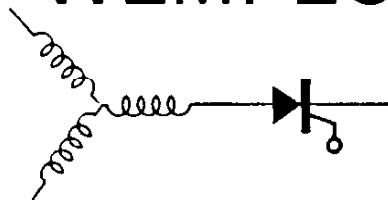




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RESEARCH REPORT  
84-5

A State Space Analysis of LCI Fed Synchronous  
Motor Drives in the Steady State

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June 1984

# A STATE SPACE ANALYSIS OF LCI FED SYNCHRONOUS MOTOR DRIVES IN THE STEADY STATE

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**Abstract** - The steady state performance of a load commutated inverter (LCI) fed synchronous motor drive is computed by means of a digital computer based state space solution of the synchronous machine equations. The computer solution permits accurate calculation of current, voltage, and torque waveforms under the assumptions of constant shaft speed and constant dc link current. Steady state operating points are characterized by the firing angle, commutation overlap angle, and the ratio of average field current to dc link current. The two angles are specified as input to the program, and the ratio of field current to dc link current is calculated. In this way, a solution is guaranteed to exist for a given operating point. The symmetry of steady state operation is exploited to calculate the appropriate boundary conditions for a given operating point directly, without the need to perform iterative calculations. Sample calculations for a 1000 hp LCI drive are presented, showing voltage, current, and torque waveforms, and steady state performance characteristics with constant commutation margin angle.

## Introduction

Load commutated inverter (LCI) fed synchronous machines are gaining acceptance as high power variable speed drives where the ability to handle large amounts of power at relatively high efficiency is important, notably in fan drives [1]. The typical LCI synchronous motor drive, Fig. 1., comprises a wound field synchronous machine supplied by a thyristor bridge inverter. The phase controlled rectifier bridge on the input side takes ac power from the mains and produces a variable dc voltage. The link inductor smooths out the ripple to provide a relatively smooth dc current to the output converter. Natural commutation of the output converter is made possible by the induced EMF of the synchronous machine.

Steady state analysis of the LCI drive has been undertaken in the past. One approach is to assume a constant dc link current and to neglect the resistances in the machine [2,3]. The methods of Ref. [2] have been found to yield accurate results in terms of the fundamental components of voltage and current [4], but do not provide detailed information on the waveforms. In addition, neglecting the resistances can make the results somewhat inaccurate at low operating frequencies, where the subtransient time constant is significant compared to the commutation time.

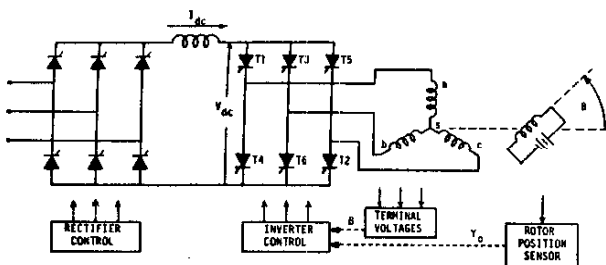


Fig. 1. Basic LCI synchronous motor drive system.

Analyses of the LCI drive that include machine resistances and finite dc link inductance are necessarily more involved than those employing simpler models. A state space formulation of the problem is used in [5], but this method requires several iterations of the numerical integrations in order to find the boundary conditions. Reference [6] employs a Fourier series expansion of the voltage and current waveforms, and requires several terms in the series for accuracy, and several iterations of the solution to meet the boundary conditions. Computation times of the order of minutes are reported for this method.

The analysis developed here permits accurate calculation of voltage, current, and torque waveforms under the assumptions of constant shaft speed and constant dc link current. The problem is formulated in terms of the synchronous machine equations (Park's equations) making it relatively easy to investigate the influence of the machine parameters on the performance of the LCI drive.

Steady state solutions are characterized by the firing angle,  $\gamma_0$ , and the commutation overlap angle,  $\mu$ . The ratio of average field current to dc link current is calculated as one of the boundary conditions necessary to guarantee a solution at the specified values of  $\gamma_0$  and  $\mu$ . The symmetry of steady state operation allows the proper boundary conditions to be found directly, without the need for iteration. This technique, which has been used previously in the study of induction motor drives [7,8], is employed here to provide a substantial reduction in the computation time necessary to obtain a solution.

