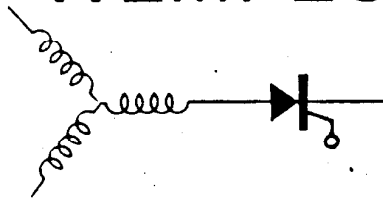




# WEMPEC



Wisconsin Electric Machines and Power Electronics Consortium

RESEARCH REPORT  
82-5

Design and Performance of a Converter  
Optimized AC Machine

T.A. Lipo and F.X. Wang  
University of Wisconsin-Madison  
Madison, Wisconsin

Department of Electrical and Computer Engineering  
University of Wisconsin-Madison  
Madison, Wisconsin 53706

June 1982

## DESIGN AND PERFORMANCE OF A CONVERTER OPTIMIZED AC MACHINE

T. A. Lipo and F. X. Wang  
University of Wisconsin-Madison  
Madison, Wisconsin

### Abstract

The design and performance of a unique concentrated winding machine specifically designed for operation with a static power converter is described. When operating in conjunction with a converter supply, the machine is theoretically capable of 15% more power output for the same active copper and iron than a conventionally designed synchronous machine of the same rating. Performance of the machine operating as a generator feeding a rectifier load is described and theoretical predictions are compared with test. The potential of such a machine operating as a motor is also described.

### Introduction

Since the advent of the thyristor, wide varieties of power conversion equipment have evolved which operate as adjustable frequency supplies for AC machines. This evolution, however, has had only a marginal effect on the design of the AC machine. In general, operation of AC machines with solid state, adjustable frequency power sources has merely influenced the selection of the design value of the conventional parameters of the machine, for example, rotor resistance and leakage reactance in the case of an induction motor or q-axis rotor resistance and magnetizing reactance in the case of the reluctance motor.

In the past, considerable attention has been directed to the development of solid state converters which seek to minimize the harmonic content of the output voltages. The object of these studies is to produce a rotating field within the machine having a minimum of time harmonics. Results relating to research concerning inverters with a low harmonic content have been extensively published<sup>1-4</sup>. On the other hand, designs of synchronous and asynchronous machines for operation with static power converters continue to involve an approximation of a sinusoidal winding distribution on the stator and rotor. This approach serves to minimize time and space harmonics. The air gap field becomes essentially sinusoidally distributed. As a result, only about one third of the active iron core is near saturation at any instant and the iron is effectively underutilized.

Instead of concentrating on the design of an inverter, it appears possible to also develop types of AC machines having a nonsinusoidal winding distribution in which flows a prescribed nonsinusoidal current. It is clear that the iron in an AC machine would be better utilized and the power density improved if the air gap flux density could have a more nearly rectangular distribution as generally encountered in a DC machine. The prescribed rectangular distribution could be achieved if the stator and rotor flux density were added together such that the resulting flux density is a nearly rectangular wave traveling in the air gap. It appears that such a structure may be able to produce higher torque with better iron utilization.

This paper describes the design, analysis and testing of a true "converter" machine. The machine consists of a stator structure having an asymmetrical

six phase connection in which a pair of three phase windings have been displaced 30 electrical degrees. Each of the stator windings are concentrated, that is, the stator phases are comprised of full pitch, concentrated windings having one slot per pole per phase. Each of the three phase windings has an independent neutral. The motor utilizes a salient pole field structure with a specially designed pole arc of 120 electrical degrees with an asymmetrical pole face taper to produce an air gap of varying length. Because of the concentrated nature of the armature winding this new machine may be designated as a concentrated winding machine (CWM machine).

### Historical Survey

Investigations concerning operation of solid-state force-commutated inverters with AC machines having uniformly distributed or concentrated windings have yet to appear in the literature. However, the possibility of specially designed machines applied to mercury-arc rectifier load-commutated systems was studied over 40 years ago. Perhaps the earliest reference relating to such machines involves the proposed design of a non-sinusoidal wave generator for a HVDC system in 1924.<sup>5</sup> In this paper a machine was described which was capable of generating a three phase set of triangle waves rather than sine waves. The author, Maxstadt, showed a diagram in his paper illustrating how the rectification of three triangle waves results in a DC output voltage having essentially no ripple. A three phase, 5 KW synchronous machine having both field and compensating windings was built and tested. The project apparently failed, however, because of the need for a step-up transformer on the AC side which tended to filter the triangle wave voltages producing a rounding of the waveshapes on the high side of the transformer. The disturbance to the desired harmonic content resulted in a ripple on the DC side of the HVDC converter of substantially the same order as that produced by the conventionally wound machine.

In 1938 Stohr wrote two classic papers devoted to the calculation of the output rating of commutatorless "inverter" motors having various connections.<sup>6-7</sup> One of the shortcomings of Stohr's work is that he was necessarily limited in scope to the then current mercury arc valve technology. As a result most of Stohr's motor configurations utilized the multi-anode single-cathode mercury pool tube then available. The conclusion of Stohr's work was that a maximum output is obtainable when the machine winding is built in a "closed" or mesh type winding with six tapped points arranged symmetrically. Stohr's conclusions, however, are limited from today's viewpoint since only one-way or bi-filar windings were considered for star connected windings and that bridge circuits were not taken into consideration.

In 1975 Leitgeb brought the work of Stohr up to date by considering other winding configurations made possible by thyristor technology, particularly the bridge configuration.<sup>8</sup> Leitgeb concludes that for phase numbers between one and three it is not the mesh connection proposed by Stohr but the star connected

circuit with bridge converters that gives the highest per unit capacity. For phase numbers greater than three the Stohr circuit was found to have a slight advantage. In both the case of Stohr and of Letigeb the presentations appear to be theoretical and correlation with experimental results is not in evidence.

One of the major weaknesses of Stohr's and of Letigeb's work is that they specifically assume highly effective damper windings. Hence, "space harmonics" above the fundamental are assumed to be cancelled out by an equal and opposite damper winding current. In effect, air gap flux was assumed sinusoidal so that the possible added benefits of an intentional rectangular flux distribution was apparently not recognized.

In 1971 Boenig reported work which specifically addresses the problem of optimizing the magnetic material in an AC motor<sup>9</sup>. His work to some extent duplicated that of Stohr and Letigeb. On the other hand, a complete system was built and tested. The motor which was designed employed tapped mesh windings in which both the stator and the rotor could be connected either as a three, six or nine phase machine. It was demonstrated that a nine phase ring type winding can produce considerably better performance in terms of efficiency and minimum harmonic content compared with a conventional three phase, six pulse operation. However, the utilization factor for the required 18 force commutated thyristors was poor since each device carried load current only 1/9 of a complete cycle resulting in a high cost design.

