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RESEARCH REPORT
81-5

Orthogonal Axis Models for Asymmetrically Connected Induction Machines

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September 1981

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Abstract - This paper presents a new method for the transient analysis of induction machines with asymmetrical connection of the stator phases. The method is based on the unified theory of electrical machines and sets forth a convenient framework for depicting the machine together with the constraints of the system to be studied. Equivalent d-q-0 circuits are deduced for a number of specific connections and results of an analog computer simulation used to illustrate the proposed procedure.

INTRODUCTION

Although usually unintended, asymmetrical operation of an induction machine is of considerable interest since such modes typically shorten the useful operating life of the machine. Such conditions can occur during normal operation following a fault on the supply line or across the motor phases. Also, unbalanced conditions occur by intentionally or inadvertantly connecting the stator in an asymmetrical manner or by introducing inequalities in the phase impedances in order to obtain a special speed-torque characteristic for starting or braking. For example, in crane drives, an asymmetric voltage control has been applied to permit a quasi-linear torque vs. slip characteristic[1]. In each case it is essential to estimate the effective stress imposed on the motor and to verify that the operating condition is within the rating of the motor.

Because of the importance of such problems many analysis techniques have arisen generally devoted to the study of a specific problem. However, a global treatment of the problem concerned with the study of both transient and steady state phenomena has not appeared in the literature. Perhaps the most systematic analysis of the problem has been given by Brown and Butler[2,3], who suggest a general method of analysis based on symmetrical components. Special attention was given to the presence of the zero sequence component and to the correct evaluation of its effect on the performance of the machine. However, the issue of transient behavior was, regrettably, not considered.

At present, the generalized theory of electrical machines incorporating orthogonal axis or dq axis theory is generally accepted as the preferred approach to almost all types of transient and steady state phenomena. Hence, it would be useful to extend this approach to also incorporate the problems associated with asymmetrically connected induction machines.

In order to demonstrate the feasibility of such an approach it is first necessary to establish the limiting assumptions associated with generalized machine theory. These assumptions are: 1) all the spatial harmonics of the MMF waveform in the air gap are neglected except for the fundamental component, 2) losses are due only to the stator and rotor copper losses so that hysteresis and eddy currents are neglected, 3) the magnetic circuit of the machine is linear which is equivalent to neglecting saturation. When the motor is supplied in an asymmetrical manner the issue of the zero sequence components must also be considered. The resultant MMF waveform produced by zero sequence voltages contains only third harmonic spatial components and its multiples due to the relative disposition of the phase windings. The superposition of this flux with the positive and negatively rotating fundamental components results in the total flux in the gap. In general, the third harmonic component and its multiples also induce rotor current. However, this effect is typically small so that if this component of rotor coupling is omitted the zero sequence flux constitutes a pure leakage component which can be solved separately from the other components of flux in the gap. If necessary, the rotor coupling effect can be approximated by introducing small modifications in the uncoupled values of zero sequence inductance and resistance. The approach remains feasible so long as assumption 3 remains valid. If necessary, saturation of both magnetizing and leakage inductances could be included[4], but at the expense of a more elaborate simulation.

The analytical method presented in this paper illustrates the key steps in defining three uncoupled groups of equations which describe the behavior of an induction machine for a wide variety of asymmetric operating conditions. In essence, the approach to be presented is a logical extension of the α, β, 0 components of E. Clarke [5]. It is shown that orthogonal axis techniques provide a straightforward method for analyzing a wide variety of asymmetries which have, in the past, required the use of symmetrical components in their solution. Whereas symmetrical components are generally restricted to steady state without fairly obscure extensions[6], the orthogonal axis approach can be used to investigate both steady state and transient behavior with equal facility. In this paper, solutions are obtained for a variety of asymmetric connections including several which have apparently not been solved by any previous analytical technique.

GENERAL METHOD OF ANALYSIS

The simplest practical configuration that can be used to represent the voltage constraints on the stator terminals of an induction machine is shown in Fig. 1. In general, the motor is normally connected to a three phase system of source voltages $e'_{ag}, e'_{bg}, e'_{cg}$ having the polarities as indicated in the figure. The source voltages are considered to be known and sinusoidal, or

$$e'_{ag} = E \cos \omega_e t \quad (1)$$

$$e'_{bg} = E \cos(\omega_e t - 2\pi/3) \quad (2)$$

$$e'_{cg} = E \cos(\omega_e t + 2\pi/3) \quad (3)$$

where ω_e is the angular speed corresponding to the source frequency. As indicated in Fig. 1 the internal impedance of each phase of the supply is also considered so that with the positive directions of voltage and current indicated,

$$e_{ag} = e'_{ag} - r_i i_{ag} - L_i \frac{di_{ag}}{dt} \quad (4)$$

$$e_{bg} = e'_{bg} - r_i i_{bg} - L_i \frac{di_{bg}}{dt} \quad (5)$$

$$e_{cg} = e'_{cg} - r_i i_{cg} - L_i \frac{di_{cg}}{dt} \quad (6)$$

82 WM 230-1 A paper recommended and approved by the IEEE Rotating Machinery Committee of the IEEE Power Engineering Society for presentation at the IEEE PES 1982 Winter Meeting, New York, New York, January 31-February 5, 1982. Manuscript submitted September 4, 1981; made available for printing November 20, 1981.

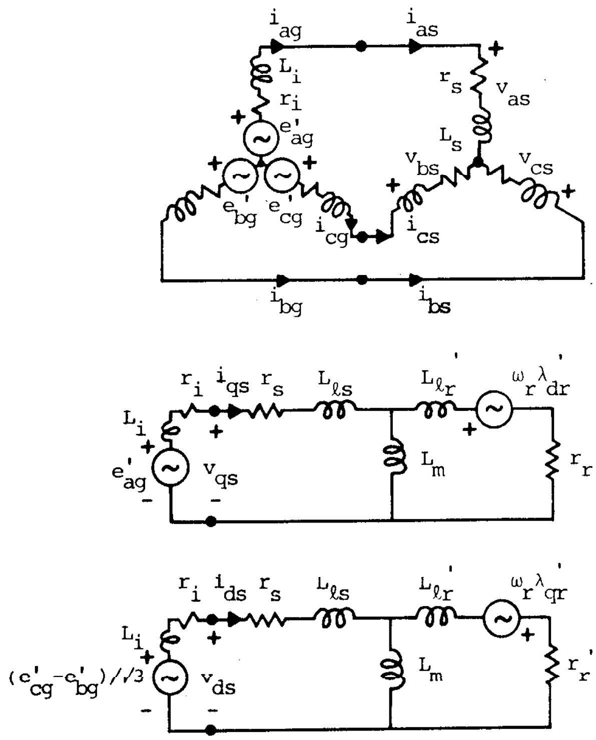


Fig. 1 Normal star connected induction machine and equivalent q and d axes circuits.

