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Modeling and simulation of Induction Motors with Saturable Leakage Reactances

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Modeling and Simulation of Induction Motors with Saturable Leakage Reactances

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***Abstract*—A new method for the transient analysis of induction machines with saturating leakage reactances is presented. The equations which define operation under this condition are arranged so that the saturation of stator and rotor leakage as well as magnetizing reactance can be readily modeled with three function generators. The method is equally applicable to the analysis of synchronous machines.**

INTRODUCTION

THE USEFUL life of an industrial drive motor is conditioned on many factors including ambient temperature, service factor load, susceptibility to surges, and severity of over- and undervoltages and voltage unbalance. One major application consideration which can cause early failures is the starting phase of operation in which the motor is accelerated from rest to its normal operating speed. Large industrial machines are often protected from starting more than several times in succession in order to prevent overheating of the machine due to the high inrush currents which flow during the starting period. The starting problem is particularly important in heavy industry where the motor is often called upon to accelerate high-inertia loads. In addition to generating heat during the starting phase, the inrush current produces torque pulsations which in some cases have been found to resonate with the mechanical load, resulting in premature failure of couplings and gear trains [1]. This problem is particulary important in starting synchronous motors in which a frequency spectrum from zero to 120 Hz is swept as the motor accelerates from rest [2].

Because of the importance of the starting problem, an accurate calculation of starting currents and the duration of the starting period is an essential application consideration. Unfortunately, the circuit parameters of an induction machine can vary widely during starting. In particular, the large inrush currents which flow in the machine during acceleration result in large values of slot leakage flux in both the stator and rotor slots which often begin to saturate the teeth of the machine. As a consequence, the leakage inductances of the machine which are normally considered constant in conventional induction motor models become functionally dependent on the machine currents. In addition, deep bar effect causes the rotor current to shift to the tops of the rotor bars during starting, resulting in an increase in rotor resistance and a decrease in leakage inductance with increasing slip frequency.

The starting problem of an induction machine is inherently a transient phenomenon. However, in the past the steady state per phase equivalent circuit has been used to calculate the starting current and torque. This approach is entirely adequate in applications in which the inertia load can be considered so large that the acceleration of the rotor is slow enough for the machine to be considered in the quasi-steady state. However, such a method entirely neglects the possible interaction of the electromagnetic pulsating starting torque with the mechanical load resonances since the rotor speed is inherently assumed constant. Also, in a large number of practical cases the acceleration of the rotor is sufficiently rapid that the quasi-steady-state analysis approach results in appreciable error. This error is particularly large when the rotor reaches the speed corresponding to breakdown, at which point the rotor is accelerating most rapidly.

When rotor speed is considered as a variable, the equations which must be solved (Stanley's equations [3]) are nonlinear. Hence solution of these equations can only be obtained by implicit integration techniques either on a digital or an analog computer. These equations have been routinely solved by either technique for many years [4], [5]. However, solution of Stanley's equations assumes that the parameters of the machine can be considered as constant. The problem of the deep bar effect is typically handled by introducing extra circuits in the rotor [6]. However, the problem of implementing saturation in the equivalent circuit has been more difficult. Although a technique has been described in the literature for modeling saturation of the air gap or magnetizing inductance [7], a corresponding model for saturation of the leakage inductances does not exist. Unfortunately, a saturation model for leakage inductance is not a straightforward extension of magnetizing inductance saturation. When implemented on an analog computer considerable difficulty arises because the leakage inductances appear as a coefficient on 12 amplifiers or integrators, whereas the magnetizing inductance appears as a coefficient on only two amplifiers. Without significant modification of the simulation model, incorporation of leakage inductance saturation becomes hopelessly complex. The computational problem is equally difficult on a digital computer since all the elements of the "L inverse" matrix which must normally be computed as part of a digital computer solution are functions of both stator and rotor leakage inductance. Enormous blocks of computing time can be consumed while iterating all elements of this matrix to

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converge on an acceptable value of leakage inductance saturation for each time step.

This paper presents a new analytical method for introducing stator and rotor leakage inductance saturation into a simulation model for an induction machine. A key step in the method is to separate both the stator and rotor leakage inductances into slot and end winding portions. Although the slot portion saturates, the end winding portion is assumed constant, thereby providing a means to isolate the leakage saturation effect and markedly simplify the complexity of the simulation. Although applied specifically to induction machines, the approach is equally applicable to the analysis of synchronous machines.

SATURATION OF LEAKAGE INDUCTANCE

In general, an induction motor is considered to have five components of leakage flux, namely, slot, zig-zag, belt, skew, and end winding leakage. The slot leakage arises from flux which crosses straight across the slot driven by the MMF of the slot itself. The zig-zag flux corresponds to that component of flux driven by an assumed sinusoidal distribution of MMF which crosses the gap but links only stator or rotor turns. The belt leakage flux is that portion of the stator or rotor flux which is driven by all harmonics not including the fundamental. This flux again crosses the air gap but again links only stator or rotor turns since the pole number of each harmonic is not the same as the fundamental. (The harmonic flux which links harmonic components of both stator and rotor is neglected in conventional analyses.) The sum of the zig-zag and belt leakage is often termed the differential leakage flux. The skew leakage inductance arises due to the difference in MMF between one side of a skewed bar and the other, thereby driving a flux component which links only the skewed winding. The end winding leakage corresponds to flux in that portion of the machine windings external to the stator and rotor core. In general the slot, zig-zag, belt, and skew leakage fluxes all complete the major portions of their paths through iron paths and therefore can be considered as saturable quantities. However, the end winding leakage flux path consists entirely or almost entirely of an air path. Hence this component of leakage flux can be assumed unsaturable and independent of saturation effects which take place in the core body.