Another interesting paper is the work of C. St. J. Lamb in 1970.<sup>10</sup> In this paper Lamb designed a machine which employed a gramme ring type concentrated winding and, in this case, the stator was slotless. The pole structure was designed with a conventional field winding and also with a small auxiliary winding on the leading edge of the pole. The auxiliary winding was arranged so that a pulse of flux to enable commutation appeared in each stator winding whenever commutation was to occur. The slotless gramme ring configuration however, resulted in a highly inefficient design.

### Comparison of Winding Designs

Although sinusoidally wound machines are almost universally employed for use with adjustable frequency converter supplies as well as in applications with fixed frequency sinusoidal supplies, the machine designer is in no way restricted to such a design. Indeed, machines could be constructed to generate counter e.m.f.s of an arbitrary waveform. However, the compatibility of these machines is generally inconsistent with the properties of the connected power converter. Figure 1 shows a table of a number of hypothetical converter machines in which it is assumed that counter e.m.f.s and line currents appear as shown in any of then ( $n > 2$ ) phases.

	AIR GAP PHASE VOLTAGE	PHASE CURRENT	$E_m$	$I_m$	AIR GAP POWER
1			1.00	1.00	1.00
2			0.637	0.707	0.900
3			1.273	1.225	1.0396
4			0.764	0.802	0.953
5			0.955	0.866	1.103
6			1.00	0.866	0.955

Fig. 1 Comparison of Six Prototype Machines.

For simplicity, it is assumed that the voltage and current waveforms are in phase. The first row of Fig. 1 illustrates the per unit amplitude of the internal air gap voltage, phase current and resulting power for a conventional sinusoidally wound machine. This machine can be used as a reference for compare the performance of a variety of unconventional machines. In order to accurately compare the nonsinusoidal machines with the conventional machine it is assumed that the RMS current and the total air gap flux in each machine is held constant. Hence, if the resistance for each winding is assumed reasonably constant, the copper and iron losses associated with the stator are essentially constant. Since the total flux is the same, the stator core cross section as well as the stator copper cross sectional area remain the same for all machines so that the machines physically have the same frame size. In addition, all machines are assumed to have the same effective gap so that the rotor excitation losses for all machines can be assumed identical. In this case the machines can also be compared on the basis of equal stator temperature rise.

The second row of Fig. 1 indicates the performance of a hypothetical machine in which the internal counter e.m.f and phase current is an ideal square wave. In principle, this type of machine is capable of the highest power output for a given voltage and current since the product of  $v \cdot i$  is a maximum equal to exactly twice the power achievable with sine wave. Design of such a machine, however, would require an enlarged core cross section as well as a wider slot area due to the increased core flux and RMS current involved. However, if the core flux remains the same the voltage amplitude must drop to  $2/\pi$  or 0.637 per unit of the conventional machine. In addition the amplitude of the phase current must be reduced by  $1/\sqrt{2}$  or 0.707 if the RMS current is to remain the same. As a result, the power producing capability of such a machine is only  $2 \cdot 0.637 \cdot 0.707$  or 0.90 per unit of that of the sine wave wound machine.

The next row of Fig. 1 shows the predicted power rating for a machine with triangular voltage and current waveshape. This type of machine corresponds to the configuration proposed by Maxstadt<sup>5</sup> and studied, effectively, by Boenig<sup>9</sup> and Lamb<sup>10</sup>. In this case the peak voltage and current can be set to 1.27 and 1.225 of the peak voltage and current for the conventional machine respectively, without exceeding the permissible value of core flux or RMS current. Note that the power produced by such an ideal machine is slightly greater, 4%, than a machine of conventional design. A machine of this type could be constructed by employing windings uniformly distributed over one pole pitch. In practice, the uniform distribution permits a reduction in the resistance per coil compared to a conventionally wound machine so that the power rating predicted for this machine could be somewhat pessimistic. Nonetheless, it appears that an improved output for such a machine could be obtained. The primary difficulty associated with such a machine is the fact that the voltage or current waveform cannot be easily derived from a static power converter in which rectangular or stepwise constant voltages or currents are more readily obtained. Impressing either rectangular voltages<sup>9</sup> or rectangular currents<sup>10</sup> on such a machine results in a harmonic mismatch in which the losses increase and the torque producing capability of the harmonic components is essentially lost.

Another machine which is somewhat easier to design is the trapezoidal winding machine, shown in the fourth row of Fig. 1. Note that the peak air gap voltage and phase current must again be reduced in order to realize the same core flux and RMS current as the conventional machine. However, since the reductions are this time not as great as the square wave machine, the power capability of the machine in this case decreases only

by 5%. A machine of this type could be readily constructed by the use of windings having a full pitch and a uniform distribution of 60 electrical degrees. Again the primary disadvantage to this type of configuration is the need to realize trapezoidally shaped voltage and current. Such waveforms are not readily derived with switching type power converters but could be obtained in high horsepower applications by the use of cycloconverters. In this case, a substantial reduction in the device rating of the cycloconverter appears possible.

The stepwise constant nature of solid state switching converters suggests another alternative machine design, the so-called CWM or concentrated winding machine, which uses full pitch concentrated windings. This machine is shown in the fifth row of Fig. 1. It can be noted that in this case the voltages and currents are quasi-rectangular in nature which is more consistent with the voltages and currents that can be easily derived from a bridge type switching power converter. The permissible voltage which can be supported without exceeding the available core flux is  $3/\pi$  or 0.955 times that of the conventional machine. The peak current must be reduced to 0.866 of the peak sine wave current to hold losses constant. Nonetheless, the power which can be derived from this machine could be as large as 1.10 times the power output from a conventionally wound sinusoidal winding machine.

As an alternative to a new winding design it is useful to consider operation with a conventional machine from the same converter power source. For simplicity, a rectangular current source is assumed which is readily approximated by either a naturally commutated or force commutated six pulse bridge having a DC current link. Because of the inherent behavior of the sinusoidally wound machine, the air gap phase voltage remains sinusoidal while the current assumes the rectangular waveform demanded by the current source bridge. The current amplitude must again be reduced to 0.866 compared to the ideal sine wave case. The consequent reduction diminishes the power producing capability of this machine to 95.4% of the power resulting when sine waves are applied to the same machine. It is apparent that the net gain realized by replacing the sinusoidally distributed machine with a concentrated winding machine is the ratio of 1.10 to 0.954. Hence, a power gain of over 15% can be derived by utilizing a properly designed concentrated winding (CWM) machine in applications normally employing quasi-rectangular voltage or current supplies such as bridge type power converter.

