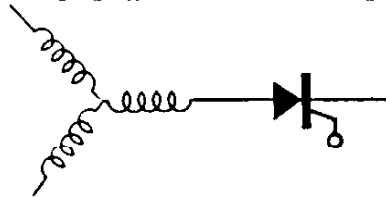




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On-Line Efficiency Optimization of a Variable  
Frequency Induction Motor Drive

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# On-Line Efficiency Optimization of a Variable Frequency Induction Motor Drive

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**Abstract**—The problems associated with the implementation of an optimal efficiency controller in variable frequency induction motor drives are examined. A simple method for minimizing, on-line, the global system losses is presented. This method is based on the adaptive control of the rotor flux in a field-oriented drive system. The effectiveness of this control strategy is verified using digital simulation.

## INTRODUCTION

**E**NERGY SAVINGS and their financial payoffs are largely responsible for the rapid spread of variable speed induction motor drives in the process industries. Indeed, variable speed operation makes possible a more efficient utilization of the mechanical power delivered by the motor. Similarly, variable flux operation makes possible a more efficient conversion of the electrical power within the motor drive. In a previous publication [1], the potential for energy savings achievable through excitation control in variable frequency induction motor drives was demonstrated, and the mechanisms yielding a reduction in losses were analyzed. The combination of stator voltage and frequency which minimizes the induction motor losses was shown to be a complex function of the operating speed and torque. Moreover, this function is also heavily influenced by the saturation level of the machine. Optimal efficiency thus cannot be achieved by implementing a fixed relation between stator frequency and voltage. Based on a detailed analysis of the data used to establish the results presented in [1], it does not appear that a conventional feedback control system using a single motor variable as an indication of efficiency would yield a satisfactory suboptimal system. These problems can be avoided through the use of a direct measurement of the power input and the implementation of an adaptive controller. As shown on Fig. 1, the adaptive controller iteratively adjusts an actuating variable  $c(t)$  related to the motor losses until it detects a minimum in the power input. At constant power output, this condition corresponds to the maximum efficiency operating point. Kusko and Galler have briefly discussed the feasibility of a similar scheme in the case of a dc drive [2].

In the following section, the advantages and constraints

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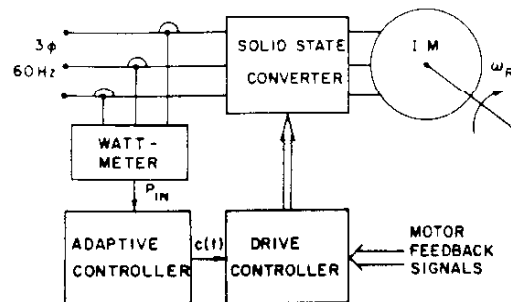


Fig. 1. Block diagram of adaptive optimal efficiency controller.

imposed by this configuration are discussed, and it is concluded that a field-oriented drive system provides an ideal vehicle for this adaptive control strategy. Characteristic loci describing variable flux operation of a field-oriented drive system in the steady state are then derived. The dynamic minimization of the power input is illustrated using digital simulation, and the accompanying variations in torque are explained using transfer functions. Finally, a method which guarantees a fast response to a large torque demand from any operating point is presented.

## DISCUSSION OF THE ADAPTIVE CONTROL STRATEGY

Besides yielding a true optimum, the adaptive control approach to the induction motor losses minimization problem has two additional advantages.

1) If the power input is measured on the source side of the rectifier, the minimization is not restricted to the motor losses but affects the entire system and thus reduces the total amount of energy consumed. Moreover, since the source voltage and current waveforms have a much smaller harmonic content than the corresponding motor waveforms, the power measurement is more accurate and easier to obtain.

2) The adaptive controller does not require knowledge of any machine parameter and can therefore be readily implemented on any motor and with different types of inverters. Such a control strategy is perfectly robust in the sense that it is totally insensitive to variations in the motor parameters such as an increase in rotor resistance due to a rise in machine temperature.

The presence of an adaptive efficiency optimization loop imposes several important constraints on the drive system.

1) The actuating variable which is adjusted to minimize the power input must be related to the motor losses by a convex

function (i.e., a function whose second derivative is never zero). Convergence will be easier if the minimum of this function is clearly defined. This will be the case if the actuating variable has a direct impact on the repartition of losses.

2) The maximum efficiency controller will tend to reduce the power delivered by the motor if a torque control loop does not maintain it at a specified level.