In order to portray the leakage flux saturation phenomenon accurately, the relative proportions of end winding and slot dependent fluxes must first be established. Calculation of the leakage inductances is best accomplished analytically or with finite elements by consideration of the winding distribution and the geometry of the slot shapes. Although saturation of the iron dependent leakage fluxes is clearly a complicated phenomenon, several analytical methods have been proposed which take into account the saturation effect on the iron-dependent leakage fluxes [8]-[10]. These methods are generally based on the calculation of unsaturated reactances which are reduced by so-called saturation factors. These saturation factors are defined as the ratio of the actual reactance to the value which would exist if no saturation occurred.

In the absence of detailed design information, the leakage saturation effect is best measured by means of a locked rotor test. If the machine also demonstrates a significant amount of deep bar effect, care must be taken to separate this frequency-dependent effect from the purely current amplitude dependence of leakage inductance saturation. The separation of current-dependent and frequency-dependent components can best be accomplished by repeating the locked rotor test at several frequencies, thereby extracting the frequency dependent from the current dependent component.

The separation of the stator and rotor components and the separation of the end winding and slot dependent portions of the leakage inductance is clearly more difficult and cannot be accomplished without laborious search coil measurements. In practice, the error is small if the stator and rotor components are apportioned equally or according to the IEEE standard [11]. The relative values of the end winding leakage inductance and the unsaturated slot iron dependent leakage inductances are equally difficult to determine. One standard method used to identify the relative proportion of end winding leakage is a locked rotor test using several machines with the same end winding configuration and having several core lengths [12]. In the absence of such a facility the values can generally be assigned by means of rough calculation. In practice, the solution of the machine equations is effectively independent of this choice so long as the sum of the end winding and slot dependent leakage inductances is always made equal to the measured total leakage inductance for any operating condition.

In general, it is then possible to assume that the total stator and rotor leakage inductances can be separated into air- and iron-dependent portions, that is

$$X_{ls} = X_{lsa} + X_{lsi} \tag{1}$$

and

$$X_{lr} = X_{lra} + X_{lri}. \tag{2}$$

The terms X_{lsi} and X_{lri} correspond to the sum of the iron-dependent slot, zig-zag, belt, and skew leakage inductances for the stator and rotor, respectively, and are assumed to saturate. The terms X_{lsa} and X_{lra} correspond essentially to the air-dependent end winding leakage inductances and are assumed constant.

The saturation effect on the iron-dependent portion of the leakage inductances can be evaluated by extending the slope-ratio method of DeMello and Walsh [7]. The method can be illustrated by reference to Fig. 1 in which the measured or calculated voltage drop across the saturable portion of a specified leakage reactance has been plotted as a function proportional to the stator current amplitude. When this proportionality constant is chosen as the unsaturated value of reactance, the curve then always intersects the origin at a slope of unity.

In general, the iron-dependent stator leakage flux which exists for any specified value of current can be written

$$\Delta\psi_{lsi}\,(\text{sat}) = K_{lsi}(\psi_{lsi})\psi_{lsi}\,(\text{unsat}) \tag{3}$$

where K_{lsi} is a saturation factor given by

$$K_{lsi}(\psi_{lsi}) = \frac{\Delta\psi_{lsi}\,(\text{sat})}{\psi_{lsi}\,(\text{unsat})}. \tag{4}$$

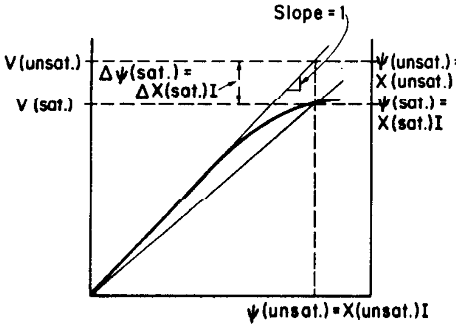

Fig. 1. Saturation curve illustrating formulation of saturation factor.

Note that the definition of this factor is effectively the reverse of the saturation factor as usually defined. That is,

$$K_{ksi} = 1 - K_s \tag{5}$$

where K_s is the usual stator saturation factor.

Since the flux is assumed to be reduced for a specified value of current, (4) can be written in the equivalent form

$$K_{lsi}(\psi_{lsi}) = \frac{\Delta X_{lsi}\,(\text{sat})\, i_s}{X_{lsi}\,(\text{unsat})\, i_s} \tag{6}$$

which can be written simply as

$$K_{lsi}(\psi_{lsi}) = \frac{\Delta X_{lsi}\,(\text{sat})}{X_{lsi}\,(\text{unsat})}. \tag{7}$$

In a similar manner it can be verified that the iron-dependent rotor leakage flux can be represented by

$$\Delta\psi_{lri}\,(\text{sat}) = K_{lri}(\psi_{lri})\psi_{lri}\,(\text{unsat}) \tag{8}$$

$$K_{lri}(\psi_{lri}) = \frac{\Delta\psi_{lri}\,(\text{sat})}{\psi_{lri}\,(\text{unsat})} \tag{9}$$

or

$$K_{lri}(\psi_{lri}) = \frac{\Delta X_{lri}\,(\text{sat})}{X_{lri}\,(\text{unsat})}. \tag{10}$$