## Modes of Operation

When the LCI drive is operating in the steady state, inverter thyristors T1-T6 fire in sequence, one every 60 electrical degrees of operation, causing the stator MMF to advance 60° with each switching event. As a consequence, a complete steady state solution can be obtained by analysis of any interval that is 60 electrical degrees in length.

The 60° interval chosen for this analysis begins when thyristor T5 turns off, and ends when thyristor T6 turns off. This interval comprises two modes of operation, which will be denoted the *conduction* and *commutation* modes. During the conduction mode, thyristors T1 and T6 are on, and the dc link current flows from phase a to phase b, as seen in Fig. 2. The commutation mode begins when thyristor T2 is gated on. A short circuit is established between phases b and c, and a circulating current  $i_k$  flows from phase c to b, eventually turning off phase b.

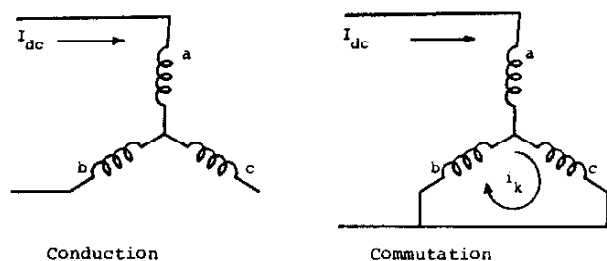


Fig. 2. Conduction and commutation modes.

At the start of the commutation interval the rotor angle is defined to be

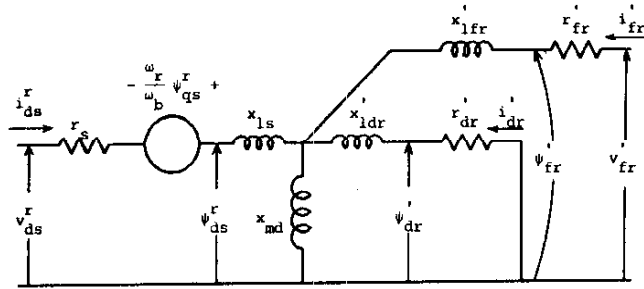
$$\theta = -\gamma_0 \quad (1)$$

and the duration of the commutation interval is given by the commutation overlap angle  $\mu$ , in electrical degrees. The firing angle  $\gamma_0$  is equivalent to the internal power factor angle neglecting overlap, i.e. it is the angle between the fundamental component of current, neglecting overlap, and the internal voltage of the synchronous machine.

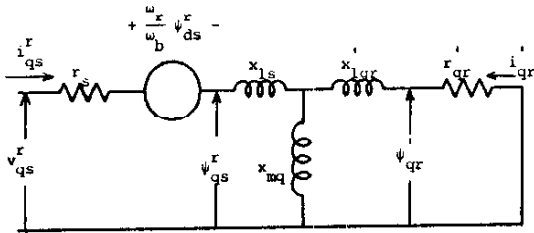
### State Equations

The standard synchronous machine model of Fig. 3, used herein, comprises a wound field with one damper winding on each rotor axis. The circuit equations for this model are expressed in the rotor reference frame as Eq. (2), below. The per unit electromagnetic torque is written as

$$T_e = \psi_{ds} i_{qs} - \psi_{qs} i_{ds} \quad (3)$$



D-Axis Equivalent Circuit



Q-Axis Equivalent Circuit

Fig. 3. Rotor reference frame equivalent circuit for synchronous machine.

where  $\psi_{ds}$  and  $\psi_{qs}$  are the stator flux linkages,

$$\psi_{ds} = x_d i_{ds} + x_{md} (i_{dr} + i_{fr}) \quad (4)$$

$$\psi_{qs} = x_q i_{qs} + x_{mq} i_{qr} \quad (5)$$

At a constant rotor speed the state of the machine is determined completely by the currents  $i_{qs}$ ,  $i_{ds}$ ,  $i_{qr}$ ,  $i_{dr}$ ,  $i_{fr}$ . However, with a constant dc link current, the stator currents contain only one independent variable, the circulating current  $i_k$ , and that is nonzero only during the commutation interval. The stator currents, in the rotor reference frame, can be written in terms of  $I_{dc}$  and  $i_k$  as

$$i_{qs} = I_{dc} \left( \cos\theta - \frac{1}{\sqrt{3}} \sin\theta \right) + \frac{2}{\sqrt{3}} i_k \sin\theta \quad (6)$$

$$i_{ds} = I_{dc} \left( \frac{1}{\sqrt{3}} \cos\theta + \sin\theta \right) - \frac{2}{\sqrt{3}} i_k \cos\theta \quad (7)$$

It proves useful in this analysis to let the dc link current be unity. Because the machine equations are linear, solutions for the same values of  $\gamma_0$  and  $\mu$ , but different values of  $I_{dc}$ , are obtained simply by scaling the normalized solution.