The phase voltages of the motor are written as

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt} \tag{7}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt} \tag{8}$$

$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt} \tag{9}$$

where the flux linkages are related to the phase currents by the following matrix equation,

$$\bar{\lambda}_{abcs} = \bar{L}_s \bar{i}_{abcs} + \bar{L}_{sr} \bar{i}_{abcr} \tag{10}$$

where

$$\bar{\lambda}_{abcs} = \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix}; \bar{i}_{abcs} = \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}; \bar{i}_{abcr} = \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

$\bar{L}_s$ is the matrix of stator self and mutual inductances, and $\bar{L}_{sr}$ is the matrix of stator-rotor mutual inductances.

The equations which describe the transient behavior of an induction machine in an orthogonal coordinate system change depending upon whether the coordinate frame is rotating or stationary. In the present paper the d-q-0 axes are assumed to be stationary (stationary reference frame) so that the d-q-0 components of voltages, currents and fluxes of the machine take on the same meaning as the α-β-0 components. With the assumption that all asymmetric or unbalanced connections occur with respect to phase a then the alignment of the stationary reference frame permits the d-axis equivalent circuit to remain unchanged from the balanced case of Fig. 1. Hence, in the succeeding analysis only the representation of the q and 0 sequence equivalent circuits of Fig. 1 need be modified and the d axis equivalent circuit of Fig. 1 is implied for each case.

The equations of transformation relating the d-q-0 stator voltages to the actual phase voltages are

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} \end{bmatrix} \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} \tag{11}$$

The independence of the d-axis voltage from the voltage across phase a can be noted. Also it can be noted that the zero sequence component differs by $\sqrt{2}$ from its usual value[5] which has been chosen to maintain consistency in the choice of scale factor in the d-q-0 components relative to the physical a-b-c components.

In the stationary reference frame the equations of the induction motor can be written in the form

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \\ v'_{qr} \\ v'_{dr} \\ v'_{0r} \end{bmatrix} = \begin{bmatrix} r_s+L_s p & 0 & 0 & L_m p & 0 & 0 \\ 0 & r_s+L_s p & 0 & 0 & L_m p & 0 \\ 0 & 0 & r_s+L_{\ell s} p & 0 & 0 & 0 \\ L_m p & -\omega_r L_m & 0 & r'_r+L'_r p & -\omega_r L'_r & 0 \\ \omega_r L_m & L_m p & 0 & \omega_r L'_r & r'_r+L'_r p & 0 \\ 0 & 0 & 0 & 0 & 0 & r'_r+L'_{\ell r} p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i'_{qr} \\ i'_{dr} \\ i'_{0r} \end{bmatrix} \tag{12}$$

where r_s and r'_r are the stator and rotor resistances, $L_{\ell s}$ and $L'_{\ell r}$ are the stator and rotor leakage inductances and L_m is the magnetizing inductance of the machine. Also, $p = d/dt$, ω_r is the rotor electrical speed and

$$L_s = L_{\ell s} + L_m \tag{13}$$

$$L'_r = L'_{\ell r} + L_m \tag{14}$$

All rotor quantities appearing in Eq. 12 have been referred to the stator by the stator/rotor turns ratio.

It is clear that the transient behavior of an induction machine can be obtained for any unbalanced condition provided the phase voltages across the windings of the machine are known since the corresponding d-q-0 voltage inputs to Eq. 12 are known by virtue of Eq. 11. For example in the case of the balanced connection of Fig. 1, the stator phase voltages can be expressed in terms of the source voltages by

$$v_{as} = \frac{2}{3} e_{ag} - \frac{1}{3} e_{bg} - \frac{1}{3} e_{cg} \tag{15}$$

$$v_{bs} = -\frac{1}{3} e_{ag} + \frac{2}{3} e_{bg} - \frac{1}{3} e_{cg} \tag{16}$$

$$v_{cs} = -\frac{1}{3} e_{ag} - \frac{1}{3} e_{bg} + \frac{2}{3} e_{cg} \tag{17}$$

where e_{ag}, e_{bg}, e_{cg} incorporate the internal voltage drop of the supply. In general, each type of asymmetry requires the derivation of a set of voltage equations (constraint equations). After Eqs. 11-12 have been used to find the d-q-0 currents, the phase currents of the machine can be solved by the inverse transformation matrix

$$\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} \tag{18}$$

UNBALANCED OPERATION

In this section four types of asymmetrical connections will be considered which lead to unbalanced operation. In most cases it is relatively easy to deduce the constraint equations for each case and the simulation can be implemented by following the procedure outlined above. Since the zero sequence component is present in all of these cases it is useful to have a qualitative picture of how this component affects performance. The equivalent circuits in d-q-0 variables can be useful for this purpose and has been deduced for each case.

Phase to neutral short circuit

Although a line to ground fault is routinely calculated by the method of symmetrical components, the possibility of a short circuit to the neutral point of the machine is a considerably more challenging analysis problem. A diagrammatic representation of this fault condition is shown in Fig. 2.

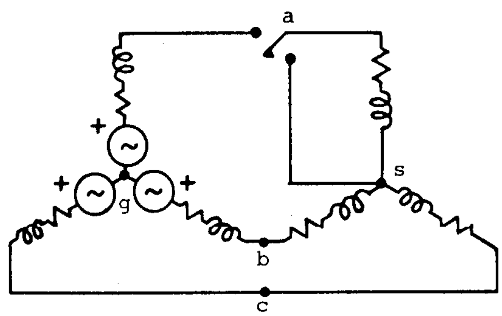


Fig. 2 Phase to neutral short circuit.

The constraint equations for this case are

$$v_{as} = 0 \tag{19}$$

$$v_{bs} = \frac{1}{2}(e_{bg} - e_{cg}) + \frac{1}{2}(r_s i_{as} + L_{\ell s}\frac{di_{as}}{dt}) \tag{20}$$

$$v_{cs} = \frac{1}{2}(e_{cg} - e_{bg}) + \frac{1}{2}(r_s i_{as} + L_{\ell s}\frac{di_{as}}{dt}) \tag{21}$$

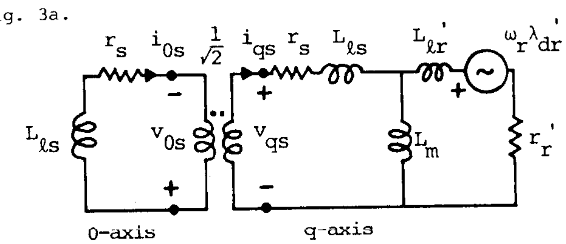
The corresponding expressions for the d-q-0 voltages are

$$v_{qs} = -\frac{r_s}{2} i_{qs} - \frac{L_{\ell s}}{2}\frac{di_{qs}}{dt} \tag{22}$$

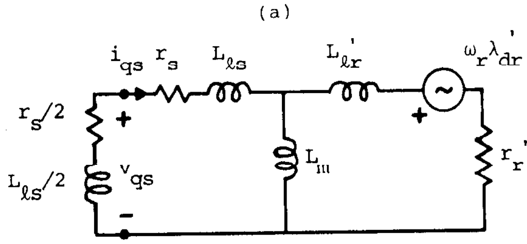
$$v_{ds} = -\frac{1}{\sqrt{3}}(e_{bg} - e_{cg}) \tag{23}$$

$$v_{0s} = -\sqrt{2}\, v_{qs} \tag{24}$$

Equation 24 suggests that the q and the 0 components are mutually coupled. The effect of this coupling can be introduced by the use of an ideal transformer as shown in Fig. 3a.