### Principle of Operation

In order to illustrate how rectangular wave operation can be achieved it is useful to first consider an elementary two pole machine. All phases are assumed to be constructed from concentrated full pitch coils so that, of necessity, the machine has one slot per pole per phase. An idealized representation of the winding arrangement is shown in Fig. 2(a). Although the number of phases that can be chosen is, in principle, arbitrary the number of phases shown in Fig. 2(a) is the practical number of six. Note that with proper labeling of the phases, the six windings can be segregated into a pair of three groups in which each of the phases of the two groups are mutually displaced by  $120^\circ$ . The two groups can be considered as mutually displaced by  $30^\circ$ . This type of winding connection is sometimes referred to as an asymmetric six phase connection. The per unit pole arc chosen is 0.66, that is, the rotor pole is assumed to span 120 electrical degrees. An equivalent representation in which the peripheral angle around the gap,  $\phi$ , is portrayed in linear fashion is shown in Fig. 2(b).

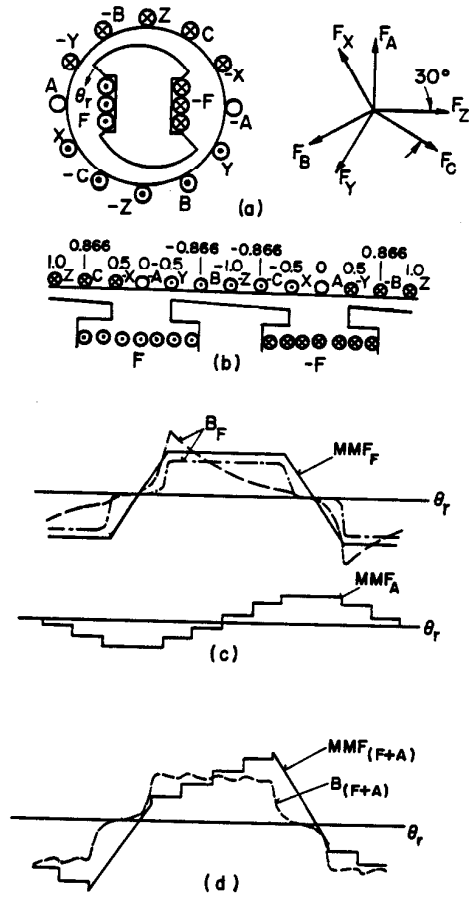


Fig. 2 Winding Configuration and Air Gap Waveforms for Concentrated Winding Machine. (a) Winding Layout for 2 Pole Equivalent Machine, (b) Rectangular Equivalent, (c) Flux Density and MMF of Field and Armature Current Components, (d) Net Resultant Flux.

If the field winding is assumed uniformly distributed over the interpolar space the resulting variation in field MMF around the periphery is shown by the trapezoidal solid line of Fig. 2(c). In general, if the field pole was designed in a conventional manner, that is, with a uniform gap under the pole arc, the corresponding flux density produced by this MMF would be rectangular in nature. This type of flux density pattern is illustrated by the dot-dashed lines in Fig. 2(c). In principle, such a waveform is precisely the variation desired if the air gap voltage is to be quasi-rectangular. Unfortunately, the air gap voltage is proportional to the field air gap flux only on open circuit. When the machine is subjected to armature load current the net flux becomes distorted by the resulting armature ampere turns. This difficulty may be overcome if the gap along the pole is properly tapered in anticipation of the armature ampere turns which will appear on load. If the pole is tapered for generating operation such that the gap increases when moving from front of the pole in the direction of rotation to rear of the pole, the air gap flux produced by the field becomes distorted as shown by the dashed line in Fig. 2(c).

Consistent with Fig. 1 it will again be assumed that the air gap voltage and phase current are in phase and that the currents supplied to the six phases of the machine are  $120^\circ$  quasi-rectangular blocks such as those

shown in the fourth row of Fig. 1. Generating operation is assumed. The current flow in the six windings are shown for the specific time instant in which the phase a current is zero and the line current is in the process of transferring from phase x to phase y. That is, the phase x current is assumed to be one half the nominal maximum current and is decreasing to zero while the phase y current is one half the nominal current and is increasing to its maximum. Current flow in the six windings a, b, c, x, y, z is therefore 0, 1, -1, 0.5, 0.5 and -1 respectively. The resulting MMF along the air gap produced by the six concentrated stator coils is shown in Fig. 2(d). It can be observed that MMF produced by the stator current is located spatially 90° behind the field MMF consistent with conventional theory. The resultant MMF is equal to the spacial superposition of the field and armature MMFs and assumes the asymmetric staircase function shown in Fig. 2(e). However, if the air gap is properly designed the effects of the tapered gap and asymmetric MMF cancel so that the resultant flux density in the gap becomes the quasi-rectangular shape shown in Fig. 2(d). Although some ripples remain due to the stator slotting and the concentrated coils, the air gap flux sensibly approximates the same shape as the input stator currents.

It can be recalled that all six windings are full pitch concentrated coils. Hence, if the shape of the air gap flux density is considered as fixed but rotating synchronously with the rotor, the voltage induced in each of the six concentrated winding stator windings will be directly proportional to the resulting flux density waveshape and also in phase with its respective current. Hence, the ideal waveshape of #5 of Fig.1 can therefore be approximated.

The close similarity of the concentrated winding machine to a conventional DC machine is, perhaps, apparent. In general, one pair of windings after the other undergo commutation in the q axis region or "neutral" zone in much the same manner as for a DC machine. The machine also encounters "armature reaction" effects similar to a DC machine. While DC machines use interpoles or pole face compensating windings to enhance commutation, the same effect is accomplished in this case by an asymmetrical air gap. Clearly the air gap taper must be carefully designed since the desired quasi-rectangular flux density waveshape can only be accomplished with a single ratio of field to armature MMF. In order that distortion to the quasi-rectangular counter e.m.f not occur under load changes, the field MMF must be changed in proportion to armature current. In effect, the concentrated winding machine functions as a type of series DC machine.