Because the circulating current  $i_k$  is nonzero only during the commutation mode, the state equations must take on somewhat different forms during the two modes:

### Conduction Mode State Equations

During the conduction mode the circulating current  $i_k$  is zero. This constraint is satisfied by the dummy state equation

$$(1 + \frac{p}{\omega_b}) i_k = 0 \quad (8)$$

Setting  $i_k = 0$  in (6) and (7) and substituting into the rotor circuit equations (the bottom three rows of (2)) yields terms proportional to  $\cos\theta$  and  $\sin\theta$ , which are the result of the constant dc link current reflected to the moving rotor. Rather than have these terms appear as time varying inputs to the system, it is desirable to generate  $\cos\theta$  and  $\sin\theta$  as auxiliary state variables  $y_1$  and  $y_2$ . Since the rotor speed and initial position are known, this is easily accomplished by solving the auxiliary equations

$$\begin{bmatrix} \frac{p}{\omega_r} & 1 \\ -1 & \frac{p}{\omega_r} \end{bmatrix} \times \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ 0 \\ 0 \\ v_{fr} \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} x_q & \frac{\omega_r}{\omega_b} x_d & \frac{p}{\omega_b} x_{mq} & \frac{\omega_r}{\omega_b} x_{md} & \frac{p}{\omega_b} x_{mq} & \frac{\omega_r}{\omega_b} x_{md} \\ -\frac{\omega_r}{\omega_b} x_q & r_s + \frac{p}{\omega_b} x_d & -\frac{\omega_r}{\omega_b} x_{mq} & \frac{p}{\omega_b} x_{md} & -\frac{\omega_r}{\omega_b} x_{mq} & \frac{p}{\omega_b} x_{md} \\ \frac{p}{\omega_b} x_{mq} & 0 & r_{qr} + \frac{p}{\omega_b} x_{qr} & 0 & r_{qr} + \frac{p}{\omega_b} x_{qr} & 0 \\ 0 & \frac{p}{\omega_b} x_{md} & 0 & r_{dr} + \frac{p}{\omega_b} x_{dr} & 0 & \frac{p}{\omega_b} x_{md} \\ 0 & \frac{p}{\omega_b} x_{md} & 0 & \frac{p}{\omega_b} x_{md} & 0 & r_{fr} + \frac{p}{\omega_b} x_{fr} \end{bmatrix} \times \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \\ i_{fr} \end{bmatrix} \quad (2)$$

The functions  $y_1$  and  $y_2$  are combined with the currents to make up an augmented state vector.

$$\mathbf{x} = [i_{qr}, i_{dr}, i_{fr}, i_k, y_1, y_2]^T \quad (10)$$

The three rotor circuit equations, the dummy state equation, (8), and the auxiliary state equations (9) can be cast in matrix form as Eq. (11), below.

By suitable manipulation of (11), the state equations can be put into a standard form,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (12)$$

where  $\mathbf{u}$  is the input vector,

$$\mathbf{u} = [V_{fr}, I_{dc}]^T \quad (13)$$

which by assumption is composed of constant elements. It should be noted that during the conduction mode, the system matrix  $\mathbf{A}$  and the input coefficient matrix  $\mathbf{B}$  are constants, i.e. the system is time-invariant.

### Commutation Mode State Equations

During the commutation interval the circulating current  $i_k$  is no longer zero, so the full expressions for the stator currents (6) and (7) must be used in the rotor circuit equations. In addition, the dummy state equation, (8), is no longer valid. The appropriate state equation for this mode is one that accounts for the short circuit between phases b and c. Transforming the stator voltage expressions to a stationary reference frame yields, after a brief struggle,