(a)



(b)

Fig. 3 (a) Equivalent Circuit, (b) Reduced Circuit.

It is apparent that the resulting circuit can be further reduced by completely eliminating the zero axis variables by referring them to the q axis side of the ideal transformer. The resulting equivalent circuit is shown in Fig. 3b. It can be noted that although the d axis circuit remains unchanged, the q axis appears as if it were short circuited with additional resistance and inductance added to the stator circuit.

Line to neutral fault

This asymmetric connection is shown in Fig. 4. It can be noted that when phase a is disconnected then no stator current flows in this phase. The constraint equations are simply

$$v_{as} = \frac{d\lambda_{as}}{dt} \tag{25}$$

$$v_{bs} = e_{bg} - e_{ag} \tag{26}$$

$$v_{cs} = e_{cg} - e_{ag} \tag{27}$$

and

$$i_{as} = 0 \tag{28}$$

Equations 25-28 together with Eqs. 11 and 12 serve as the defining set of system equations for this connection and can be used to devise an appropriate equivalent circuit.

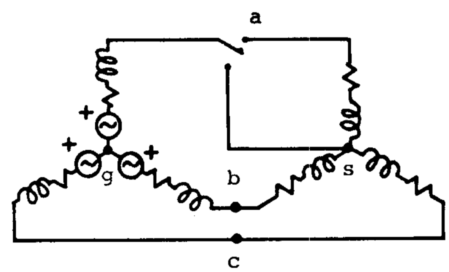


Fig. 4 Line to neutral short circuit.

The corresponding expressions for the d-q-0 voltages are

$$v_{qs} = \frac{2}{3} e_{ag} - \frac{1}{3} e_{bg} - \frac{1}{3} e_{cg} + \frac{2}{3}\frac{d\lambda_{mq}}{dt} \tag{29}$$

$$v_{ds} = -\frac{1}{\sqrt{3}}(e_{bg} - e_{cg}) \tag{30}$$

$$v_{0s} = \frac{-2\sqrt{2}}{3} e_{ag} + \frac{\sqrt{2}}{3} e_{bg} + \frac{\sqrt{2}}{3} e_{cg} + \frac{\sqrt{2}}{3}\frac{d\lambda_{mq}}{dt} \tag{31}$$

where

$$\lambda_{mq} = L_m (i_{qs} + i'_{qr}) \tag{32}$$

Equation 28 suggests a constraint relationship between the q and 0 axis currents of the form

$$i_{qs} = -\frac{1}{\sqrt{2}} i_{0s} \tag{33}$$

Again the zero sequence axis equations can be absorbed into the q axis circuit so that the connection can be effectively represented by the conventional d axis circuit together with the q axis circuit shown in Fig. 5. It can be noted that the d and q axis equivalent circuits again become unsymmetric so that a representation in a rotating reference frame would imply time dependent parameters.

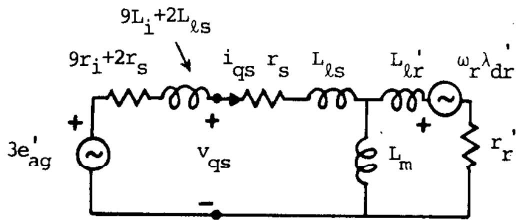


Fig. 5 q-axis equivalent circuit of line to neutral fault incorporating effects of zero sequence component.

Backward phase connection

One unusual connection which can arise when both terminals of each phase winding are made externally available is an inadvertant backward connection of one phase as shown in Fig. 6. The equations which now relate the phase voltages of the motor to the source voltages are:

$$v'_{as} = -\frac{2}{3}e_{ag} + \frac{1}{3}e_{bg} + \frac{1}{3}e_{cg} + \frac{2}{3}\frac{d\lambda_{mq}}{dt} \quad (34)$$

$$v_{bs} = -\frac{1}{3}e_{ag} + \frac{2}{3}e_{bg} - \frac{1}{3}e_{cg} - \frac{2}{3}\frac{d\lambda_{mq}}{dt} \quad (35)$$

$$v_{cs} = -\frac{1}{3}e_{ag} - \frac{1}{3}e_{bg} + \frac{2}{3}e_{cg} - \frac{2}{3}\frac{d\lambda_{mq}}{dt} \quad (36)$$

The d-q-0 voltages are expressed by

$$v_{qs} = -\frac{2}{9}e_{ag} + \frac{1}{9}e_{bg} + \frac{1}{9}e_{cg} + \frac{8}{9}\frac{d\lambda_{mq}}{dt} \quad (37)$$

$$v_{ds} = -\frac{1}{\sqrt{3}}(e_{bg} - e_{cg}) \quad (38)$$

$$v_{0s} = -\frac{4\sqrt{2}}{9}e_{ag} + \frac{2\sqrt{2}}{9}(e_{bg}+e_{cg}) - \frac{2\sqrt{2}}{9}\frac{d\lambda_{mq}}{dt} \quad (39)$$

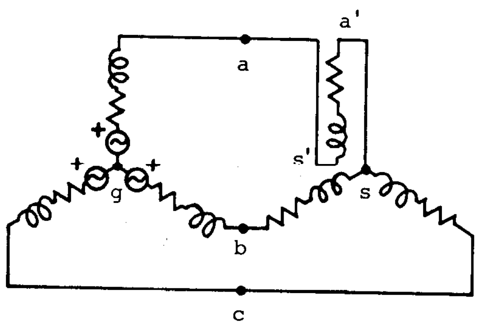


Fig. 6 Backward connected phase.

Examination of Eqs. 37 and 39 indicate that again the q and 0 axes components are interrelated. The constraint equation is

$$i_{0s} = 2\sqrt{2}\, i_{qs} \quad (40)$$

The zero axis component can again be eliminated and the q axis circuit modified appropriately to form the equivalent circuit of Fig. 7. The d axis circuit is again that of Fig. 1.

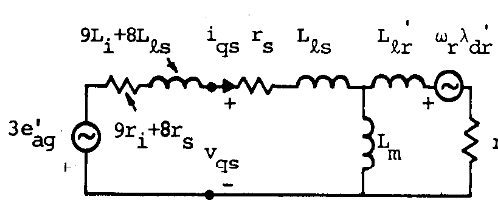


Fig. 7 q-axis equivalent circuit of backward phase connection including effects of zero axis components.