3) Ideally, the loss minimization loop and the torque control loop should be decoupled. Although total decoupling is probably impossible, it can be approximated by making the response time of the torque loop much shorter than the interval of time separating two iterations of the adaptive controller. If this requirement is not satisfied, the torque control loop will be unable to maintain the power output at its original level.

4) It was shown in [1] that the flux level must be reduced for maximum efficiency at light load. Such a reduction slows down the system response to a load change and decreases the peak torque which can be developed by the motor. This problem must be addressed if the drive is to maintain satisfactory dynamic performances.

Among the many ac drive control methods suggested in the literature, direct field orientation satisfies best these constraints. The rotor flux is indeed one of the controlled variables and provides direct control of the distribution of losses. Moreover, field orientation realizes the best decoupling achieved so far between the flux and torque control loops, simplifying greatly the design of the adaptive controller. Finally, the dynamics of a field-oriented drive are simple, and the implications of operation at reduced flux can therefore be easily understood. The next section is devoted to the analysis of field-oriented drive systems operating at variable flux.

#### FIELD ORIENTATION WITH VARIABLE ROTOR FLUX

Since the principle of field-oriented control of induction machines has been described in many references, only the most significant results will be repeated here. A more detailed treatment using identical notation can be found in [3]. In field-oriented drive systems, the stator current is controlled as a vector quantity using either its magnitude and phase or, more commonly, its components along the two axes of a Cartesian reference frame. The position of this reference frame is arbitrary, but the equations turn out to be simpler if a synchronously rotating reference frame with its  $d$  axis locked on the rotor flux vector is used. Setting

$$\lambda_{qr} = 0 \quad (1)$$

in the machine equations expressed in the synchronously rotating reference frame (see Appendix I), one finds the slip frequency needed to maintain the field orientation of the reference frame:

$$\omega_s = \omega_e - \omega_r = \frac{r_r}{L_r} \frac{L_m i_{qs}}{\lambda_{dr}} \quad (2)$$

Further manipulations of these equations then yield a relation

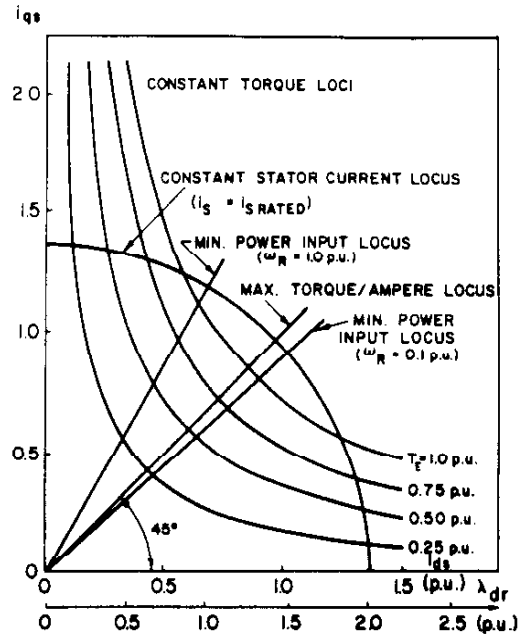


Fig. 2. Characteristic loci of 7.5-hp induction machine in field-oriented reference frame, neglecting saturation.

describing the dynamic behavior of the rotor flux:

$$(r_r + L_r p) \lambda_{dr} = r_r L_m i_{ds} \quad (3)$$

and another giving the instantaneous electromagnetic torque:

$$T_e = \frac{L_m}{L_r} \lambda_{dr} i_{qs} \quad (4)$$

Equations (3) and (4) show that the rotor flux and the torque are controlled by the  $d$ - and  $q$ -axis components of the stator current, respectively. Although a change in the magnitude of the rotor flux modifies the torque versus  $q$ -axis current relation, it does not affect the field orientation and the resulting decoupling if (2) is respected.

In the steady state, (3), (4), and (2) become

$$\lambda_{dr} = L_m i_{ds} \quad (5)$$

$$T_e = \frac{L_m}{L_r} \lambda_{dr} i_{qs} = \frac{L_m^2}{L_r} i_{ds} i_{qs} \quad (6)$$

$$\omega_s = \frac{r_r}{L_r} \frac{i_{qs}}{i_{ds}} \quad (7)$$

These equations suggest the graphical interpretation of the steady-state motor behavior shown on Fig. 2 for the unsaturated case. Fig. 2 and the other numerical results presented in this paper are established for the 7.5-hp motor described in Appendix II.