$$\begin{aligned} 0 &= \frac{1}{\sqrt{3}}(v_{cs} - v_{bs}) = \frac{r_s}{\sqrt{3}}(I_{dc} - 2i_k) \\ &- \frac{1}{\sqrt{3}}[x_d + x_q + (x_d - x_q)\cos 2\theta] \frac{p}{\omega_b} i_k \\ &- x_{mq} \sin \theta \frac{p}{\omega_b} i_{qr} + x_{md} \cos \theta \frac{p}{\omega_b} (i_{dr} + i_{fr}) \\ &+ \frac{2}{\sqrt{3}} \frac{\omega_r}{\omega_b} (x_d - x_q) \sin 2\theta i_k \\ &- \frac{\omega_r}{\omega_b} x_{mq} \cos \theta i_{qr} - \frac{\omega_r}{\omega_b} x_{md} \sin \theta (i_{dr} + i_{fr}) \\ &+ \frac{\omega_r}{\omega_b} (x_d - x_q) (\cos 2\theta - \frac{1}{\sqrt{3}} \sin 2\theta) \end{aligned} \quad (14)$$

The last term in (14), in its present form, would appear as a time varying input to the system. To eliminate this undesired behavior the trigonometric terms are factored into the equivalent form

$$\begin{aligned} \cos 2\theta - \frac{1}{\sqrt{3}} \sin 2\theta &= [\cos \theta - (1 + \frac{1}{\sqrt{3}}) \sin \theta] \cos \theta \\ &+ [(1 - \frac{1}{\sqrt{3}}) \cos \theta - \sin \theta] \sin \theta \end{aligned} \quad (15)$$

In this form the double frequency terms appear explicitly in terms of the auxiliary state variables, with time varying coefficients.

The commutation mode state equations can be cast in matrix form as Eq. (16), below. The abbreviations  $S\theta$  and  $C\theta$  in (16) represent  $\sin \theta$  and  $\cos \theta$ , respectively. By suitable matrix manipulation, (16) can also be put into standard form,

$$\begin{bmatrix} r_{qr} + \frac{p}{\omega_b} x_{qr} & 0 & 0 & 0 & -\frac{1}{\sqrt{3}} \frac{\omega_r}{\omega_b} x_{mq} & -\frac{\omega_r}{\omega_b} x_{mq} \\ 0 & r_{dr} + \frac{p}{\omega_b} x_{dr} & \frac{p}{\omega_b} x_{md} & 0 & \frac{\omega_r}{\omega_b} x_{md} & -\frac{1}{\sqrt{3}} \frac{\omega_r}{\omega_b} x_{md} \\ 0 & \frac{p}{\omega_b} x_{md} & r_{fr} + \frac{p}{\omega_b} x_{fr} & 0 & \frac{\omega_r}{\omega_b} x_{md} & -\frac{1}{\sqrt{3}} \frac{\omega_r}{\omega_b} x_{md} \\ 0 & 0 & 0 & 1 + \frac{p}{\omega_b} & \frac{p}{\omega_r} & 0 \\ 0 & 0 & 0 & 0 & \frac{p}{\omega_r} & 1 \\ 0 & 0 & 0 & 0 & -1 & \frac{p}{\omega_r} \end{bmatrix} \times \begin{bmatrix} i_{qr} \\ i_{dr} \\ i_{fr} \\ i_k \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_{fr} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} r_{qr} + \frac{p}{\omega_b} x_{qr} & 0 & 0 & \frac{2}{\sqrt{3}} x_{mq} (C\theta \frac{\omega_r}{\omega_b} + S\theta \frac{p}{\omega_b}) & -\frac{1}{\sqrt{3}} \frac{\omega_r}{\omega_b} x_{mq} & -\frac{\omega_r}{\omega_b} x_{mq} \\ 0 & r_{dr} + \frac{p}{\omega_b} x_{dr} & \frac{p}{\omega_b} x_{md} & \frac{2}{\sqrt{3}} x_{md} (S\theta \frac{\omega_r}{\omega_b} - C\theta \frac{p}{\omega_b}) & \frac{\omega_r}{\omega_b} x_{md} & -\frac{1}{\sqrt{3}} \frac{\omega_r}{\omega_b} x_{md} \\ 0 & \frac{p}{\omega_b} x_{md} & r_{fr} + \frac{p}{\omega_b} x_{fr} & \frac{2}{\sqrt{3}} x_{md} (S\theta \frac{\omega_r}{\omega_b} - C\theta \frac{p}{\omega_b}) & \frac{\omega_r}{\omega_b} x_{md} & -\frac{1}{\sqrt{3}} \frac{\omega_r}{\omega_b} x_{md} \\ -x_{mq} (S\theta \frac{p}{\omega_b} + C\theta \frac{\omega_r}{\omega_b}) & x_{md} (C\theta \frac{p}{\omega_b} - S\theta \frac{\omega_r}{\omega_b}) & x_{md} (C\theta \frac{p}{\omega_b} - S\theta \frac{\omega_r}{\omega_b}) & \frac{1}{\sqrt{3}} [(x_d + x_q + (x_d - x_q) \cos 2\theta) \frac{p}{\omega_b} \\ & & & + 2 \frac{\omega_r}{\omega_b} (x_d - x_q) \sin 2\theta - 2r_s] & \frac{\omega_r}{\omega_b} (x_d - x_q) [C\theta \frac{\omega_r}{\omega_b} (x_d - x_q) [-S\theta \\ & & & & - (1 + \frac{1}{\sqrt{3}}) S\theta] & + (1 - \frac{1}{\sqrt{3}}) C\theta] \\ 0 & 0 & 0 & 0 & \frac{p}{\omega_r} & 1 \\ 0 & 0 & 0 & 0 & -1 & \frac{p}{\omega_r} \end{bmatrix} \times \begin{bmatrix} i_{qr} \\ i_{dr} \\ i_{fr} \\ i_k \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_{fr} \\ -\frac{r_s}{\sqrt{3}} I_{dc} \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