Partially open circuited ground connection

Figure 8 illustrates the case where a three phase ground connection has opened resulting in only phase a being solidly grounded. The defining equations assume the form

$$v'_{as} = e_{ag} \quad (41)$$

$$v_{bs} = \frac{1}{2}(e_{bg} - e_{ag} - e_{cg}) + \frac{1}{2}(r_s i_{as} + L_{\ell s}\frac{di_{as}}{dt}) \quad (42)$$

$$v_{cs} = \frac{1}{2}(e_{cg} - e_{ag} - e_{bg}) + \frac{1}{2}(r_s i_{as} + L_{\ell s}\frac{di_{as}}{dt}) \quad (43)$$

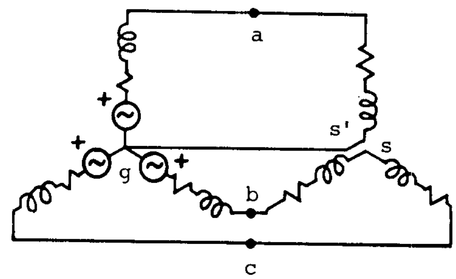


Fig. 8 Partially open circuited ground connection.

It can be noted from Fig. 8 that the constraint on current is

$$i_{bs} + i_{cs} = 0 \quad (44)$$

After some effort it can be shown that the voltages in the d-q-0 system are

$$v_{qs} = e_{ag} - \frac{1}{2}(r_s i_{qs} + L_{\ell s}\frac{di_{qs}}{dt}) \quad (45)$$

$$v_{ds} = -\frac{1}{\sqrt{3}}(e_{bg} - e_{cg}) \quad (46)$$

$$v_{0s} = r_s i_{0s} + L_{\ell s}\frac{di_{0s}}{dt} \quad (47)$$

The current constraint, Eq. 44 , becomes

$$i_{0s} = \frac{1}{\sqrt{2}} i_{qs} \quad (48)$$

Equations 45-48 lead to the equivalent circuit of Fig. 9. Again the need for a zero axis equivalent circuit has been eliminated by incorporating these terms in the q axis equivalent circuit. The d axis circuit again remains unchanged from Fig. 1.

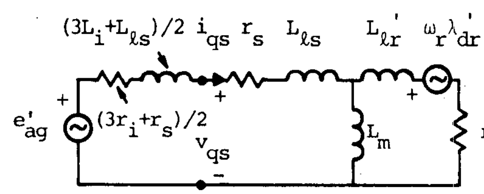


Fig. 9 q-axis equivalent circuit corresponding to partially open circuited ground connection.

OTHER CONNECTIONS

Thus far, four abnormal connections have been presented in order to illustrate the flexibility of the d-q-0 method for analyzing steady state and transient unbalanced operation. It clear that many if not all of what can be considered as typical unbalances can also be studied by this method. A number of these cases appear in the book by Clarke[5]. For reference purposes several of these connections are summarized in Fig. 10. Each connection is characterized by the property that the zero axis components are absent.

The equations which relate the d-q-0 voltages of the machine are summarized below.

Line to line fault, machine phase open circuited

$$v_{qs} = \frac{d\lambda_{mq}}{dt} \quad (49)$$

$$v_{ds} = \frac{1}{\sqrt{3}} (e_{cg} - e_{bg}) \quad (50)$$

$$v_{0s} = 0 \quad (51)$$

Line to line fault, supply phase open circuited

$$v_{qs} = \frac{1}{3} (e_{bg} - e_{cg}) \quad (52)$$

$$v_{ds} = \frac{1}{\sqrt{3}} (e_{cg} - e_{bg}) \quad (53)$$

$$v_{0s} = 0 \quad (54)$$

Line to ground fault

$$v_{qs} = -\frac{1}{3} (e_{bg} + e_{cg}) \quad (55)$$

$$v_{ds} = \frac{1}{\sqrt{3}} (e_{cg} - e_{bg}) \quad (56)$$

$$v_{0s} = 0 \quad (57)$$

From the above equations, the q axis circuits can be deduced for each connection. In each case the d axis circuit is the same as for the symmetric connection. Since the zero sequence circuit is not excited, it is not needed in the analysis of these connections.

Delta connection

Although induction machines are often connected in delta, this connection is rarely studied specifically in the literature. Most often, the windings are replaced by "equivalent" star connected windings and the behavior of the resulting machine is said to be "equivalent" to the delta connected machine. It is clear that such equivalence can be valid only for the symmetric connection that is Fig. 1, and is not valid even in this case unless the zero axis component which circulates in the delta can be neglected. Although apparently not mentioned specifically in the literature, d-q-0 analysis can be applied with equal facility to delta connected machines without recourse to an equivalent star connection. Moreover, both balanced and asymmetric delta connections can be studied by the method presented in this paper.

In particular, Fig. 11 shows two of many possible delta connections of the stator windings, whose d-q-0 voltages are:

Conventional delta connection

$$v_{qs} = e_{ag} - e_{bg} \quad (58)$$

$$v_{ds} = -\frac{1}{\sqrt{3}} (e_{ag} + e_{bg} - 2e_{cg}) \quad (59)$$

$$v_{0s} = 0 \quad (60)$$

Backward delta phase connection

$$v_{qs} = -\frac{1}{3} (e_{ag} - e_{bg}) \quad (61)$$

$$v_{ds} = -\frac{1}{\sqrt{3}} (e_{ag} + e_{bg} - 2e_{cg}) \quad (62)$$

$$v_{0s} = -2e_{ag} + 2e_{bg} \quad (63)$$

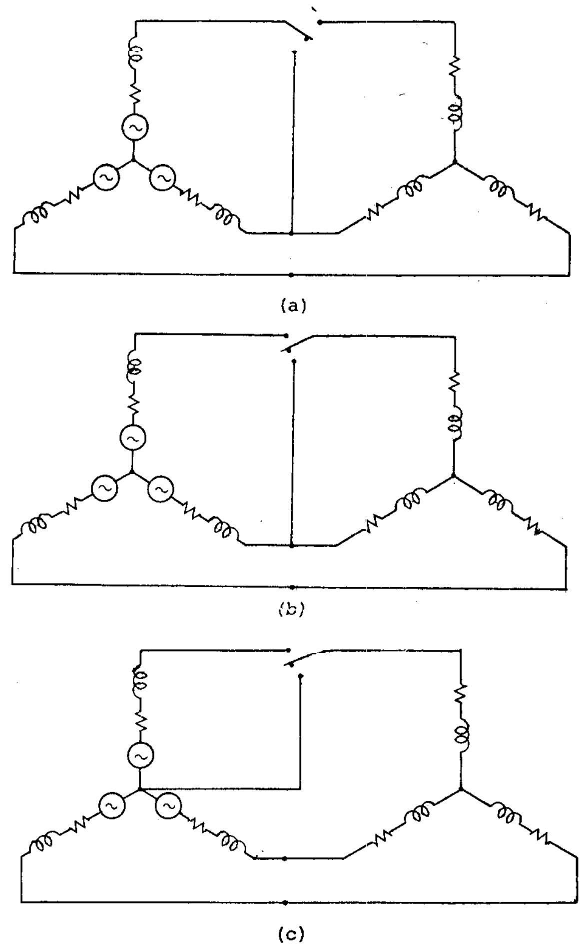


Fig. 10 Unbalanced operation with symmetric connection. (a) line to line fault, machine phase open circuited, (b) line to line fault, supply phase open circuited, (c) line to ground fault.