If the  $d$ - and  $q$ -axis components of the stator current are used as abscissa and ordinate, respectively, (6) represents a family of rectangular hyperbolas, the parameter of the family being the value of the torque developed by the motor. A second scale has been provided for the abscissae because (5) clearly indicates that the rotor flux is constant along vertical

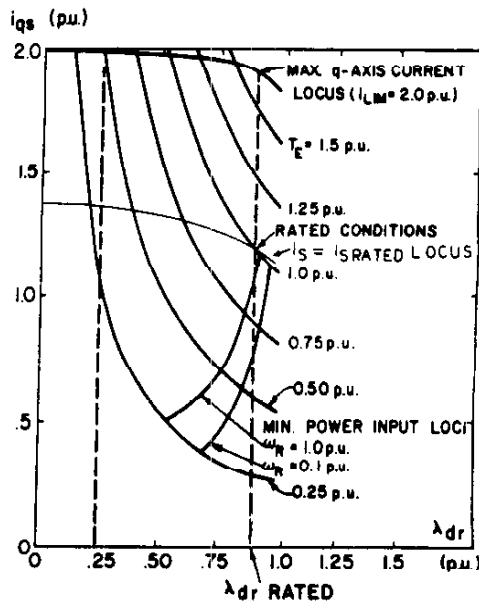


Fig. 3. Characteristic loci of 7.5-hp induction machine in field-oriented reference frame, including saturation.

lines. Since the  $d$  and  $q$  axes are orthogonal, the magnitude of the stator current is constant on circles centered at the origin.

Equation (7) indicates that the slip frequency is constant on any straight line through the origin. It has been shown in [1] that the slip frequency for maximum efficiency is independent of the load torque if saturation is neglected. The power input is thus also minimum on a straight line through the origin, the slope of this line being a function of the rotor speed.

The torque developed per ampere of stator current is maximum at the points where a constant torque locus is tangent to a constant current magnitude locus. The locus of these points is a straight line at  $45^\circ$ , and hence the maximum torque per ampere locus is also a constant slip frequency locus. Equation (7) then shows that this value of slip frequency is independent of torque and speed and is equal to the inverse of the rotor time constant.

In practice, the parameters  $L_m$  and  $L_r$  are affected by saturation, and the characteristic loci do not have the geometric simplicity displayed in Fig. 2. However, distortion can be minimized if, as shown in Fig. 3, the rotor flux is taken as the abscissa.

Since, at constant rotor flux, the torque is directly proportional to the  $q$ -axis stator current, the operating point can move along a vertical line of Fig. 3 as fast as the  $q$ -axis current can be changed. On the other hand, (3) shows that the relation between the rotor flux and the  $d$ -axis stator current involves the rotor time constant. The operating point will thus move much more slowly horizontally than vertically on Fig. 3. For best dynamic performance, a conventional field-oriented drive would therefore operate along the vertical line corresponding to rated rotor flux. The objective of the maximum efficiency controller is to push the operating point along a constant torque characteristic up to the intersection with the minimum power input locus corresponding to the rotor speed.

## SYSTEM SIMULATION

### Description of the Model

The system represented in Fig. 4 was chosen for a digital simulation study of the proposed maximum efficiency controller. The induction motor is modeled in terms of the flux linkages expressed in the synchronously rotating reference frame [4]. Saturation is taken into account using the method described in [5], and the losses are computed as in [1] except for the harmonic and stray losses which are neglected. The motor is fed by a current-regulated pulsewidth-modulated inverter [6]. As a first approximation, this type of inverter can be modeled as supplying a voltage proportional to the current error [7].

For the purpose of this analysis, field orientation is assumed ideal and is realized by forcing the slip frequency relation (2). A rotor flux feedback signal and a torque feedback signal are assumed to be available for regulation purposes. The flux and torque regulators are of the proportional plus integral type, and their outputs are the reference values of the  $d$ - and  $q$ -axis components of the stator current, respectively. The rotor flux reference is determined by the adaptive controller ( $\mu P$ ) based on the power measurement  $P_{in}$ . The gains of the torque controller have been adjusted to yield, at rated flux, a torque response comparable to the response of an actual drive system described in the literature [8]. The flux regulator has been tuned to give a fast response while avoiding excessive operation at the current limit.