It is apparent that the conventional method of combating armature reaction in DC machines, namely pole face compensating windings, could also be employed in this machine. The detrimental effects of armature reaction could be cancelled out in much the same manner as for a DC machine. The use of interpole windings to enhance commutation also appears feasible. However, the added expense associated with either of these approaches appear to make them less attractive. Nonetheless, the feasibility of these methods could be the basis for further study.

#### Design and Test of Prototype Machine

In order to verify the predictions which have been set forth concerning the performance of a concentrated winding machine, a prototype four pole version was constructed and studied in some detail. As mentioned previously, a machine with any number of phases can be constructed. However, in practice the phase number is best chosen as a multiple of three in order to benefit from the inherent advantages of a 6 pulse converter bridge configuration. Although a simple three phase

connection is possible, the number of slots dictated by concentrated windings necessitates the use of only 12 slots for a four pole machine. However, this slot number appears to be too small for good thermal integrity and also results in windings having a high leakage reactance which degrades performance. The characteristics of the CWM machine improves as the number of phases increases. If the phase number is chosen as 12 the number of slots for a four pole machine becomes 48, which is identical to the number often employed in a conventional design. The use of six phases is probably the minimum number needed to obtain reasonably good behavior and therefore this number was selected for the study.

The final design for the machine incorporates four poles with six phases concentrated in 24 slots. The machine is configured in the dual wye connection illustrated in Fig. 2 with two independent neutrals. Each phase consists of two circuits which can be connected either in parallel or series. Each coil is a full pitch winding having 15 turns. The rotor poles, spanning 120 electrical degrees, are constructed with a spiral shape such that the physical air gap varies linearly from 0.032 to 0.162 inches from front to rear.

Fig. 3 shows the open circuit saturation curve for this machine measured at a speed of 800 RPM. The value of the phase voltage plotted as the ordinate is the RMS voltage read from a conventional RMS reading iron vane galvanometer. This curve is helpful in obtaining a measure of the saturation that was present during the many tests. In Fig. 4 is shown the waveform obtained across one of the phase windings when the machine is operating on open circuit. A triangular shaped wave similar to the predicted air gap flux density waveshape of Fig. 2 can be observed. In addition to the triangular shape, notches also appear which can be traced to the effects of the stator slot openings. Since these notches act to induce an undesirable ripple component of current they should be minimized in future designs. In the prototype machine the notches are somewhat larger than necessary since the machine was constructed with a rather large slot opening which could be substantially reduced. A slight skewing of either the stator or rotor lamination could also be used if the problem warrants.

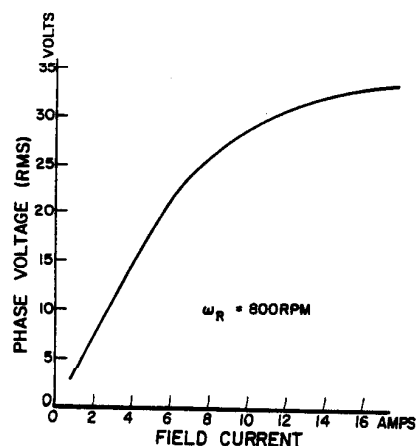


Fig. 3 Open Circuit Saturation Curve of Concentrated Winding Machine for Parallel Circuit Connection.

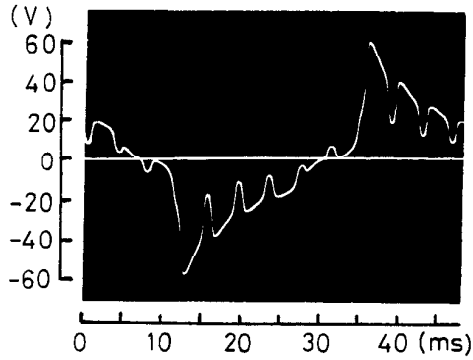


Fig. 4 Open Circuit Phase Voltage, Series Circuit Connection at 660 RPM with  $I_f = 4.5$  A. Scales: 20 V/div., 5 ms/div.

Figure 5 illustrates the circuit connection used to test the prototype machine under load. It can be observed that the machine is connected to diode bridges connected in series. The bridges are connected to supply a pure resistive load. The DC link inductor, normally employed to smooth the DC line current, was not used. Figure 6(a) shows a scope trace of the DC link voltage when the concentrated winding machine is operated as a generator supplying power to the series connected bridges. The load resistance  $R_d$  for these tests was  $0.525 \Omega$  and the machine was connected in the parallel circuit configuration.

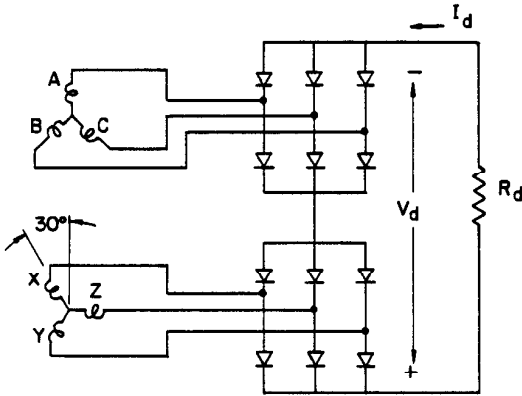


Fig. 5 Diode Bridge-Resistive Load Circuit.

In order to compare the behavior of the new CWM machine to more conventional technology a six phase synchronous machine of conventional design was implemented by connecting two three phase machines in tandem. Figure 6(a) shows a similar scope trace obtained under the same operating conditions. A comparison of these two traces clearly shows a marked reduction on DC ripple using the CWM machine. An order of magnitude improvement of about four times in amplitude and a twice infrequency is apparent. The harmonics remaining on the DC voltage derived from the CWM are primarily due to slot harmonics and could be probably reduced further by smaller slot openings. This result clearly demonstrates an important benefit of the concentrated winding construction since the filtering requirements needed to produce a smooth DC current are markedly reduced. The results obtained from this test indicate the need for a link inductor of only about 1/8 that required for generator of conventional design.