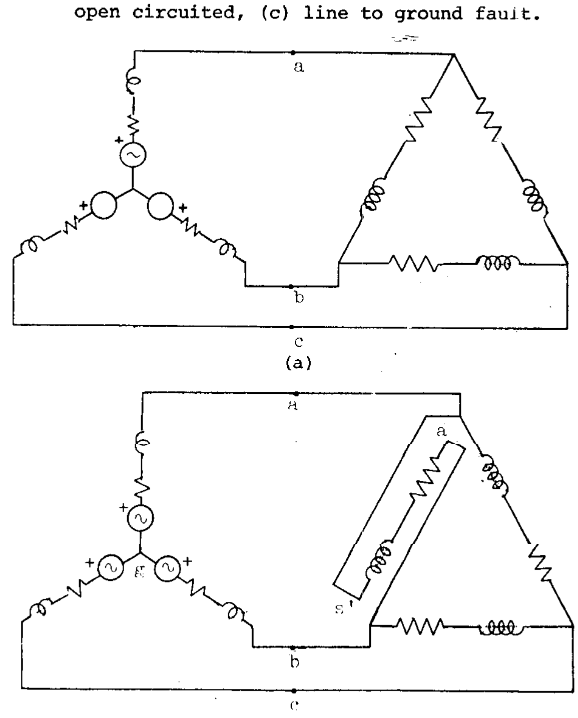


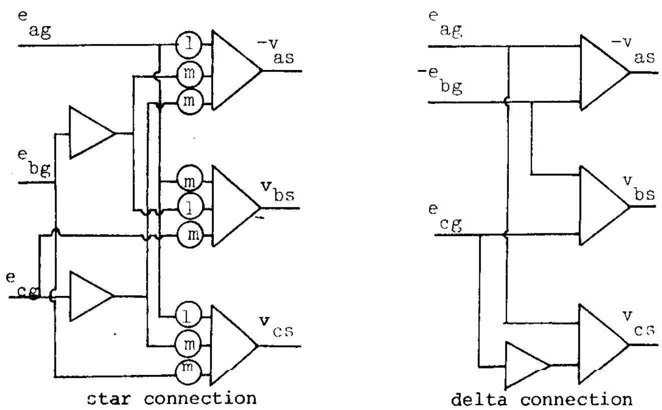
Fig. 11 Delta connections, (a) conventional connection, (b) backward phase connection.

COMPUTER SIMULATION

A computer simulation of an induction machine with any of the asymmetrical connections described in this paper can be implemented from Eqs. 11, 12, 18 and the constraint equations which must be identified for each case. Equations 11, 12 and 18 are simulated by the analog computer diagram shown in Fig. 12 where Eq. 11 is used to generate the d-q-0 voltages applied to the motor model and Eq. 12 is implemented to obtain the resulting phase currents. It can be noted that the simulation of the q axis components is more extensive than that for the d axis in order to develop the derivatives of the magnetizing flux and stator current which appear in the constraint equations for several of the asymmetric connections.

A final step in the simulation is to develop suitable expressions for the terminal voltages which appear as inputs in the upper left hand portion of Fig. 12. This can be accomplished by lumping the source impedance with the motor stator impedance or, preferably, by inserting a fictitious resistor R of large ohmic value so as to provide a means for writing three additional differential equations which can be used to solve for the terminal voltages. The placement of this resistor in phase a is shown in Fig. 13. From this figure it is possible to write the following equations:

$$i_{ag} = \frac{1}{L_i}\int (e'_{ag} - e_{ag} - r_i i_{ag})\, dt \qquad (64)$$



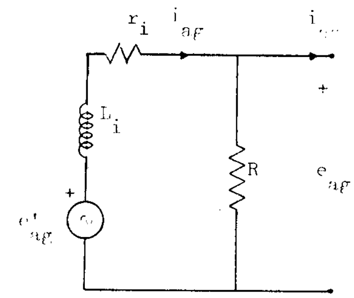


Fig. 13 Approximate model to simulate terminal conditions.

