | -156.1 | 3.201 | 0. | - |
| 6.051 | $\omega_e$ | -32.0 | 2.056 | $1.5\omega_e$ | 164.9 |
| 0.627 | $5\omega_e$ | -65.4 | 0.308 | $4.5\omega_e$ | -91.5 |
| 0.313 | $5.5\omega_e$ | 127.5 | 0.089 | $6\omega_e$ | 78.1 |
| 0.169 | $6.5\omega_e$ | -60.0 | 0.200 | $7.5\omega_e$ | 39.4 |
| 0.335 | $7\omega_e$ | 109.3 | 0.083 | $10.5\omega_e$ | 96.2 |
| 0.142 | $11\omega_e$ | 102.3 | 0.017 | $12\omega_e$ | -62.0 |
| 0.073 | $11.5\omega_e$ | -71.4 | 0.062 | $13.5\omega_e$ | -130.1 |
| 0.052 | $12.5\omega_e$ | 106.4 | 0.005 | $16.5\omega_e$ | -118.3 |
| 0.102 | $13\omega_e$ | -79.3 | .0004 | $18\omega_e$ | -47.0 |

One of the primary concerns in the application of reluctance motors is the losses incurred with an inverter source both during steady-state synchronous operation and during runup. One commonly used approach for calculating losses during runup is to assume the rotating MMF sees an equivalent "transient" reactance, which is the average value of the $d$- and $q$-axis transient reactances. Induction-motor theory is then used to calculate the motor losses. Fig. 8 shows the result of such a calculation compared with the more exact approach using harmonic balancing. In both cases the source is again a square wave voltage source inverter. The losses for the equivalent induction motor case were calculated using state variables [9]. Three dashed lines are shown corresponding to an equivalent induction motor having the reluctance motor $d$-axis parameters, $q$-axis parameters, and the average of the $d$-$q$ axes parameters. The solid line represents the results using harmonic balancing. Note that the losses are quite closely approximated by an induction motor with "average" parameters until the motor reaches half speed. At this point the losses increase rapidly and for lower slips the motor losses are better approximated by using the $d$-axis parameters

Another application problem of concern is asynchronous torque pulsations. It is evident from Fig. 7 that large pulsating torques occur during run-up. These pulsations can have detrimental effects on connected loads or gear sets. Most severe is the pulsating component having a frequency of twice slip frequency $2\omega_e - 2\omega_r$. One commonly used method of calculating this torque pulsation also uses the simpler induction-motor theory. Again, since the synchronously rotating MMF alternately encounters the $d$- and $q$-axis impedances during run-up, it is assumed that the torque developed by the motor alternates between the "induction-motor torque" produced by the $d$-axis parameters and that produced by the $q$-axis parameters. The pulsating torque is then approximated as the difference between these two torques.

Fig. 9 indicates how this approximation compares with the more accurate approach. Again, both curves include the effects of an actual square wave inverter source. Note that the approximate method is rather poor indication of the pulsating torques which actually occur. A discrepancy of 40 percent occurs at zero speed. The large low-frequency pulsations which appear as the motor approaches pull-in is not predicted by the approximate method.

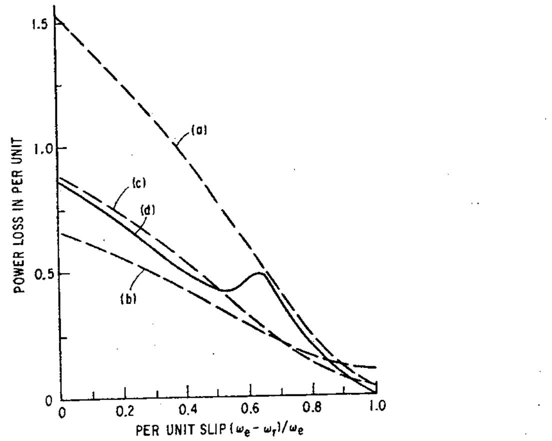


Fig. 8. Per unit losses versus slip. Losses of equivalent induction motor. (a) Using $d$-axis parameters. (b) Using $q$-axis parameters. (c) Using average of $d$ and $q$ axis parameters. (d) Exact solution using harmonic balance.

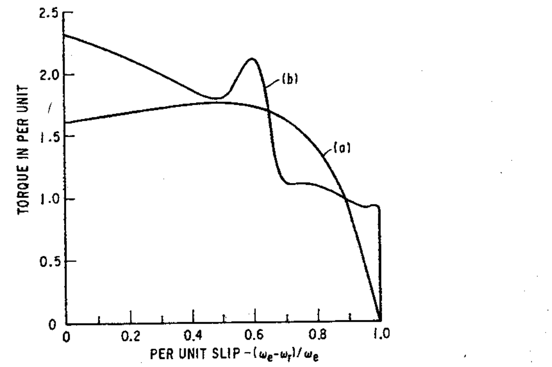


Fig. 9. Amplitude of pulsating torque at twice slip frequency. (a) Approximate result using induction motor theory. (b) Exact result using $d$-$q$ harmonic balance.

## CONCLUSION

A $d$-$q$ axis time-domain approach to the analysis of steady-state behavior of salient pole ac machines has been presented. Using this theory, machine currents, fluxes, and torques can be calculated for applied voltages having an arbitrary periodic waveform. The method is soundly based on the principles of linearity and harmonic superposition and is developed directly from Park's $d$-$q$ equations. Employing real variables exclusively and making full use of matrix theory the method is ideally suited to machine computation. Both synchronous and asynchronous operation can be analyzed with the same theory, eliminating the need for additional variable transformations such as symmetrical components and forward-backward components.