INDUCTION MOTOR EQUATIONS INCLUDING SATURATION EFFECTS

The equations used to define the transient behavior of an induction machine are generally represented in an orthogonal coordinate system which is either rotating or stationary. When the angular speed ω of the rotating axes is not specified, these equations can be written in the form [4]

$$v_{qs} = r_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega\lambda_{ds} \tag{11}$$

$$v_{ds} = r_s i_{ds} + \frac{d\lambda_{ds}}{dt} - \omega\lambda_{qs} \tag{12}$$

$$v_{0s} = r_s i_{0s} + \frac{d\lambda_{0s}}{dt} \tag{13}$$

$$v_{qr} = r_r i_{qr} + \frac{d\lambda_{qr}}{dt} + (\omega - \omega_r)\lambda_{dr} \tag{14}$$

$$v_{dr} = r_r i_{dr} + \frac{d\lambda_{dr}}{dt} - (\omega - \omega_r)\lambda_{qr} \tag{15}$$

$$v_{0r} = r_r i_{0r} + \frac{d\lambda_{0r}}{dt}. \tag{16}$$

The voltages v_{qs}, v_{ds}, and v_{0s} represent the applied stator voltages in the orthogonal coordinate system and are related algebraically to these voltages. The voltages v_{qr}, v_{dr}, and v_{0r} as well as all other rotor variables are referred to the stator by the effective stator/rotor turns ratio. The equations of transformation relating the d-q-0 stator and rotor voltages to the actual applied voltages are given in [4]. When the machine rotor windings are shorted, such as for a squirrel-cage machine, the rotor voltages are identically equal to zero. Also, when the machine is fed from a three-wire system without a neutral return, the currents i_{0s} and i_{0r} are identically zero so that (13) and (16) can often be omitted from consideration. However, for the purpose of generality, these equations will be included throughout this analysis.

The flux linkages which appear in (11)–(16) are related to the currents by

$$\lambda_{qs} = L_{ls} i_{qs} + L_m(i_{qs} + i_{qr}) \tag{17}$$

$$\lambda_{qr} = L_{lr} i_{qr} + L_m(i_{qs} + i_{qr}) \tag{18}$$

$$\lambda_{ds} = L_{ls} i_{ds} + L_m(i_{ds} + i_{dr}) \tag{19}$$

$$\lambda_{dr} = L_{lr} i_{dr} + L_m(i_{ds} + i_{dr}) \tag{20}$$

$$\lambda_{0s} = L_{ls} i_{0s} \tag{21}$$

$$\lambda_{0r} = L_{lr} i_{0r} \tag{22}$$

where the leakage inductances are now assumed to be comprised of both an iron-dependent and air-dependent portions, that is

$$L_{ls} = L_{lsa} + L_{lsi} \tag{23}$$

and

$$L_{lr} = L_{lra} + L_{lri}. \tag{24}$$

The flux linkages λ_{md} and λ_{mq} in (17)–(20) represent the mutual fluxes which link both the stator and rotor and are given by

$$\lambda_{mq} = L_m(i_{qs} + i_{qr}) \tag{25}$$

$$\lambda_{md} = L_m(i_{ds} + i_{dr}) \tag{26}$$

where L_m is the magnetizing inductance and is also assumed to be saturable.

It is useful to define flux linkages corresponding to only that portion of the leakage fluxes which do not saturate, or

$$\lambda_{lqsa} = L_{lsa} i_{qs} \tag{27}$$

$$\lambda_{ldsa} = L_{lsa} i_{ds} \tag{28}$$

$$\lambda_{l0sa} = L_{lsa} i_{0s} \tag{29}$$

$$\lambda_{lqra} = L_{lra} i_{qr} \tag{30}$$

$$\lambda_{ldra} = L_{lra} i_{dr} \tag{31}$$

$$\lambda_{l0ra} = L_{lra} i_{0r}. \tag{32}$$

Upon substituting (23), (24), and (27)-(32) in (17)-(22) and solving for the six machine currents, it is possible to write the result as

$$i_{qs} = [\psi_{qs} - \psi_{mq}(\text{sat}) - \psi_{lqsi}(\text{sat})]/X_{lsa} \tag{33}$$

$$i_{ds} = [\psi_{ds} - \psi_{md}(\text{sat}) - \psi_{ldsi}(\text{sat})]/X_{lsa} \tag{34}$$

$$i_{0s} = [\psi_{0s} - \psi_{l0si}(\text{sat})]\ X_{lsa} \tag{35}$$

$$i_{qr} = [\psi_{qr} - \psi_{mq}(\text{sat}) - \psi_{lqri}(\text{sat})]/X_{lra} \tag{36}$$

$$i_{dr} = [\psi_{dr} - \psi_{md}(\text{sat}) - \psi_{ldri}(\text{sat})]/X_{lra} \tag{37}$$

$$i_{0r} = [\psi_{0r} - \psi_{l0ri}(\text{sat})]/X_{lra}. \tag{38}$$

In (33)-(38) the inductances have been replaced by equivalent reactances, i.e., $X_{lsi} = \omega_b L_{lsi}$ where ω_b is a base or reference angular velocity generally taken to be 377 rad/s. Similarly, the flux linkages λ have been replaced by modified flux linkages ψ where, for example, $\psi_{lqsi} = \omega_b \lambda_{lqsi}$. Also the functional dependence of the saturable flux linkages has been explicitly indicated.