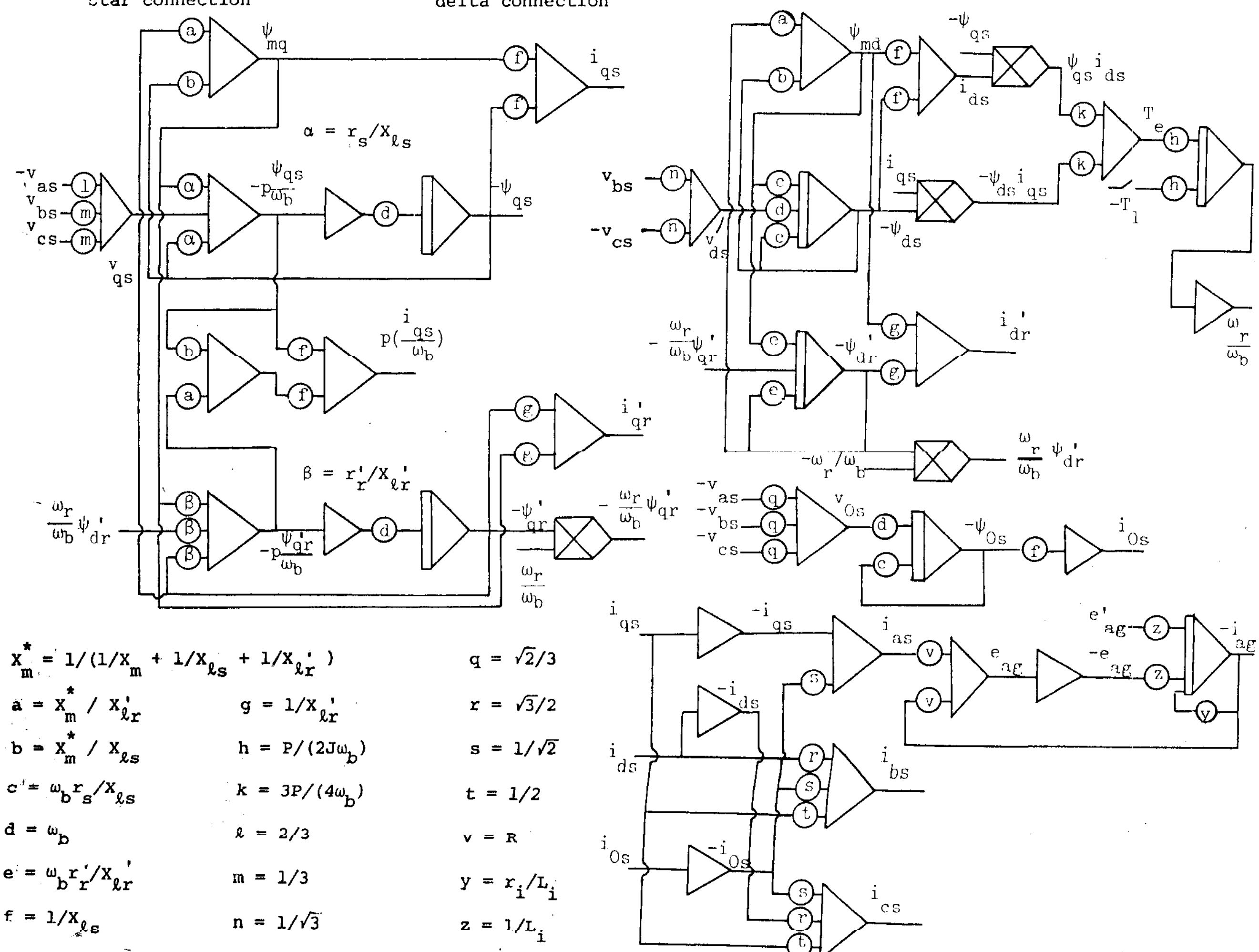


Fig. 12 Computer simulation of induction machine suitable

$$c_{ag} = R(i_{ag} - i_{as}) \qquad (65)$$

The calculation of the terminal voltages on the remaining phases follow the same approach. The simulation of the phase a terminal voltage appears in the lower right hand side of Fig. 12. Figure 12 shows a complete simulation for the balanced case, Fig. 1 and Fig. 11a. Although too space consuming to document here, the modifications to Fig. 12 needed to accomodate each of the asymmetric connections is straightforward.

Figures 14 and 15 show start up transients for nominal star and delta connected machines having the following parameters: stator and rotor resistances r_s = 0.0788 Ω, r_r = 0.0408 Ω, magnetizing reactance X_m=9.33Ω (unsaturated), stator and rotor reactances $X_{\ell s}$= 0.2122 $X_{\ell r}$ =0.4632 Ω, bus internal reactance and resistance X_i = 0.1061 Ω, r_i = 0.0 Ω, rotor inertia J = 0.31 kg·m^2. These values have been multiplied by three in order to obtain the delta connected machine parameters. The machine is rated at 230 volts, 60 Hz, 25 HP. It can be noted that basic electromechanical response of the two motors is identical as expected with the $\sqrt{3}$ factor clearly evident in the voltage and current waveforms.

Figures 16-19 show key results of a computer study devoted to four of the asymmetrical connections described previously. In particular, Fig. 16 shows the machine voltages, currents and torque for the line to neutral fault of Fig. 4. The double frequency oscillations in the torque trace due to the unbalanced condition can be clearly seen.

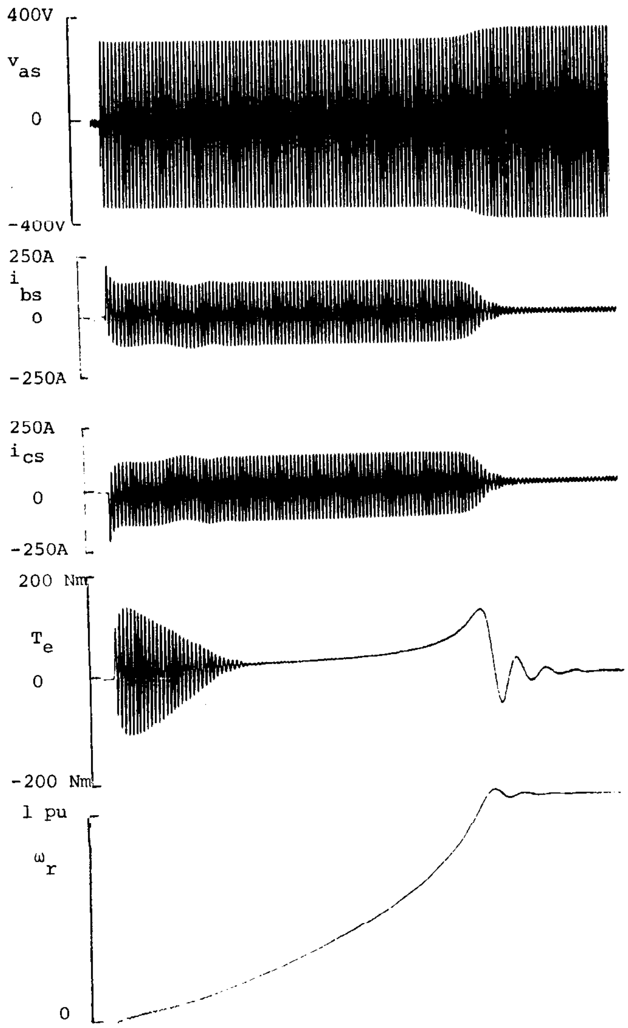


Fig. 14 Delta connection starting,balanced case.

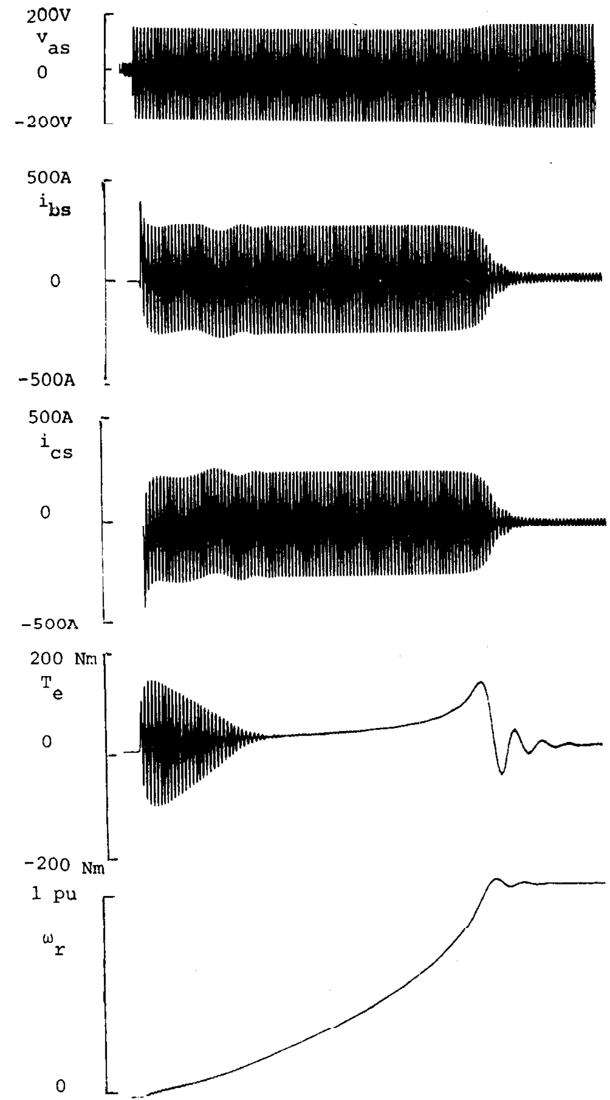


Fig. 15 Starting transient for star connected machine with balanced conditions.

Figure 17 concerns the case of a sudden short from phase to neutral, Fig. 2, when the motor is operating at rated full load. Note that since the torque after the fault is not sufficient to support the load so that the motor quickly brakes to a halt. Also interesting is the current induced in phase a which decays to zero since the cemf in the winding disappears when the motor stops.