### Efficiency Optimization

Fig. 5 illustrates the operation of the maximum efficiency controller. Initially, the motor operates at rated speed, 0.25-pu torque, and rated flux. At  $t = 0$ , the controller reduces the flux reference by 0.04 pu. After 0.5 s, the rotor flux has reached its new steady-state value, the torque controller has compensated for the flux reduction, and the power input measurement has stabilized at a value smaller than the initial reading. The controller thus continues to reduce gradually the flux reference until it detects, after the ninth step, a leveling in the power input. At that point, the direction of the search is inverted, and the step size is reduced to close in more precisely on the optimum. Subsequent direction changes take place each time the power input increases between two consecutive steps. The step size cannot be reduced below a certain limit because the system must maintain enough sensitivity to react to slow variations in the power output. Thus once the controller has determined the location of the optimum, it must continue to impose small variations in the rotor flux reference in order to adapt rapidly to any change in the operating conditions.

In this example, the overall minimization process takes approximately 7 s, but this duration could be curtailed if the rotor flux was reduced in bigger steps. However, as shown on Fig. 5(c), each rotor flux reduction is accompanied by a torque pulsation. Since the amplitude of these pulsations is proportional to the change in flux, a compromise has to be found between the length of the minimization process and the smoothness of the torque as a function of time. A better understanding of these torque transients can be gained from

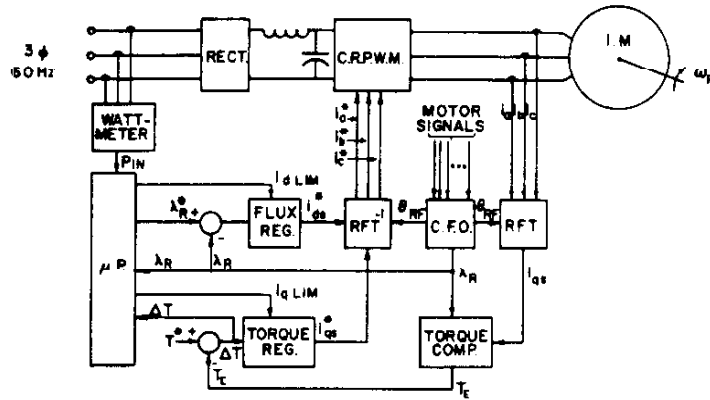


Fig. 4. Block diagram representation of proposed optimal efficiency controller.