$$\dot{\mathbf{x}} = \mathbf{A}'\mathbf{x} + \mathbf{B}'\mathbf{u} \quad (17)$$

During the commutation mode, the system matrix  $\mathbf{A}'$  and input coefficient matrix  $\mathbf{B}'$  are time varying, being functions of the rotor angle  $\theta$ . This is an inevitable consequence of having an unbalanced component of stator current  $i_s$  flowing in circuits that are in motion relative to the rotor, which because of saliency is asymmetric.

### Calculation of Initial Conditions

The symmetry of steady state operation provides a means to compute the boundary conditions for the steady state operating point in closed form, without resorting to iterative calculations. This calculation requires that an expression be derived relating the initial values of the state variables to the final values, at the end of the  $60^\circ$  interval. The symmetry constraints are then applied to solve for the initial values of the rotor currents  $i_{qr}$ ,  $i_{dr}$ ,  $i_{fr}$ , and for the field voltage  $V_{fr}$ .

For any linear, time invariant system with constant inputs, for example the system described by (12), the state of the system at time  $t_1$  is related to the initial state by

$$\mathbf{x}(t_1) = e^{\mathbf{A}t_1}\mathbf{x}(0) + \lambda(t_1,0)\mathbf{B}\mathbf{u} \quad (18)$$

where  $e^{\mathbf{A}t_1}$  is the state transition matrix, and  $\lambda(t_1,0)$  is the integral,

$$\lambda(t_1,0) = \int_0^{t_1} e^{\mathbf{A}(t_1-t)} dt \quad (19)$$

Let the entire solution interval be divided as shown in Fig. 4. If we denote the initial state as  $\mathbf{x}_0$ , the state at the mode transition, where  $t = T$ , can be written as

$$\mathbf{x}_1 = e^{\mathbf{A}_0 T}\mathbf{x}_0 + \lambda_0 \mathbf{B}_0 \mathbf{u}_0 \quad (20)$$

where the subscript 0 on the matrices indicates that they were evaluated for the conduction mode. (i.e. the  $0^{\text{th}}$  increment in Fig. 4.)

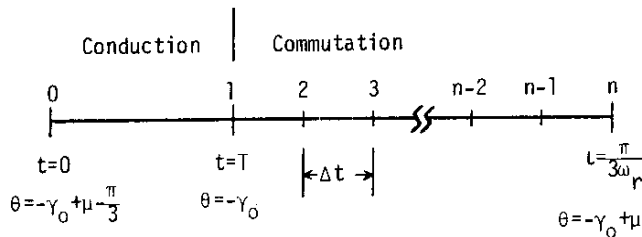


Fig. 4. Discretization of solution interval for initial condition calculation.