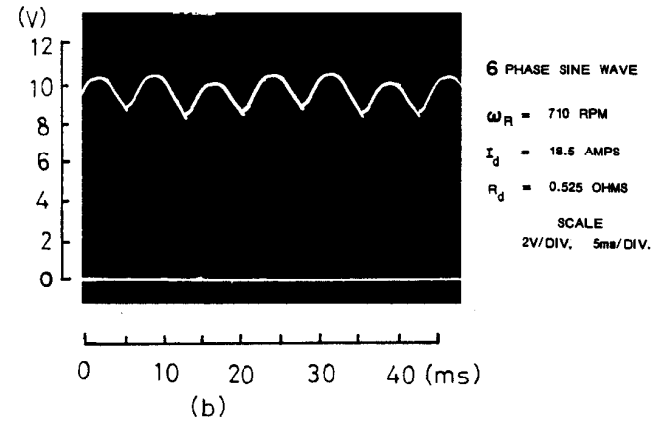
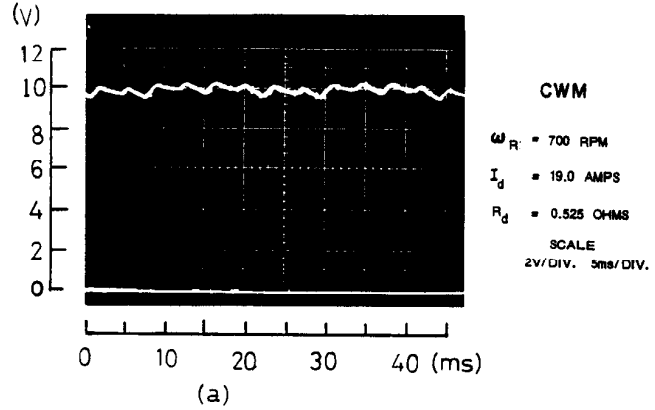


Fig. 6 Comparison of DC Output Voltage Between Conventional Six Phase Machine and Concentrated Winding Machine; Parallel Circuit Connection.

The series of oscillograms contained in Fig. 7 show the performance of the CWM machine for three widely spaced speeds for a fixed resistance load, again, no link inductor is employed in the circuit. The connection of the machine in this case and in subsequent traces in this section is for the series circuit configuration. Since the armature reaction increases with increasing speed, the field excitation must also be increased appropriately with speed to maintain the required series field operation. Note that the rectangular current waveshape can be maintained over the full range of speed. However, the influence of the slot ripple clearly becomes more prevalent at the lower speeds.

Figure 8 shows the results when the behavior of the machine under various types of loads is studied. The machine in this case was connected in the series circuit configuration. Note that the phase current maintains its characteristic quasi-rectangular shape over a wide range of load impedances while the phase voltages change somewhat but also maintains a reasonably rectangular shape. It can be observed that the slotting effect on phase voltage is considerably stronger in the case of the R-L load whereas the current for this case is nearly an ideal rectangle. Since the DC link current and AC current of the conducting phase are identical when commutation is not occurring, the flat topped rectangular AC current wave indicates that the DC load current is also free of ripple harmonics. The highest ripple current occurs with the R-C load which appears to be understandable. It is interesting to observe that with the same R-C load, the current tends to lead the voltage similar to the case of sine wave excitation.

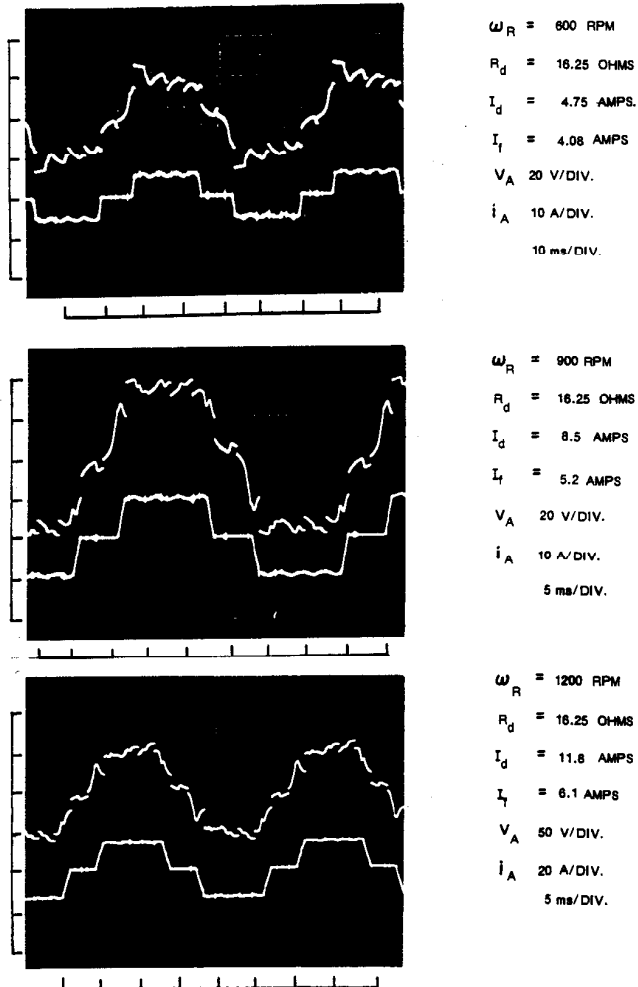


Fig. 7 Effect of Speed on CWM Voltage and Current Waveforms.

Thusfar the CWM prototype machine has not been tested due to the need for either a counter e.m.f. commutated or force commutated six phase inverter. This phase of the study, however, is in progress. Since the machine is not equipped with damper windings operation with a force commutated current source inverter (CSI) will probably lead to high spike voltages at the machine terminals and may not be feasible. However, operation from a load commutated system appears practical although the machine must operate at a slight leading power factor in order to have sufficient voltage available for commutation. Operation at a significantly leading power factor may require redesign of the rotor pole shape. It is clear amortisseur windings could also be added to the CWM to enhance commutation in the same manner as conventional machines. However, the current in the short circuited windings would possibly distort the rectangular counter e.m.f. waveform. The feasibility of amortisseur windings is not clear and warrants further study.