When the currents are eliminated from (11)-(16) by use of (33)-(38), these equations can be solved for the time derivatives and then integrated to form

$$\psi_{qs} = \int \omega_b \left[v_{qs} + \frac{r_s}{X_{lsa}} (\psi_{mq}(\text{sat}) + \psi_{lqsi}(\text{sat}) - \psi_{qs}) - \left(\frac{\omega}{\omega_b}\right) \psi_{ds} \right] dt \tag{39}$$

$$\psi_{ds} = \int \omega_b \left[v_{ds} + \frac{r_s}{X_{lsa}} (\psi_{md}(\text{sat}) + \psi_{ldsi}(\text{sat}) - \psi_{ds}) + \left(\frac{\omega}{\omega_b}\right) \psi_{qs} \right] dt \tag{40}$$

$$\psi_{qr} = \int \omega_b \left[v_{qr} + \frac{r_r}{X_{lra}} (\psi_{mq}(\text{sat}) + \psi_{lqri}(\text{sat}) - \psi_{qr}) - \frac{(\omega - \omega_r)}{\omega_b} \psi_{dr} \right] dt \tag{41}$$

$$\psi_{dr} = \int \omega_b \left[v_{dr} + \frac{r_r}{X_{lra}} (\psi_{md}(\text{sat}) + \psi_{ldri}(\text{sat}) - \psi_{dr}) + \frac{(\omega - \omega_r)}{\omega_b} \psi_{qr} \right] dt \tag{42}$$

$$\psi_{0s} = \int \omega_b \left[v_{0s} + \frac{r_s}{X_{lsa}} (\psi_{l0si}(\text{sat}) - \psi_{0s}) \right] dt \tag{43}$$

$$\psi_{0r} = \int \omega_b \left[v_{0r} + \frac{r_r}{X_{lra}} (\psi_{l0ri}(\text{sat}) - \psi_{0r}) \right] dt. \tag{44}$$

In order to solve these equations completely, we must develop independent expressions for the saturated values of the magnetizing and iron-dependent leakage fluxes. In order to solve for these quantities it is useful first to solve for those values which would exist were saturation not to occur, that is, if the saturation curves followed the air gap line. When (33), (34), (36), and (37) are substituted into (25) and (26) using the unsaturated values of mutual flux linkages, we obtain

$$\psi_{mq}(\text{unsat}) = X_m^* \psi_{qs}/X_{lsa} + X_m^* \psi_{qr}/X_{lra} + X_m^* \left(\frac{1}{X_{lsa}} + \frac{1}{X_{lra}}\right) \Delta\psi_{mq} - X_m^* \psi_{lqsi}(\text{sat})/X_{lsa} - X_m^* \psi_{lqri}(\text{sat})/X_{lra} \tag{45}$$

$$\psi_{md}(\text{unsat}) = X_m^* \psi_{ds}/X_{lsa} + X_m^* \psi_{dr}/X_{lra} + X_m^* \left(\frac{1}{X_{lsa}} + \frac{1}{X_{lra}}\right) \Delta\psi_{md} - X_m^* \psi_{ldsi}(\text{sat})/X_{lsa} - X_m^* \psi_{ldri}(\text{sat})/X_{lra} \tag{46}$$

where

$$X_m^* = 1/[1/X_m(\text{unsat}) + 1/X_{lsa} + 1/X_{lra}]. \tag{47}$$

When unsaturated, the iron-dependent leakage flux linkages are related to the machine currents by

$$\psi_{lqsi}(\text{unsat}) = X_{lsi}(\text{unsat})\, i_{qs} \tag{48}$$

$$\psi_{ldsi}(\text{unsat}) = X_{lsi}(\text{unsat})\, i_{ds} \tag{49}$$

$$\psi_{l0si}(\text{unsat}) = X_{lsi}(\text{unsat})\, i_{0s} \tag{50}$$

$$\psi_{lqri}(\text{unsat}) = X_{lri}(\text{unsat})\, i_{qr} \tag{51}$$

$$\psi_{ldri}(\text{unsat}) = X_{lri}(\text{unsat})\, i_{dr} \tag{52}$$

$$\psi_{l0ri}(\text{unsat}) = X_{lri}(\text{unsat})\, i_{0r}. \tag{53}$$

When (33)-(38) are substituted into (48)-(53), again using the unsaturated values of flux linkages, the following equations can be obtained for leakage components of iron-dependent flux linkages

$$\psi_{lqsi}(\text{unsat}) = (X_{ls}^*/X_{lsa})[\psi_{qs} + \Delta\psi_{lqsi}(\text{sat}) - \psi_{mq}(\text{sat})] \tag{54}$$

$$\psi_{ldsi}(\text{unsat}) = (X_{ls}^*/X_{lsa})[\psi_{ds} + \Delta\psi_{ldsi}(\text{sat}) - \psi_{md}(\text{sat})] \tag{55}$$

$$\psi_{l0si}(\text{unsat}) = (X_{ls}^*/X_{lsa})[\psi_{0s} + \Delta\psi_{l0si}(\text{sat})] \tag{56}$$

$$\psi_{lqri}\,(\text{unsat}) = (X_{lr}^*/X_{lra})[\psi_{qr} + \Delta\psi_{lqri}\,(\text{sat}) - \psi_{mq}\,(\text{sat})] \quad (57)$$