Figure 18 shows the voltage, current,torque and speed starting transients when the winding of phase a is connected backwards as in Fig. 6. In particular these traces show the latter portion of the starting transient when the motor speed reaches rated speed. In this case the machine is again unloaded. Note that with this connection the motor runs in the reverse direction with a large double frequency torque pulsation appearing when the motor reaches synchronism.

Figure 19 shows a starting transient for the case where ground connection is partially open circuited as in Fig. 8.

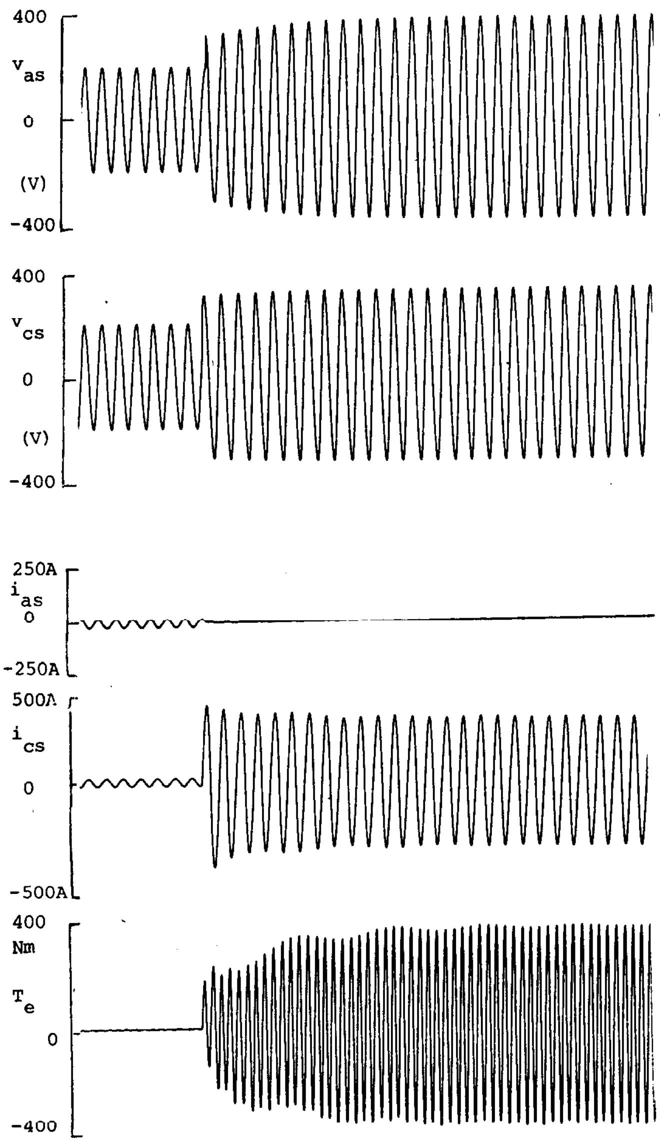


Fig. 16 Transients for sudden line to neutral fault

CONCLUSIONS

Asymmetrical connection of induction motors, either accidental or intentional, occurs sufficiently often in motor application to warrant an accurate calculation of transient as well as steady state behavior. In the past asymmetric connections have been investigated by the method of symmetrical components. However, this approach is lacking in that it can only deal with steady state phenomena. This paper has presented a new and novel method for the transient analysis of induction machines with asymmetrically connected stator phases. The method is based on the orthogonal axis theory of electrical machines. Four specific asymmetric connections have been studied which, to the authors' knowledge, have never been reported in the literature. In each case, the orthogonal axis approach has been shown to lead to a simple, straightforward equivalent circuit which is readily simulated. Although this study has been restricted to balanced sinusoidal supplies, it is clear that this approach can be readily extended to accomodate unbalanced or non sinusoidal supplies. Specifically, this procedure can be extended cases wherein the motor is used in a drive incorporating a solid state voltage inverter.

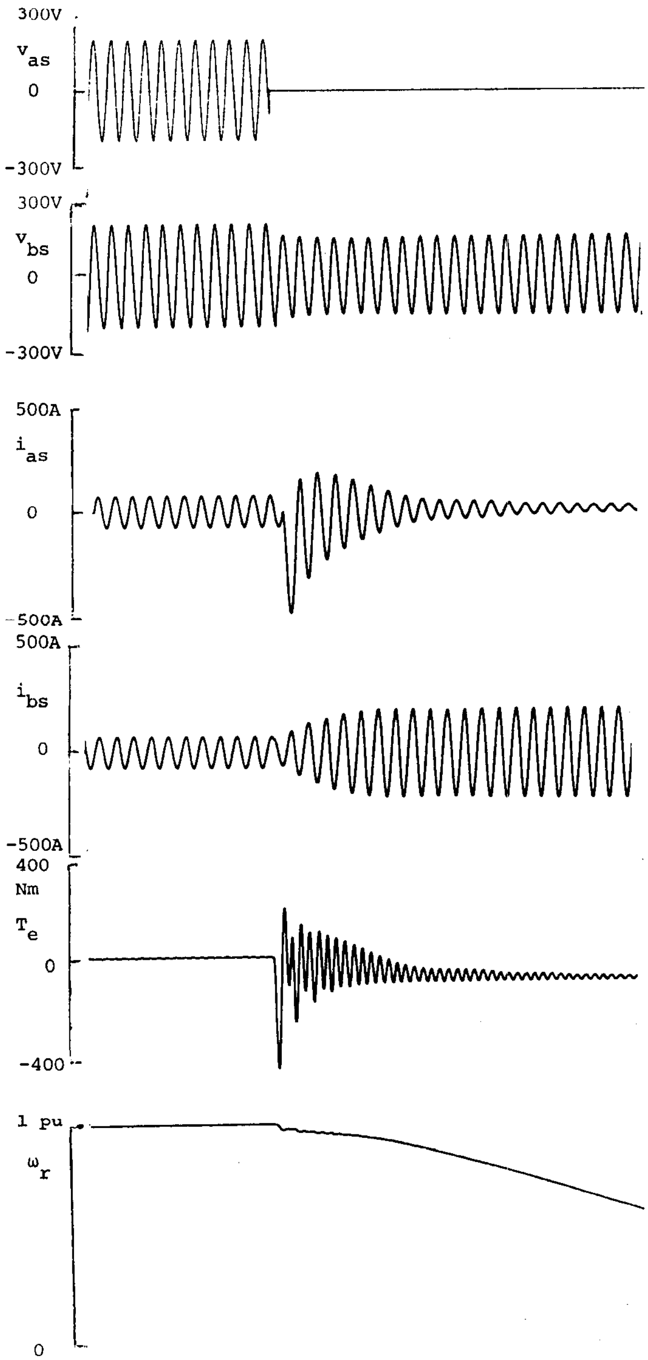


Fig. 17 Transients for phase to neutral fault.

ACKNOWLEDGEMENTS

The work reported in this paper was completed while the authors were associated with Purdue University, West Lafayette Indiana. This work has been partially supported by the Electrical Machines Group of the Italian National Research Council.