#### Correlation of Theory with Test Results

Because of the unique design of the concentrated winding machine, an analytical treatment predicting behavior of this machine is a formidable task. In con-

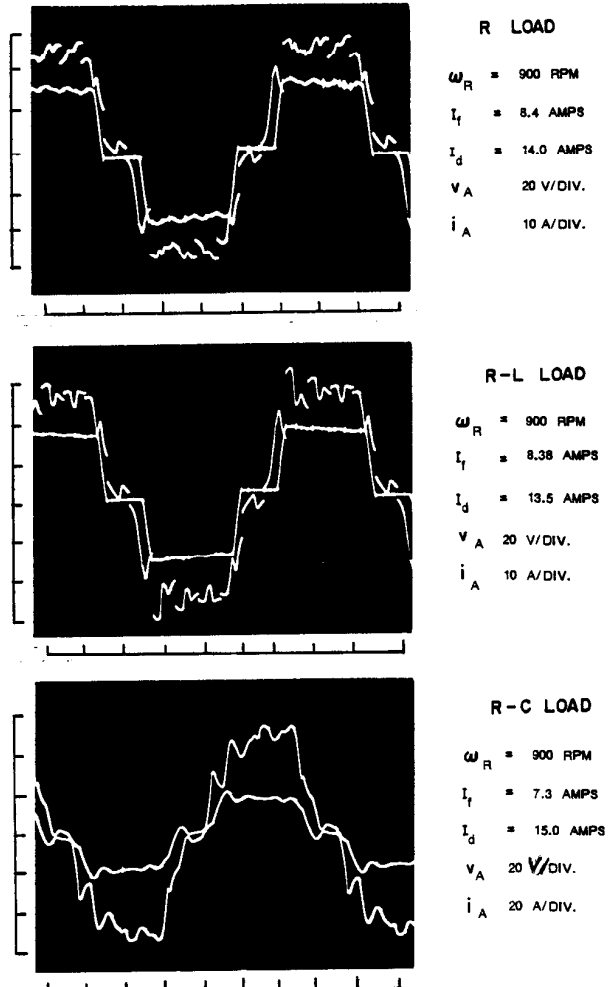


Fig. 8 Effect of Load Impedance on Voltage and Current Waveforms of CWM.

ventional analysis of AC machines, the coupled three phase circuit equations are typically referred to orthogonal stationary or rotating axes in which the equivalent circuits of transformed variables are not coupled inductively but only by so-called speed voltages. In this manner the system equations become remarkably simplified. Unfortunately, a key assumption in the transformation from three phase to orthogonal axes variables is that the windings are sinusoidally distributed. The assumption of sinusoidally distributed windings implies that the corresponding MMFs and consequent air gap fluxes are also distributed sinusoidally around the airgap periphery. While a valid approximation for conventional machines, replacement of concentrated windings (square wave windings) with equivalent sinusoidal windings is clearly not valid if a reasonable approximation of the voltage and current waveshapes are to be obtained.

Since the CWM machine is a six phase rather than a three phase machine and it must operate in conjunction with a solid-state converter supply, the analytical problem becomes even more complex. In general, the problem to be solved concerns the solution of a differential equation containing six stator and one rotor variable in which inductive coupling exists between all windings. The equations to be solved are, in matrix form:

$$v = R i + X \frac{p}{\omega_b} i + \frac{1}{\omega_b} \frac{d\theta_r}{dt} \frac{dX}{d\theta_r} i \quad (1)$$

$$T_e = \frac{p}{2\omega_b} \left[ i^t \frac{dX}{d\theta_r} i + i^t X \frac{di}{d\theta_r} \right] \quad (2)$$

where the superscript "t" denotes the transpose and R and X are the 7 x 7 matrices

$$R = \begin{bmatrix} r_s & 0 & 0 & \cdot & 0 \\ 0 & r_s & 0 & \cdot & \cdot \\ 0 & 0 & r_s & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & r_s & 0 \\ 0 & \cdot & \cdot & 0 & r_f \end{bmatrix} \quad (3)$$

and

$$X = \begin{bmatrix} X_{aa} & X_{ab} & X_{ac} & X_{ax} & X_{ay} & X_{az} & X_{af} \\ X_{ab} & X_{bb} & X_{bc} & X_{bx} & X_{by} & X_{bz} & X_{bf} \\ X_{ac} & X_{bc} & X_{cc} & X_{cx} & X_{cy} & X_{cz} & X_{cf} \\ X_{ax} & X_{bx} & X_{cx} & X_{xx} & X_{xy} & X_{xz} & X_{xf} \\ X_{ay} & X_{by} & X_{cy} & X_{xy} & X_{yy} & X_{yz} & X_{yf} \\ X_{az} & X_{bz} & X_{cz} & X_{xz} & X_{yz} & X_{zz} & X_{zf} \\ X_{af} & X_{bf} & X_{cf} & X_{xf} & X_{yf} & X_{zf} & X_{ff} \end{bmatrix}$$

It is convenient to use the reactance matrix X for purposes of computation. The reactance matrix is defined as the product of the inductance matrix L times the base angular frequency  $\omega_b$ , a scalar. The vectors i and v are current vector of state variables and the input voltage vector respectively given by

$$i^t = [i_a, i_b, i_c, i_x, i_y, i_z, i_f] \quad (5)$$

and

$$v^t = [v_a, v_b, v_c, v_x, v_y, v_z, v_f] \quad (6)$$

Since all but one of the reactances in the matrix X are not constant but functions of the angular position of the rotor, the solution of these equations is formidable. However, by the reciprocity theorem the inductance matrix is symmetrical so that the number of independent inductance functions contained in the in-

ductance matrix numbers 28. Fortunately, sufficient symmetry exists so that only seven inductances (or reactances) need be derived in detail and the remainder obtained by appropriate phase shift in the argument involving the spacial position of the rotor. The seven reactance functions required are:  $X_{aa}$ ,  $X_{af}$ ,  $X_{ab}$ ,  $X_{ax}$ ,  $X_{ay}$ ,  $X_{az}$ , and  $X_{ff}$ . Since the third term in Eq. 1 involves the derivative with respect to angular displacement of the rotor, the quantities  $dX_{aa}/d\theta_r$ , ...,  $dX_{ff}/d\theta_r$  must also be derived.

The mutual inductance between any two windings can be measured by exciting one of the windings with sinusoidal voltage with all other windings open circuited. The ratio of the open circuit secondary voltage to the current in the excited primary winding corresponds to the inductive mutual reactance between the two phases in question for a particular angular position. The functional variation of the mutual inductance with rotor angle can be constructed by obtaining a series of such measurements at various rotor angles. The self inductances of the windings can also be derived in this manner.

The theoretical prediction of the same inductance variations is clearly a more formidable task and is not within the scope of this paper. However, reasonably accurate predictions can be made by the use of permeance functions. Figure 9 shows the correlation that has been obtained for the mutual inductance between phase a and the field winding f.

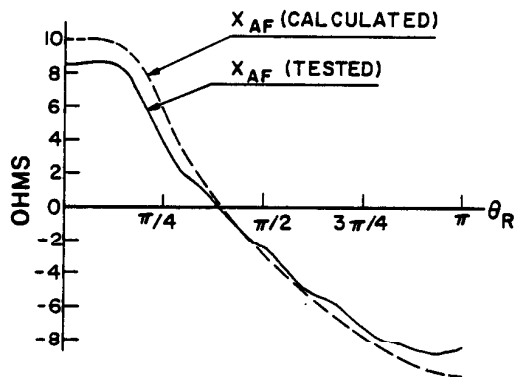


Fig. 9 Comparison of Measured and Calculated Values of Mutual Inductance  $X_{af}$  as a Function of the Rotor Electrical Displacement Angle  $\theta_r$ .