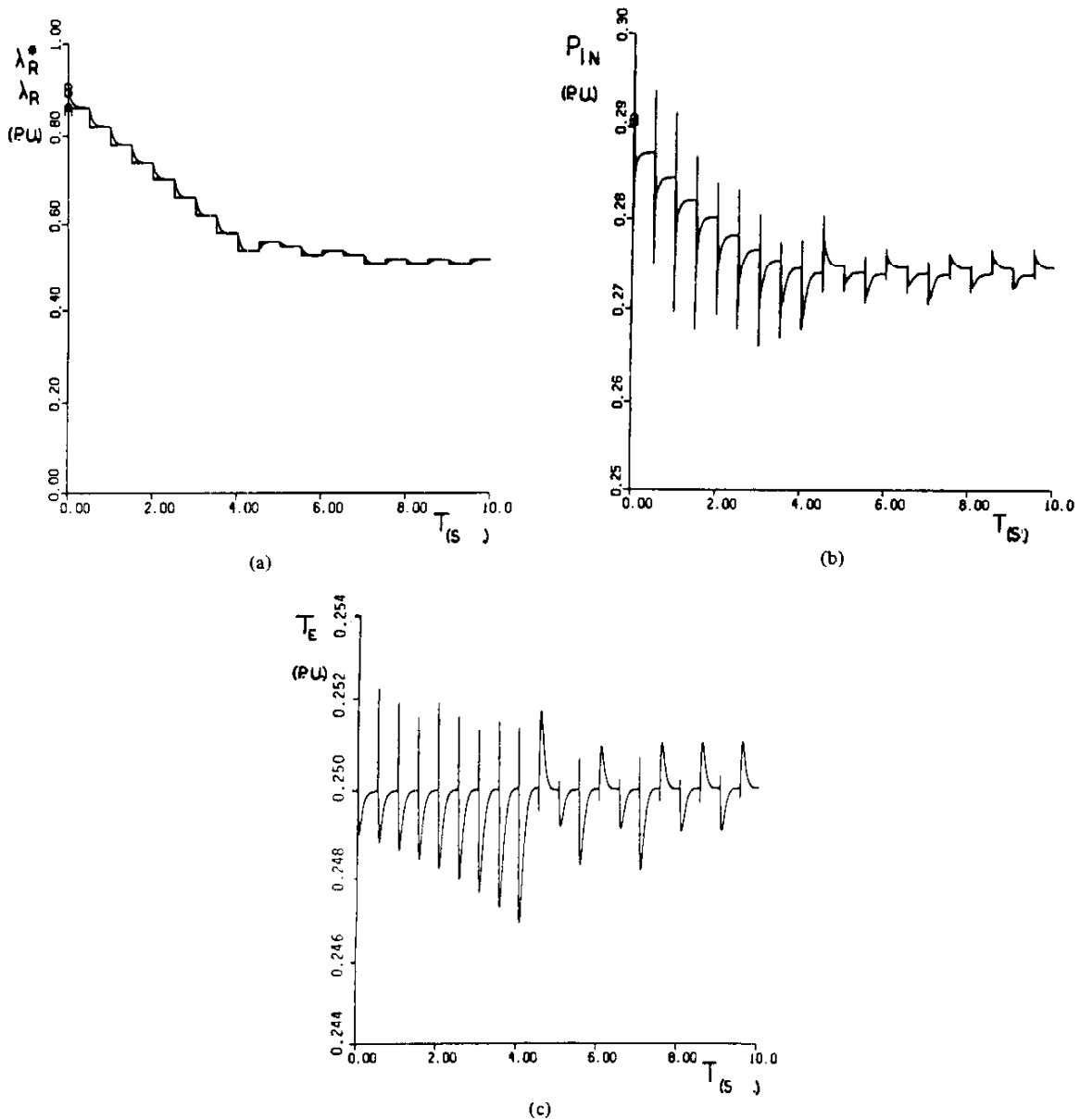


Fig. 5. Digital simulation of typical power input minimization process. Motor operates at rated speed and 0.25-pu torque. (a) Rotor flux reference and rotor flux. (b) Power input. (c) Torque.

the derivation of a small signal transfer function. Linearizing (4) around an operating point  $T_{e0}$ ,  $i_{qs0}$ ,  $\lambda_{dr0}$ , one obtains

$$\Delta T_e = \alpha \cdot \lambda_{dr0} \cdot \Delta i_{qs} + \alpha \cdot i_{qs0} \cdot \Delta \lambda_{dr} \quad (8)$$

with

$$T_{e0} = \alpha \cdot \lambda_{dr0} \cdot i_{qs0} \quad (9)$$

and

$$\alpha = \frac{L_m}{L_r} \quad (10)$$

If an ideal current regulator is assumed, the  $q$ -axis component of the stator current is equal to the reference value specified by the torque regulator:

$$i_{qs} = i_{qs}^* = K_T \cdot \left( \frac{p + K_{TI}}{p} \right) \cdot (T_e - T^*) \quad (11)$$

For small variations around the equilibrium point, (11) becomes

$$\Delta i_{qs} = -K_T \cdot \left( \frac{p + K_{TI}}{p} \right) \cdot \Delta T_e \quad (12)$$

Inserting (12) in (8), one obtains after some manipulations

$$\Delta T_e = \frac{T_{e0}}{1 + \alpha \lambda_{dr0} K_T} \frac{p}{p + \alpha_T} \frac{\Delta \lambda_{dr}}{\lambda_{dr0}} \quad (13)$$

where the pole of this transfer function is given by

$$\alpha_T = \frac{\alpha \lambda_{dr0} K_T K_{TI}}{1 + \alpha \lambda_{dr0} K_T} \quad (14)$$

Equations (13) and (14) clearly show that a reduction in the rotor flux level  $\lambda_{dr0}$  has three effects:

- it increases the relative size of the step change  $\Delta \lambda_{dr} / \lambda_{dr0}$ ;
- it increases the gain  $T_{e0} / (1 + \alpha \lambda_{dr0} K_T)$ ; and
- it moves the pole of the transfer function toward the real axis, reducing the damping of the system.

These effects explain why the torque and power input pulsations become larger and wider as the system approaches the optimum for a reduced value of the flux.