REFERENCES

1. M.G. Say,"Alternating Current Machines", Wiley, New York, 1978.
2. J.E. Brown and O.I. Butler,"A general method of analysis of three phase induction motors with asymmetrical primary connections", Proc. I.E.E., vol. 100, Part II, pp. 25-34, February 1953.

3. J.E. Brown and O.I. Butler,"The zero sequence parameters and performance of three phase induction motors", Proc. I.E.E., vol 101, Part IV, pp. 219-230, February 1954.

4. T.A. Lipo and A. Consoli,"Modelling and simulation of induction motors with saturable leakage reactances", I.E.E.E. Industry Applications Society Annual Meeting Conference Record, October 5-9, 1981.

5. E. Clarke,"Circuit Analysis of A-C Power Systems, Volume I, Symmetrical and Related Components", John Wiley & Sons, Inc., New York, 1943.

6. W.V. Lyon,"Transient Analysis of Alternating-Current Machinery" (book) J. Wiley & Sons, New York, 1954.

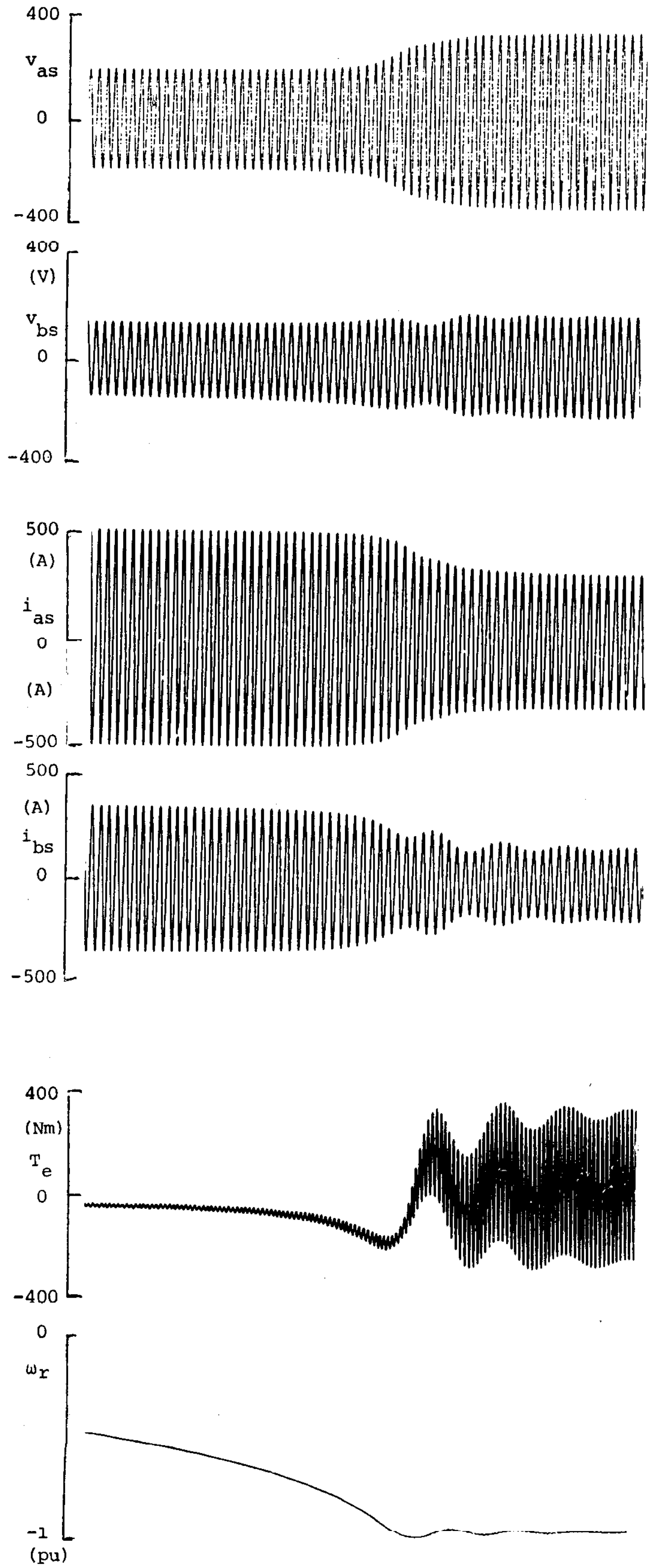


Fig.18 Portion of starting transient with backward connected phase.

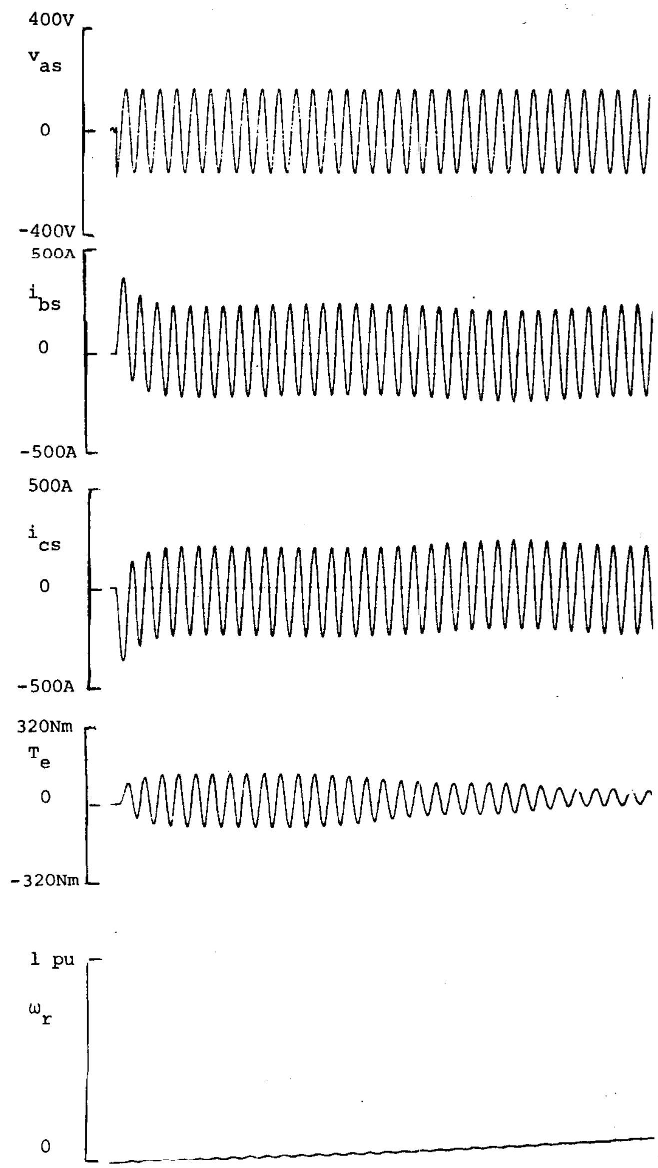


Fig. 19 Start up transient with partially opened ground connection.