The system is time varying during the commutation mode, so no such simple calculation exists to find the final value of the state variables. Although it can be shown that a closed form solution exists for the time variant system [9], such solutions are generally impossible to find. However, reasonable accuracy can be obtained by dividing the commutation interval into small increments, say one degree, and treating the system as time invariant over each increment. The value of the state vector at the end of any increment is related to the value at the end of the previous increment by

$$\mathbf{x}_n = e^{\mathbf{A}_{n-1}\Delta t}\mathbf{x}_{n-1} + \lambda_{n-1}\mathbf{E}_{n-1}\mathbf{u} \quad (21)$$

where the state transition matrix and its integral are evaluated at the midpoint of each increment. Successive applications of (21) yield an expression for the final state, at the end of the  $60^\circ$  interval, in terms of the initial state,

$$\mathbf{x}_n = \mathbf{D}\mathbf{x}_0 + \mathbf{C}_{n-1}\mathbf{u} \quad (22)$$

where

$$\mathbf{D} = e^{\mathbf{A}_{n-1}\Delta t} e^{\mathbf{A}_{n-2}\Delta t} \dots e^{\mathbf{A}_1\Delta t} e^{\mathbf{A}_0 T} \quad (23)$$

and  $\mathbf{C}_{n-1}$  is given recursively by

$$\mathbf{C}_{n-1} = e^{\mathbf{A}_{n-1}\Delta t}\mathbf{C}_{n-2} + \lambda_{n-1}\mathbf{B}_{n-1} \quad (24)$$

$$\mathbf{C}_0 = \lambda_0 \mathbf{B}_0 \quad (25)$$

The top four rows of (22) are used to solve for the unknown currents and the field voltage. Symmetry of steady state operation requires that

$$i_{qr_n} = i_{qr_0} \quad (26)$$

$$i_{dr_n} = i_{dr_0} \quad (27)$$

$$i_{fr_n} = i_{fr_0} \quad (28)$$

and the commutation constraint requires

$$i_{kn} = 1.0 \text{ per unit.} \quad (29)$$

These relations are substituted into the top four rows of (22) to obtain four equations in  $i_{qr}$ ,  $i_{dr}$ ,  $i_{fr}$ , and  $V_{fr}$ . Solution of those four equations is carried out by usual linear algebra techniques.

### Time Step Solution

Once the initial conditions have been found, it is a straightforward matter to integrate the state equations to find values for the state variables at convenient time intervals. In particular, if the time interval is chosen to be the same as the increment for the initial condition calculation, the state transition matrices can be stored and used again, resulting in some saving of computation time.

Full-cycle ( $360^\circ$ ) waveforms can easily be reconstructed from the waveforms calculated over  $60^\circ$ . Rotor quantities (currents and flux linkages) and electromagnetic torque simply repeat every  $60^\circ$ . Stator quantities (phase voltage, current, and flux linkages) exhibit half wave and three phase symmetry, e.g.

$$v_{as}(\theta + \pi) = -v_{as}(\theta) \quad (30)$$

and

$$v_{bs}(\theta + \frac{\pi}{3}) = -v_{bs}(\theta) \quad (31)$$

$$v_{cs}(\theta + \frac{\pi}{3}) = -v_{cs}(\theta) \quad (32)$$

$$v_{cs}(\theta + \frac{\pi}{3}) = -v_{as}(\theta) \quad (33)$$

## Sample Calculations

The steady state performance of a 1000 hp LCI synchronous motor drive is calculated for an operating point characterized by  $\gamma_0 = 55^\circ$ ,  $\mu = 18^\circ$ , and dc link current of 0.68 per unit. The parameters of the machine are given in the Appendix, and all calculated quantities are given in per unit, with the machine rating as base. The computation time for this data point is estimated at approximately 1.5 seconds of CPU, on a DEC VAX-11/780. No effort was made to optimize the efficiency of the program.

Fig. 5. shows voltage and current waveforms for one of the inverter thyristors. From this plot it is possible to deduce the thyristor voltage stress, the rate of forward current rise, and the commutation margin angle,  $\delta$ , which indicates how long the device is reverse biased before it must support forward voltage.

Commutation notches in the inverter voltage are clearly visible in Fig. 6. The deviation of the inverter voltage from the average value can be used to estimate the current ripple that would occur with finite dc link inductance. The electromagnetic torque, Fig. 7., and field current, Fig. 8., both exhibit the sixth harmonic ripple components caused by inverter operation. These components are useful for identifying additional field heating caused by ripple current and provide a tool for evaluating the severity of torque oscillations imposed on the connected mechanical system.