On Fig. 5(c) one can also observe narrow torque spikes in the direction opposite to the step changes in the rotor flux reference. A decrease in the rotor flux reference results indeed in a sudden decrease in  $i_{ds}$  and in the  $d$ -axis stator flux linkage. Thus it also results in a reduction of the  $q$ -axis counter EMF and hence in an increase of  $i_{qs}$ . This increase creates the torque spikes and cannot be immediately compensated due to the nonideal nature of the stator current regulator.

### System Response to a Large Torque Demand

Since the maximum efficiency controller will often force the motor to operate at reduced flux, the question of the system response to a large torque demand must be addressed. For a given value of the flux, the maximum current available for

torque production is given by

$$i_{qs}^2 = i_{lim}^2 - \left( \frac{\lambda_{dr}}{L_r} \right)^2 \quad (15)$$

where  $i_{lim}$  is the current limit of the inverter. This locus is plotted at the top of Fig. 3 for  $i_{lim} = 2.0$  pu and permits the determination of the peak torque achievable for a given value of rotor flux. For example, if  $\lambda_{dr} = 0.25$  pu, the torque cannot exceed 0.50 pu. When facing a large torque demand from a low flux operating point, the temptation is great to let the controller force the current to the maximum given by (15). However, this strategy would be disastrous if the torque demand cannot be met with the existing flux level, since there would then be no component of current left to increase the flux. A practical solution to this problem consists in imposing an artificial limit on  $i_{qs}$  and thereby using the balance of the stator current to build up the flux. Fig. 6(a) shows how, using this method, the torque can be increased smoothly from 0 to 1.5 pu in 0.20 s, with an initial flux level of 0.50 pu. In this example the limit on  $i_{qs}$  is given by

$$i_{qlim} = 1.25 + 0.75 \cdot \lambda_{dr} \quad (16)$$

and the flux reference is reset at its rated value when  $i_{qs}$  reaches 95 percent of this limit (Fig. 6(b)).

### INFLUENCE OF MOTOR SIZE AND TYPE

As mentioned previously, an important benefit of this type of controller is that performance is unaffected by the value of the machine parameters. However, the minimum interval of time separating two iterations of the adaptive controller is determined by the system response to a step change in flux. This response is influenced primarily by the rotor time constant  $L_r / r_r$ , which increases significantly with the motor rating. Thus, under similar current constraints, the minimization process will take longer for large machines. Similarly, this duration will be shorter for motors having a larger rated slip.

### SUMMARY

An adaptive control strategy for minimizing induction motor losses has been described. This strategy has the following advantages:

- it yields the true minimum in the total system losses, and
- it is robust and does not require a detailed knowledge of the motor parameters.

This maximum efficiency controller relies on the operation at variable flux of a field-oriented drive system. This mode of operation has been analyzed using characteristic loci and digital simulation. It was demonstrated that this type of controller is capable of minimizing the losses of a 7.5-hp drive system in approximately 7 s, with very small transients in torque. A faster convergence could be achieved at the expense of larger disturbances in torque.

The construction of a prototype of this maximum efficiency controller has been undertaken, and the authors hope to report soon on its actual performance.