$$\psi_{ldri}\,(\text{unsat}) = (X_{lr}^*/X_{lra})[\psi_{dr} + \Delta\psi_{ldri}\,(\text{sat}) - \psi_{md}\,(\text{sat})] \quad (58)$$

$$\psi_{l0ri}\,(\text{unsat}) = (X_{lr}^*/X_{lra})[\psi_{0r} + \Delta\psi_{l0ri}\,(\text{sat})] \quad (59)$$

where the quantities X_{ls}^* and X_{lr}^* are given by

$$X_{ls}^* = 1/[1/X_{lsa} + 1/X_{lsi}\,(\text{unsat})] \quad (60)$$

$$X_{lr}^* = 1/[1/X_{lra} + 1/X_{lri}\,(\text{unsat})]. \quad (61)$$

The saturated values of magnetizing and iron-dependent leakage flux linkages can now be determined by means of saturation factors. Saturation of the magnetizing flux is dependent upon the instantaneous amplitude of flux in the air gap, that is,

$$\psi_m\,(\text{unsat}) = \sqrt{\psi_{mq}^2\,(\text{unsat}) + \psi_{md}^2\,(\text{unsat})}. \quad (62)$$

If the saturation factor K_m is assumed to be known, then the saturated value of magnetizing flux linkage can be obtained from (3), that is,

$$\Delta\psi_m\,(\text{sat}) = K_m(\psi)\psi_m\,(\text{unsat}). \quad (63)$$

The d and q axis components can also be found by use of the same saturation factor, that is,

$$\Delta\psi_{mq}\,(\text{sat}) = K_m(\psi_m)\psi_{mq}\,(\text{unsat}) \quad (64)$$

$$\Delta\psi_{md}\,(\text{sat}) = K_m(\psi_m)\psi_{md}\,(\text{unsat}). \quad (65)$$

The saturated flux linkages, used in (39)-(44) are then simply

$$\psi_{mq}\,(\text{sat}) = \psi_{mq}\,(\text{unsat}) - \Delta\psi_{mq}\,(\text{sat}) \quad (66)$$

$$\psi_{md}\,(\text{sat}) = \psi_{md}\,(\text{unsat}) - \Delta\psi_{md}\,(\text{sat}). \quad (67)$$

The saturated values of stator and rotor iron-dependent leakage flux linkages are found in a similar manner except that the magnitude of the unsaturated value of flux linkage must be determined from the vector sum of all three flux components. Hence

$$\psi_{lsi}\,(\text{unsat}) = \sqrt{\psi_{lqsi}^2\,(\text{unsat}) + \psi_{ldsi}^2\,(\text{unsat}) + \psi_{l0si}^2\,(\text{unsat})}. \quad (68)$$

The saturation factor K_{lsi} can then be found from (7), and the saturated values of ψ_{lqsi}, ψ_{ldsi}, and ψ_{l0si} found in a manner analogous to (66) and (67). The calculation of the saturated rotor leakage flux linkages also follow the same approach.

COMPUTER SIMULATION

Equations (33)-(66) form the basis for a computer simulation of an induction machine with both saturated magnetizing and leakage reactances. These equations can be used to generate the analog computer diagram shown in Fig. 2. In order to avoid unnecessary complication, this simulation diagram shows the implementation for the case where the rotating axes are stationary (stationary reference frame). In this case the d, q, 0 components take on the same meaning as the conventional α, β, 0 components. The quantities K_m, K_{lsi}, and K_{lri} shown as inputs to multipliers in Fig. 2 are the saturation factors. When these factors can be approximated sufficiently accurately by straight lines, they can be developed in analog computer fashion from the simple computer simulation shown in Fig. 3. Specifically, Fig. 3 shows a mechanization of the saturation factor K_{lsi}. Implementation of the other saturation factors clearly follow by analogy. Although not mentioned explicitly in this paper, a complete simulation of machine transients also involves the computation of the electromagnetic torque as well as an appropriate representation of the connected mechanical load. Computation of torque with saturated parameters does not differ from standard techniques and is included in Fig. 2.

It is important to observe that the coefficients of all potentiometer settings in Fig. 2 are constant and that the saturation effects have been completely isolated to three function generators. Implementation of saturation in this manner is now clearly a straightforward process. In practice, leakage path saturation generally takes place primarily in either the stator or rotor teeth, and sufficient accuracy can often be maintained by modeling only one of the two leakage path saturation effects.

It is evident that although an analog computer approach has been emphasized in this paper, the equations which have been derived can be implemented on a digital computer with equal facility. The simplification of the computer equations will in this case result in a substantial reduction in computational time. However, because of the nature of digital computation, iterations will still be required to establish proper values of saturated magnetizing and leakage fluxes for each time step.