The method was applied to the analysis of a reluctance motor drive supplied by a static square wave voltage converter. The solutions for synchronous and asynchronous operations were shown to compare favorably with conventional state variable and simulation techniques.

## APPENDIX

It is useful to define the following four matrices, each of which expresses the vector cross product between various components of flux and current:

$$\bar{X}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad (75)$$

$$\bar{X}_2 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \quad (76)$$

$$\bar{X}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (77)$$

$$\bar{X}_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \quad (78)$$

If $\bar{0}_4$ denotes a $4 \times 4$ matrix of zeros, then the torque components are defined by

$$T_{e1\alpha} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \cos (k-l)\omega_e t \, \bar{I}_k{}^t \bar{C}_{1\alpha} \bar{\Psi}_l \quad (79)$$

where

$$\bar{C}_{1\alpha} = \begin{bmatrix} \bar{X}_1 & \bar{0}_4 \\ \bar{0}_4 & \bar{X}_1 \end{bmatrix},$$

$$T_{e1\gamma} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sin (k+l)\omega_e t \, \bar{I}_k{}^t \bar{C}_{1\gamma} \bar{\Psi}_l \quad (80)$$

where

$$\bar{C}_{1\gamma} = \begin{bmatrix} \bar{X}_2 & \bar{0}_4 \\ \bar{0}_4 & \bar{X}_2 \end{bmatrix},$$

$$T_{e2\alpha} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \cos (k+l)\omega_e t \, \bar{I}_k{}^t \bar{C}_{2\alpha} \bar{\Psi}_l \quad (81)$$

where

$$\bar{C}_{2\alpha} = \begin{bmatrix} \bar{0}_4 & \bar{X}_3 \\ \bar{X}_3 & \bar{0}_4 \end{bmatrix},$$

$$T_{e2\gamma} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sin (k+l)\omega_e t \, \bar{I}_k{}^t \bar{C}_{2\gamma} \bar{\Psi}_l \quad (82)$$

where

$$\bar{C}_{2\gamma} = \begin{bmatrix} \bar{0}_4 & \bar{X}_4 \\ \bar{X}_4 & \bar{0}_4 \end{bmatrix},$$

$$T_{e3\alpha} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \cos [(k+l)\omega_e - 2\omega_r] t \, \bar{I}_k{}^t \bar{C}_{3\alpha} \bar{\Psi}_l \quad (83)$$

where

$$\bar{C}_{3\alpha} = \begin{bmatrix} \bar{X}_3 & \bar{0}_4 \\ \bar{0}_4 & \bar{0}_4 \end{bmatrix},$$

$$T_{e3\gamma} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sin [(k+l)\omega_e - 2\omega_r] t \, \bar{I}_k{}^t \bar{C}_{3\gamma} \bar{\Psi}_l \quad (84)$$

where

$$\bar{C}_{3\gamma} = \begin{bmatrix} \bar{X}_4 & \bar{0}_4 \\ \bar{0}_4 & \bar{0}_4 \end{bmatrix},$$

$$T_{e4\alpha} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \cos [(k+l)\omega_e + 2\omega_r] t \, \bar{I}_k{}^t \bar{C}_{4\alpha} \bar{\Psi}_l \quad (85)$$

where

$$\bar{C}_{4\alpha} = \begin{bmatrix} \bar{0}_4 & \bar{0}_4 \\ \bar{0}_4 & \bar{X}_3 \end{bmatrix},$$

$$T_{e4\gamma} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sin [(k+l)\omega_e + 2\omega_r] t \, \bar{I}_k{}^t \bar{C}_{4\gamma} \bar{\Psi}_l \quad (86)$$

where

$$\bar{C}_{4\gamma} = \begin{bmatrix} \bar{0}_4 & \bar{0}_4 \\ \bar{0}_4 & \bar{X}_4 \end{bmatrix},$$

$$T_{e5\alpha} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \cos [(k-l)\omega_e - 2\omega_r] t \, \bar{I}_k{}^t \bar{C}_{5\alpha} \bar{\Psi}_l \quad (87)$$

where

$$\bar{C}_{5\alpha} = \begin{bmatrix} \bar{0}_4 & \bar{X}_1 \\ \bar{0}_4 & \bar{0}_4 \end{bmatrix},$$

$$T_{e5\gamma} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sin [(k-l)\omega_e - 2\omega_r[ t \, \bar{I}_k \bar{C}_{5\gamma} \bar{\Psi}_l \quad (88)$$

where

$$\bar{C}_{5\gamma} = \begin{bmatrix} \bar{0}_4 & \bar{X}_2 \\ \bar{0}_4 & \bar{0}_4 \end{bmatrix},$$

$$T_{e6\alpha} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \cos\left[(k-l)\omega_e + 2\omega_r\right] t \, \bar{I}_k{}^t \bar{C}_{6\alpha} \bar{\Psi}_l \quad (89)$$

where

$$\bar{C}_{6\alpha} = \begin{bmatrix} \bar{0}_4 & \bar{0}_4 \\ \bar{X}_1 & \bar{0}_4 \end{bmatrix},$$

$$T_{e6\gamma} = \frac{1}{2} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sin\left[(k-l)\omega_e + 2\omega_r\right] t \, \bar{I}_k{}^t \bar{C}_{6\gamma} \bar{\Psi}_l \quad (90)$$

where

$$\bar{C}_{6\gamma} = \begin{bmatrix} \bar{0}_4 & \bar{0}_4 \\ \bar{X}_2 & \bar{0}_4 \end{bmatrix}.$$

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