Operation of the LCI drive at the minimum margin angle  $\delta$  needed for safe commutation results in the highest power factor at the machine terminals and the best utilization of the machine windings [2]. Performance calculations for this type of operating scheme were carried out in [4]. For various values of firing angle  $\gamma_0$ , an iterative technique was used to find the steady state operating point (the value of  $\mu$ ) that yielded the desired value of commutation margin angle. Use of a binary search technique limited the number of iterations to a maximum of six for each operating point.

Steady state performance curves for the LCI drive are shown in Figs. 9-10, for operation at rated speed and a margin angle of  $\delta = 10^\circ$ . Fig. 9. shows the average torque, fundamental ac voltage, and inverter voltage as a function of dc link current. Extreme overvoltage can be noted at low values of dc link current, indicating the need for some sort of field current control. Fig. 10 shows a similar performance curve, with the field current adjusted to maintain the voltage at the rated value. The linearity of the solution made it possible to obtain the two plots from the same set of steady state solutions, simply by scaling the results to achieve either constant field current or constant voltage.

## Summary

This paper has described a new, computationally efficient method for predicting the steady state behavior of an LCI synchronous motor drive. The solution does not neglect stator or rotor resistances so that the results remain valid even at low operating frequencies.

The assumption of constant dc link current results in considerable simplification of the state equations, and permits a significant part of the initial condition calculation to be carried out in one step. The use of symmetry to find the boundary conditions without the need for iteration reduces the computation time and would probably make this method feasible even for personal sized computers.

The extension of this work to the problem of the finite dc link inductor can be carried out, essentially by employing the techniques used in integrating the commutation mode state equations. This work is in progress at this writing, and will be reported on at a later date.

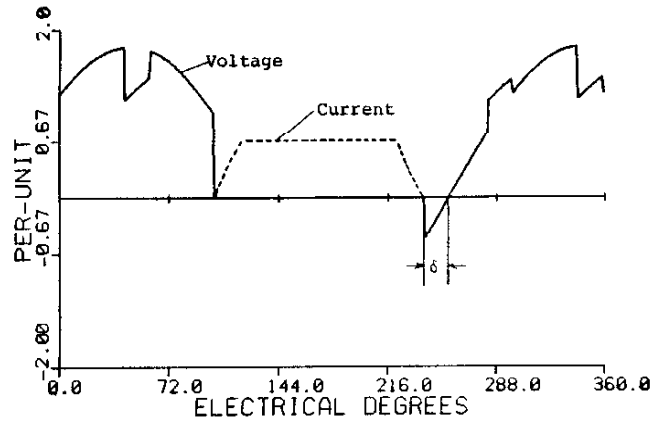


Fig. 5. Thyristor current and voltage.

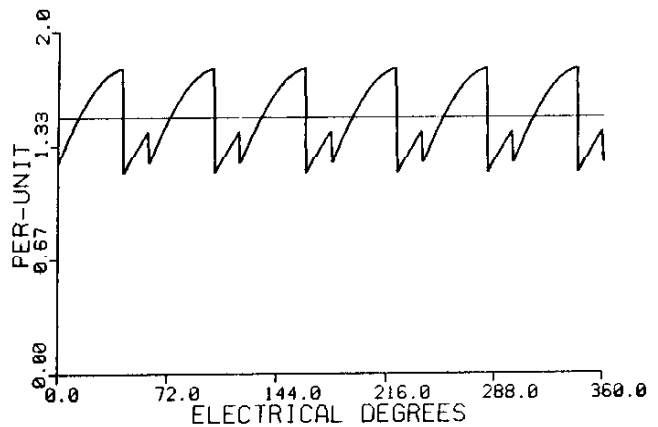


Fig. 6. Inverter voltage.

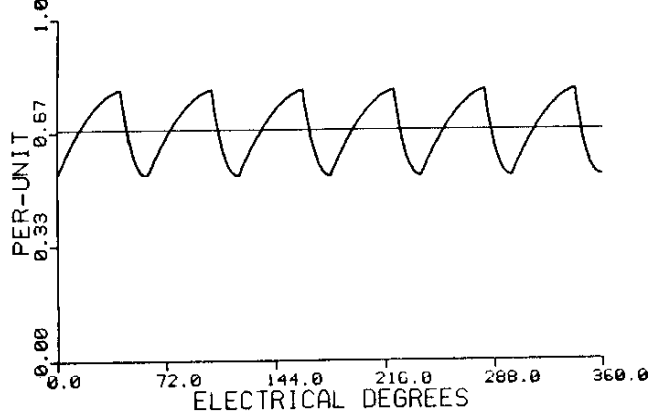


Fig. 7. Electromagnetic torque.