COMPARISON OF COMPUTED AND EXPERIMENTAL RESULTS

In order to verify the simulation approach which has been developed in this paper, experimental studies were conducted to compare theory with test. The machine chosen for this study was a three-phase three-wire 230-V squirrel-cage machine rated at 5 hp. The motor was designed for submersible pump operation. Since this class of machines is designed to fit in drilled wells, the ratio of stack length to air gap diameter is much larger than conventionally designed machines with the consequent effect that the iron-dependent leakage inductances are a much larger proportion of the total leakage inductance. The measured no-load and locked-rotor saturation curves are shown in Figs. 4 and 5, respectively. On the basis of these tests the following parameters of this machine were established: stator resistance $r_s = 0.4122\ \Omega$, rotor resistance $r_r = 0.4976\ \Omega$, magnetizing reactance X_m (unsat) = 15.7 Ω stator, and rotor air dependent leakage reactance X_{lsa}, X_{lra} = 0.15 Ω, stator and rotor iron-dependent leakage reactance X_{lsi} (unsat), X_{lri} (unsat) = 0.95 Ω, rotor inertia J = 0.11 kg·m^2. Note that, for convenience, the saturation factors calculated from

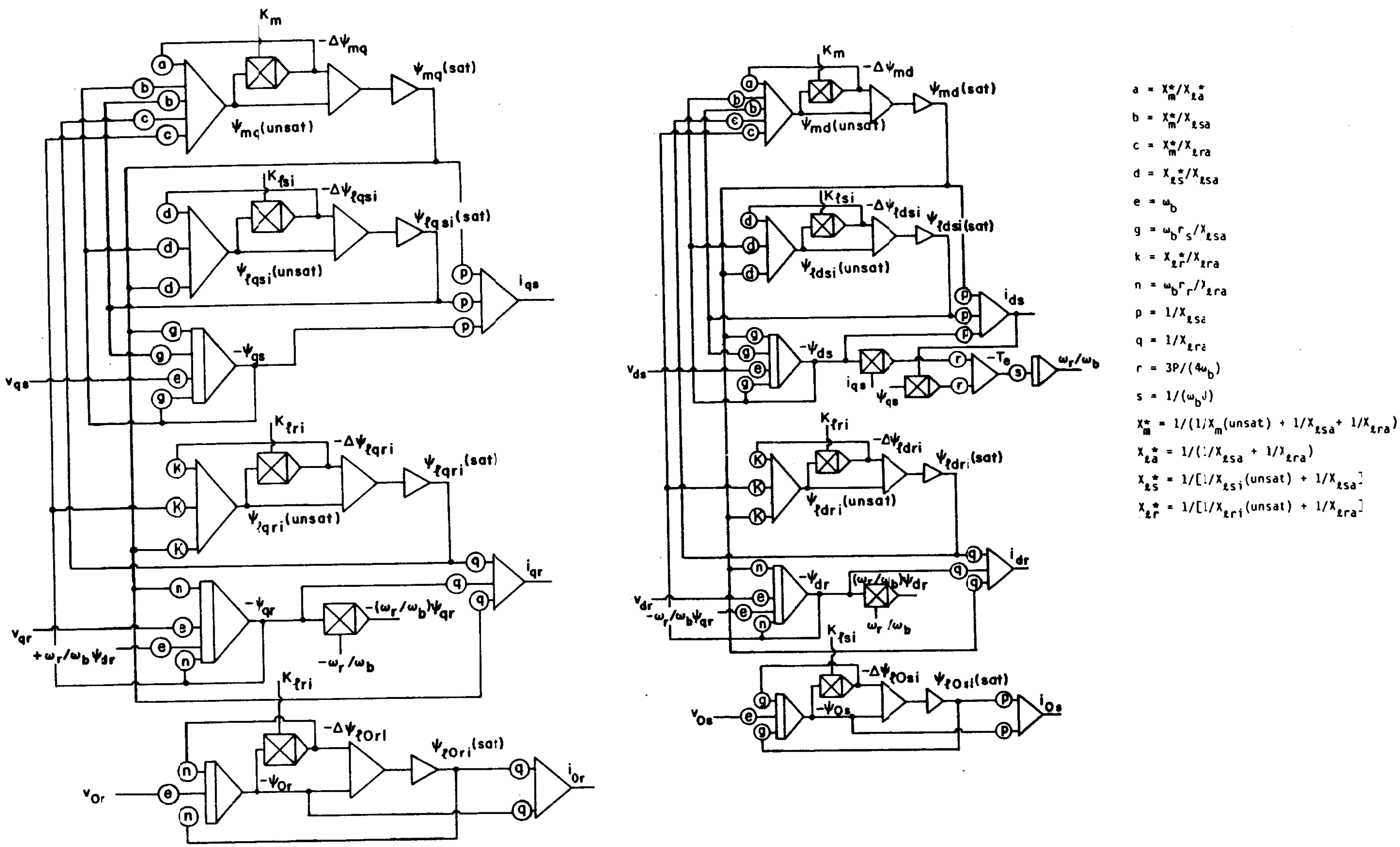

Fig. 2. Simulation diagram for modeling of induction machine with saturating magnetizing and leakage reactance, stationary reference frame.

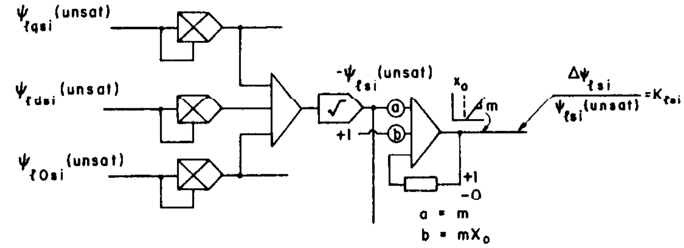

Fig. 3. Simulation diagram to model saturation factor K_{lsi}.