Although the solution of Eqs. 1 and 2 only requires that the resistance and reactance matrices R and X be known, the solution can be much more efficiently computed if the derivative of the reactance matrix,  $dX/d\theta_r$  in Eqs. 1 and 2 are also predetermined. Differentiation of the analytical prediction of the inductive reactance, the dashed line of Fig. 9 in the case of  $X_{af}$ , is readily carried out. Although the direct differentiation of the measured reactance function is possible, magnification of measurement errors by differentiating make the results not feasible for practical use. Examination of Eq. 1, however, suggests that any desired inductance derivative function, for example  $dX_{af}/d\theta_r$ , can be determined by exciting one of the windings with a DC current. The voltage measured at the open circuited terminal of the secondary winding in question while the rotor is rotated at constant speed divided by the primary current is a measure of the angular derivative of the mutual reactance between these two windings.



Shown in Fig. 10 is a comparison of the analytical result obtained by differentiating the calculated inductance function with the measured result obtained by test. Good correlation is evident except for the notches in the measured waveform resulting from the slot harmonics which were neglected in the analytical prediction. It is important to mention that the open circuit voltage is essentially produced by the term  $i_f \frac{dL_{af}}{d\theta_r}$ . Hence, if  $i_f$  is constant the open circuit voltage must be proportional to  $\frac{dL_{af}}{d\theta_r}$ . Comparison of Fig. 4 with Fig. 10 confirms this result.

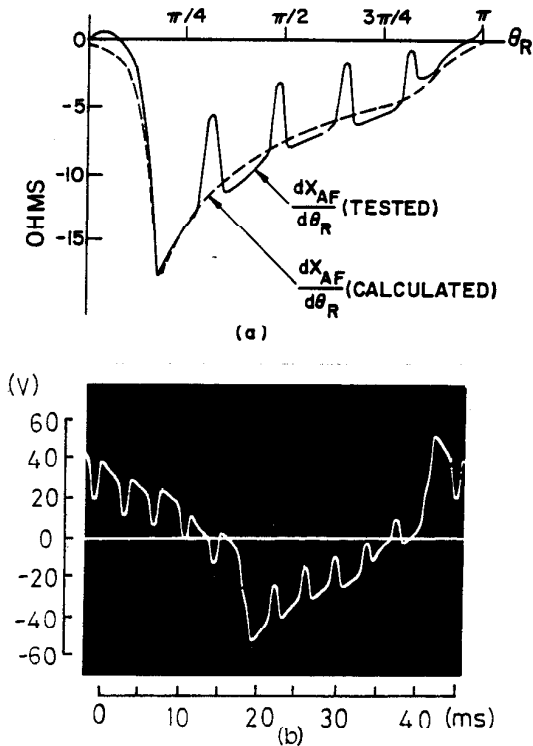


Fig. 10 (a) Comparison of Tested and Calculated Values of the Spatial Derivative  $\frac{dX_{AF}}{d\theta_R}$ , (b) Showing Open Circuit Voltage Induced at the Terminals of Phase a with Constant DC Field Current.

Similar correlation has been obtained for the remaining seven inductance functions and their derivatives. A detailed treatment of the computation of winding inductances and the solution of Eqs. 1 and 2 is beyond the scope of this paper. However, in general the voltage induced in any phase winding can be broken down into several components 1) the resistive  $iR$  drop, 2) transformer voltages arising from time varying stator currents, 3) the transformer voltage resulting from time variation in field current, 4) speed voltages which appear due to coupling between stator phases and 5) the speed voltage induced from the field circuit. Term 5 corresponds to the no-load open circuit voltage while term 3 is, in effect, the voltage contribution due to armature reaction.

Figure 11 shows the contributions of the five terms to the terminal voltage for a specific operating condition shown in Fig. 7b, that is,  $i_f = 8.4$  A,  $\omega_r = 900$  RPM and the DC load resistance  $R_d = 12.7 \Omega$ . In particular, the reactance functions measured from test were used. For simplicity, the current measured from test was taken as the input and the various voltage

drops computed from Eq. 1.

It can be noted that the stator speed voltage and field transformer voltage, terms 3 and 4, which are normally small in a conventionally designed DC machine are by no means negligible in this case. Also ripples occur in the voltage components which cannot be due to slotting since this effect has been neglected in Fig. 10. This behavior can be attributed to two factors, first the rather large number of ampere turns which must be commutated during each switching instant of the bridge and secondly the sizable commutating voltage required due to the leakage reactance involved during the commutation.

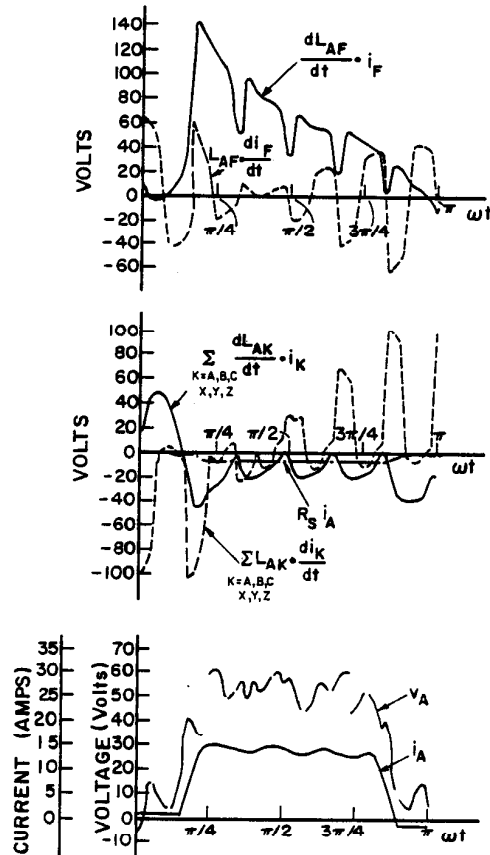


Fig. 11 Illustrating Components of the CWM Phase Voltage Under Load and the Corresponding Phase Current.