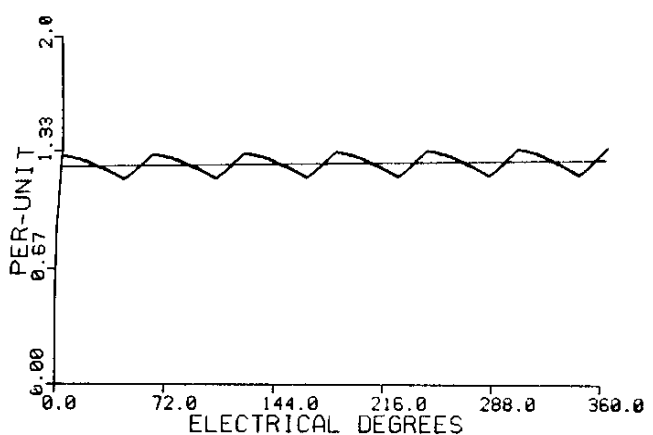


Fig. 8. Field current.

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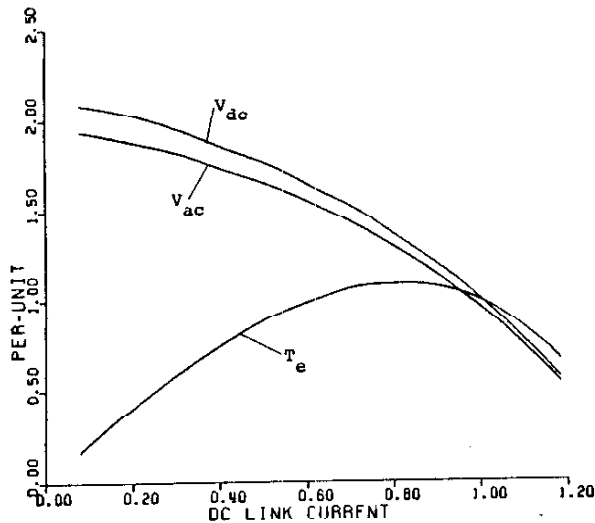


Fig. 9. Steady state performance at rated speed, rated field current,  $10^\circ$  margin angle.

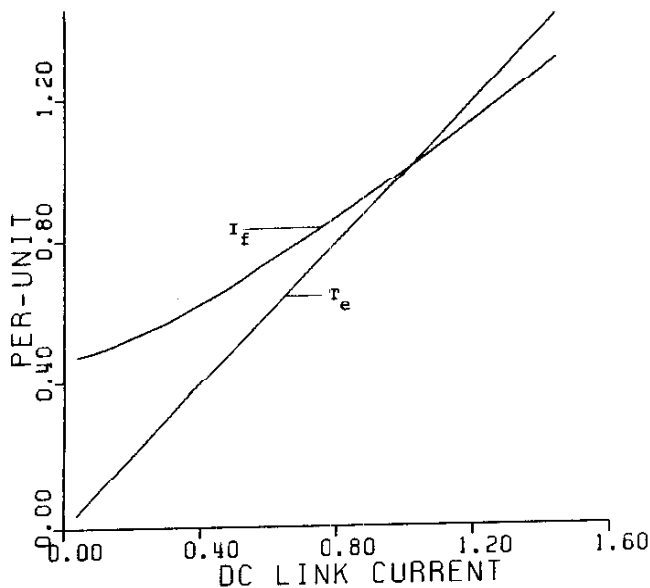


Fig. 10. Steady state performance at rated speed, rated voltage,  $10^\circ$  margin angle.

## Acknowledgement

The authors wish to acknowledge the financial support provided by the Wisconsin Electric Machines and Power Electronics Consortium (WEMPEC), and by General Electric Company.

## Appendix

### Synchronous machine data

Rating:

1000 hp, 4kV, 4 pole, 60 Hz.

Per unit parameters:

$$x_{md} = 1.28 \quad \tau_s = 0.0051$$

$$x_{mq} = 0.770 \quad \tau_{dr} = 0.085$$

$$x_{is} = 0.0932 \quad \tau_{fr} = 0.001$$

$$x_{idr} = 0.096 \quad \tau_{qr} = 0.032$$

$$x_{ifr} = 0.1838$$

$$x_{iqr} = 0.115$$