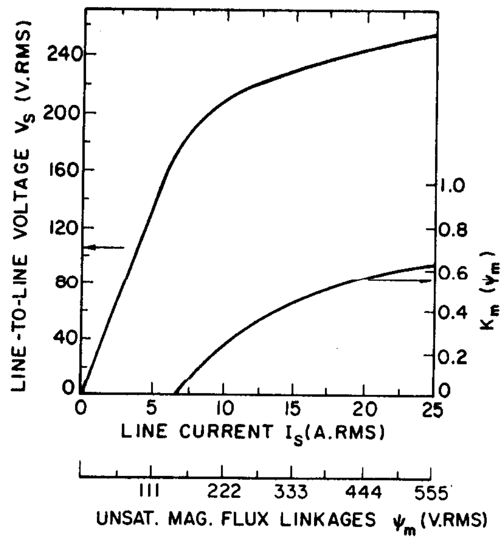

Fig. 4. No-load saturation curve and saturation factor K_m for tested machine.

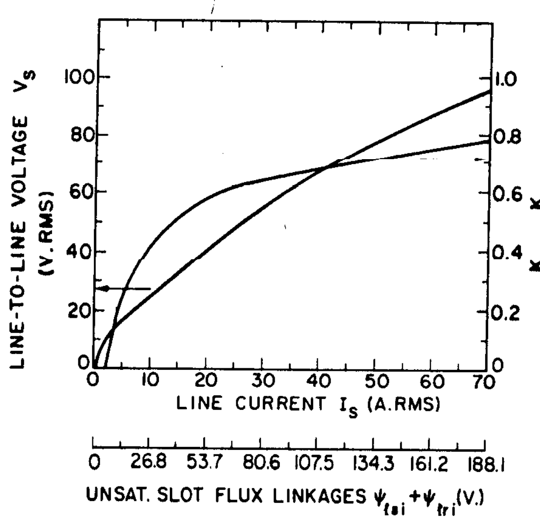

Fig. 5. Locked-rotor saturation curve and saturation factors K_{lsi} and K_{lri} for tested 5-hp machine.

the no-load and locked-rotor test were approximated slightly in the region where the actual saturation curve deviates from the air gap line. In general, extreme accuracy is not necessary for acceptable results.

Fig. 6 shows the line current and rotor speed measured with a digital sampling oscilloscope when the machine was accelerated from rest at 230 V without any connected load. It can be observed that acceleration of this machine is extremely rapid since the rotor is basically long and tubular in shape and therefore has a relatively small inertia. In Fig. 7 is a computer trace of the same condition using the simulation model of Figs. 2 and 3. A close correlation of both line current and speed is clearly evident.

The computer traces shown in Fig. 8 have been included to illustrate the importance of the saturation effect during the acceleration period of this motor. In this case the motor was simulated having constant values of stator and rotor leakage reactance. In particular, the values were selected by using the

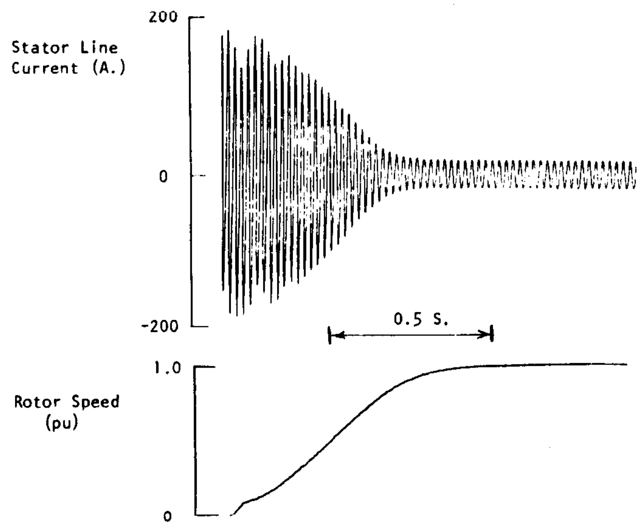

Fig. 6. Measured stator current and rotor speed of test machine during free acceleration from rest.

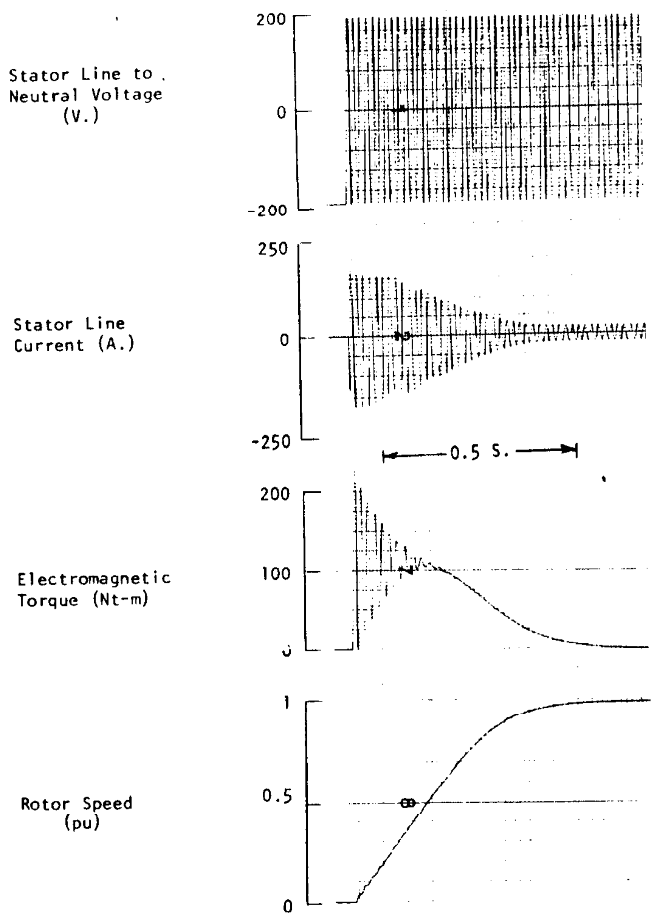

Fig. 7. Analog computer traces for free acceleration of test machine using magnetizing and leakage reactance saturation factors.