The computation of winding voltages using measured rather than calculated values of machine reactances can be carried out in a similar manner. Figure 12 gives a comparison of the resulting terminal phase voltages for the case of Fig. 7b. In particular, the waveform measured from test has been reproduced on Fig. 10 and the computed values of phase voltage using both the measured and the tested values of inductance superimposed. Note that the results obtained with measured values of inductance are reasonably accurate and sufficient to confidently predict behavior at other operating points. The voltages computed with the calculated reactances is not as accurate and additional work is perhaps in order to incorporate the effects of stator slotting.

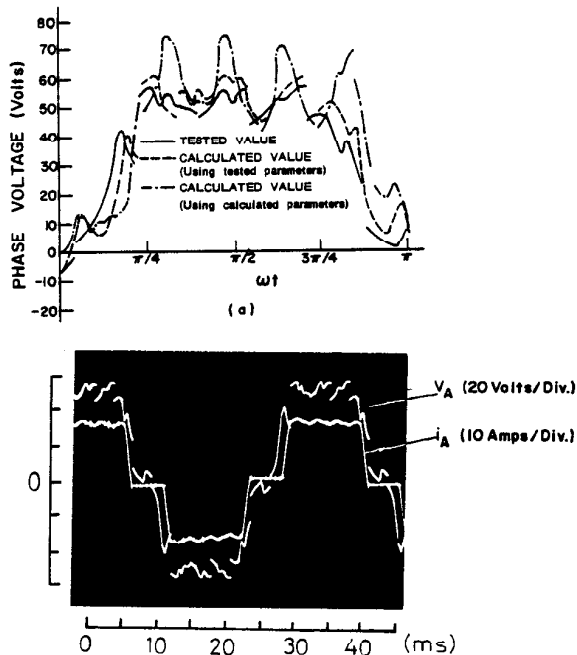


Fig. 12 (a) Comparison of Calculated Phase Voltage with Tested Results for Typical Operating Conditions, (b) Corresponding Scope Trace of Experimental Results.

A plot of the electromagnetic torque computed by Eq. 2 for the case of Fig. 7b is shown in Fig. 13. The computation uses the measured values of machine reactances. Because of the difficulty in instrumenting such a test a corresponding measurement of torque was not attempted. A harmonic torque pulsation of six times the operating frequency characteristic of machines which operate from a six pulse converter bridge can be observed. Ripple harmonics induced by the slots also appear which could become a problem at sufficiently low rotational speed. However, the magnitude of the torque harmonics appear to be qualitatively the same as with a machine of conventional design.

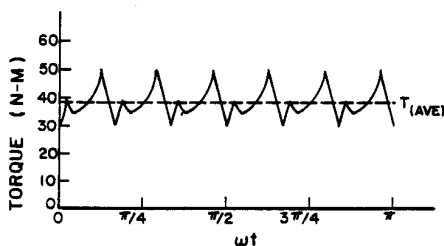


Fig. 13 Instantaneous Electromagnetic Torque Corresponding to Operating Conditions of Fig. 12.

### Conclusion

This paper has described the design, analysis and performance of a unique new concentrated winding machine specifically designed for operation from a converter supply. The machine is ideally capable of supplying 15% more power than a conventional machine of the same frame size and temperature rise. Although this ideal figure will probably not be reached in practice, some improvement in performance appears likely.

In addition, the new machine has been demonstrated to require a far smaller DC link inductor, a major cost item in many converter applications. Although wound field type machines have been discussed in this paper, permanent magnet configurations are clearly also possible. Such machines should find potential application as a generator in applications such as automobile alternators, shipboard power supplies and other isolated systems in which low cost or light weight DC supplies are of importance. The machine design also appears to have considerable potential as a motor in an adjustable frequency drive system. The feasibility of such an application is presently under investigation.

### Acknowledgements

Mr. F. X. Wang is a Visiting Scholar from the People's Republic of China. The authors wish to thank the General Electric Company for constructing the prototype machine and specifically to W. R. Oney and R. Keys who were responsible for fabricating the rotor and stator of the machine respectively. The authors are grateful for many useful discussions with GE personnel, particularly J. Franz who suggested a version of the trapezoidally wound machine. The authors also wish to acknowledge the industrial sponsors of the Wisconsin Electric Machine and Power Electronics Consortium (WEMPEC) program at Wisconsin which supported a portion of this work.

### References

1. F. G. Turnbull, "Selected Harmonic Reduction in Static dc-ac Inverters", *IEEE Trans. on Communication Electronics*, vol. 83, July 1964, pp. 374-78.
2. H. S. Patel and R. G. Hoft, "Generalized Techniques of Harmonic Elimination and Voltage Control in Thyristor Inverters. Part I, Harmonic Elimination", *IEEE Trans. on Ind. Appl.*, vol. IA-9, May/June 1974, pp. 310-317.
3. H. S. Patel and R. G. Hoft, "Generalized Techniques of Harmonic Elimination and Voltage Control in Thyristor Inverters. Part II, Voltage Control", *IEEE Trans. on Ind. Appl.*, Vol. IA-10, Sept./Oct. 1974, pp. 666-673.
4. G. S. Buja and G. B. Indri, "Optimal Pulsewidth Modulation for Feeding AC Motors", *IEEE Trans. on Ind. Appl.*, Vol. IA-13, Jan./Feb. 1977, pp. 38-44.
5. F. W. Maxstadt, "Obtaining Steady High-Voltage Direct Current from a Thermionic Rectifier Without a Filter", *Trans. AIEE*, Nov. 1924, pp. 1055-1057.
6. M. Stohr, "The Rated Output of Commutatorless Inverter Motors Having a Simple Six-Phase Connection", *Archiv. fur Elektrotechnik*, vol. 32, 1938, pp. 691-720 (In German).
7. M. Stohr, "The Rating of Brushless Converter Motors with Improved Motor Connections", *Archiv. fur Elektrotechnik*, vol. 32, 1938, pp. 767-784 (In German).
8. W. Leitgeb, "The Unit Capacity of Converter-Fed Synchronous Machines in Connection with Various Number of Phases and Types of Circuits", *Archiv. fur Elektrotechnik*, vol. 57, June 1975, pp. 71-84 (In German).
9. H. Boenig, "Optimization of Magnetic Materials Utilization in Semiconductor Commutated Electric Machines", Ph.D. Thesis, University of Wisconsin, 1971, 150 pp.
10. C. St. J. Lamb, "Analysis and Testing of a Direct-Voltage Induced e.m.f. Commutated Thyristor Motor", *Proc. IEEE*, vol. 117, No. 10, October 1970, pp. 1975-1985.