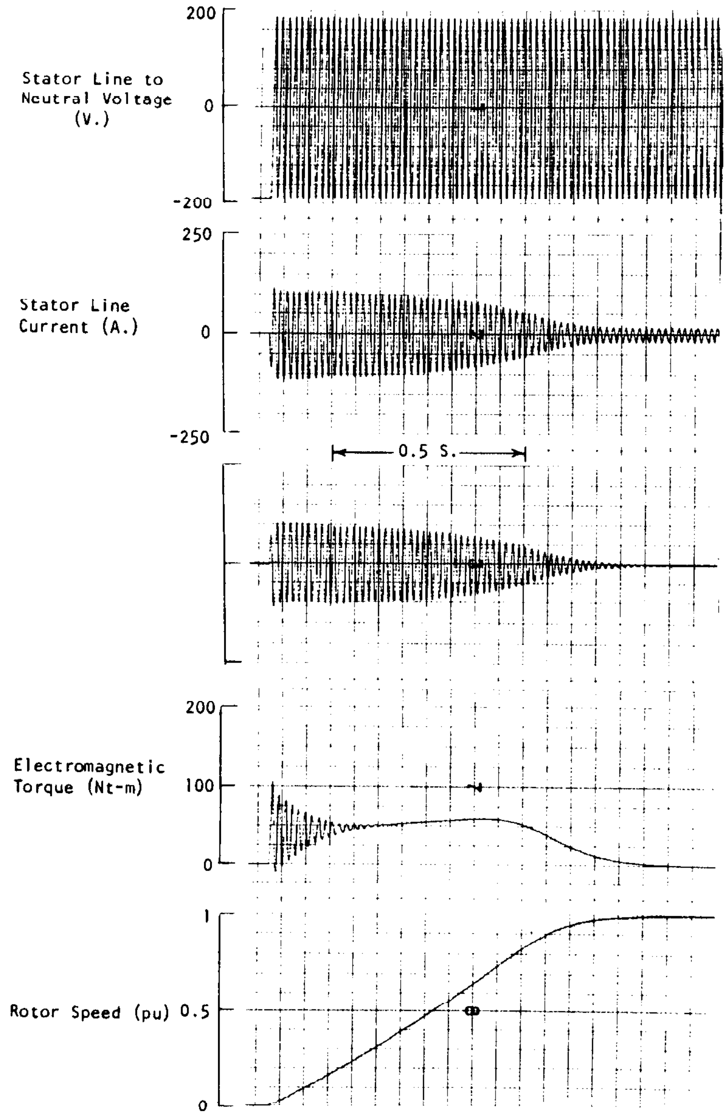

Fig. 8. Analog computer traces for free acceleration of test machine using constant full load measured values of magnetizing and leakage reactance.

locked-rotor test curves of Fig. 5 and finding the slope of a line which passes through the rated value of line current (17.5 A) resulting in X_{lsi} (sat) $= X_{lri}$ (sat) $= 0.4\ \Omega$. A substantial error in the solution is apparent. It is clear that much closer correlation could be obtained by selecting constant values of leakage reactance which correspond not to rated current but to the inrush current. Although good correlation could now be obtained during the starting phase, the behavior model of the motor over the normal range of operating slip speeds will now be in error since the starting value of leakage reactance is too small for good correlation at normal levels of current.

Note that several key assumptions have been made in the derivation of the motor simulation. In particular, it has been assumed that the reactance to zero sequence current is identically equal to the stator and rotor leakage reactance. In practice, this is only approximately true since the belt, zig-zag, and skew leakage reactances can differ appreciably due to the different MMF distribution for the zero sequence component. If necessary, this effect could be included in the analysis by using a modified value of iron-dependent reactance in the determination of the zero sequence flux linkages (56) and (59) and weighting the zero sequence component different than the d and q axis component in the determination of the unsaturated value of slot leakage flux in (66). In most applications this problem is not of concern since three-wire systems are commonly employed, and the zero sequence components of current are identically zero. Also, it has been assumed that the main air gap flux does not contribute markedly to the saturation of the leakage component of flux. This assumption has been made by some researchers [8], [9] and not others [10]. Should this effect become important, the structure of the simulation could change markedly. However, if the functional dependence of the leakage flux on the magnetizing flux could be determined analytically, this effect could be incorporated by making the saturated value of leakage flux dependent on the unsaturated values of both magnetizing and leakage flux.

CONCLUSION

In many motor applications an accurate calculation of the inrush current and accelerating time cannot be accomplished by traditional methods since the motor accelerates too rapidly to be considered in the quasi-steady state. In this case a true transient simulation is needed to obtain a faithful representation of the acceleration process. In the past, only crude approximations have been available to calculate these important parameters. This paper has presented a new and novel method for an accurate transient simulation of a machine with saturating leakage reactances as well as saturating magnetizing reactance. The method is believed to involve a minimum of computer components and/or computational steps. The algorithm is computationally stable with either analog or digital computational tools. The method can be readily extended to problems involved with the starting of synchronous machines. This technique should find widespread acceptance with those engineers engaged in the analysis and application of high horsepower industrial motor drives.

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