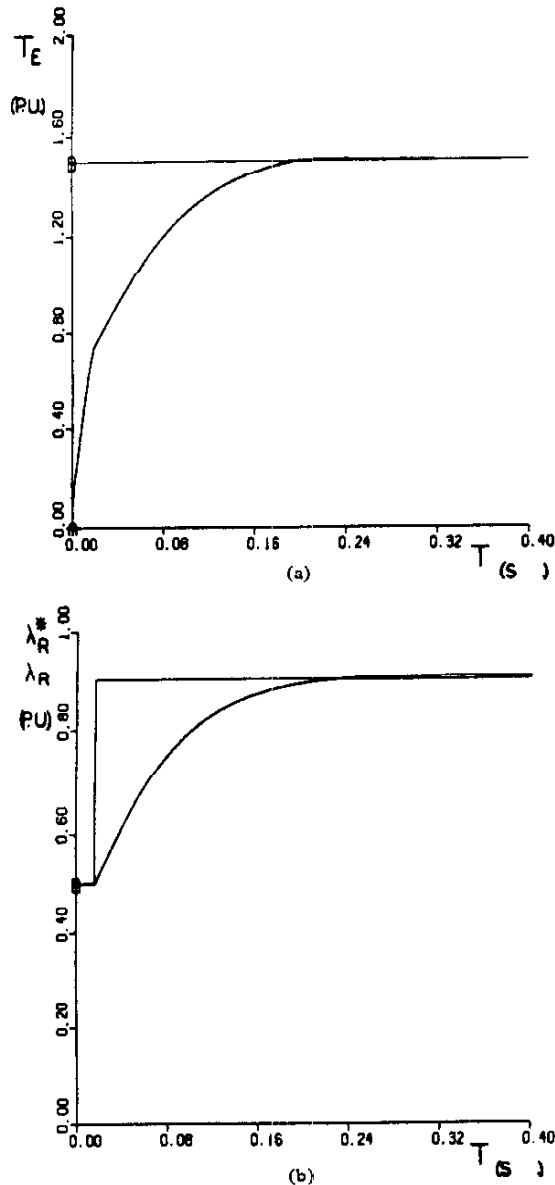


Fig. 6. System response to large step increase in torque reference. System operates at rated speed and initially at 0.5-pu flux and zero torque. (a) Torque. (b) Rotor flux.

#### APPENDIX I

#### INDUCTION MACHINE EQUATIONS EXPRESSED IN A SYNCHRONOUSLY ROTATING REFERENCE FRAME

All quantities are in per unit:

$$v_{qs} = r_s i_{qs} + p \lambda_{qs} + \omega_e \lambda_{ds} \quad (A1)$$

$$v_{ds} = r_s i_{ds} + p \lambda_{ds} - \omega_e \lambda_{qs} \quad (A2)$$

$$0 = r_r i_{qr} + p \lambda_{qr} + (\omega_e - \omega_r) \lambda_{dr} \quad (A3)$$

$$0 = r_r i_{dr} + p \lambda_{dr} - (\omega_e - \omega_r) \lambda_{qr} \quad (A4)$$

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \quad (A5)$$

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr} \quad (A6)$$

$$\lambda_{qr} = L_m i_{qs} + L_r i_{qr} \quad (A7)$$

$$\lambda_{dr} = L_m i_{ds} + L_r i_{dr} \quad (A8)$$

$$T_e = \frac{L_m}{L_r} (\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}) \quad (A9)$$

#### APPENDIX II

#### INDUCTION MOTOR DATA

The nameplate data are as follows: 7.5 hp, 208/220/440 V, 21/20/10 A, 1725 r/min, 60 Hz, three phase, 1.15 SF. Per unit parameters are

$$r_s = 0.023 \quad L_{ls} = 0.1045$$

$$r_r = 0.014 \quad L_{lr} = 0.1045$$

$$r_{m0} = 24.57 \quad L_{m0} = 1.4686.$$

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**Donald W. Novotny** (M'62-SM'77) for a photograph and biography please see page 570 of this issue.



**Thomas A. Lipo** (M'64-SM'71) received the B.E.E. and M.S.E.E. degrees from Marquette University, Milwaukee, WI, in 1962 and 1964, respectively, and the Ph.D. degree in electrical engineering from the University of Wisconsin in 1968. He was an NRC postdoctoral fellow at the University of Manchester Institute of Science and Technology, Manchester, England, during 1968-1969.

From 1969 to 1979 he was an Electrical Engineer in the Power Electronics Laboratory of Corporate Research and Development of the General Electric Company, Schenectady, NY. While at General Electric, he helped pioneer the computer simulation of many types of converter systems including cycloconverters, pulsewidth modulation voltage inverters, current-source ASCI inverters, third harmonic commutated CSI inverters, and load commutated converters. He has also been heavily engaged in the development of algorithms for control of solid-state converter drives for which he has received several IEEE Prize Paper Awards

including corecipient of the 1984 Best Paper Award for the IAS Transactions. He holds six patents with one additional pending. He has published over 60 technical papers, contributing to the analysis and design of a wide range of industrial applications including ac drives for ball mills, pumped hydro, excavators, as well as traction drives for transit cars, locomotives, and off-highway vehicles. He is currently a Professor in the Department of Electrical and Computer Engineering, University of Wisconsin-Madison.

Dr. Lipo is Chairman of the IAS Industrial Drives Committee and serves on the IAS Fractional and Integral Horsepower Subcommittee, the IES Drives Committee and the PES Synchronous Machine Subcommittee, Electric Machine Theory Subcommittee, and Induction Machine Subcommittee of which he is a past Chairman. He has also served on the Program Committee for the IEEE Power Electronics Specialists Conference for the past seven years and was Program Chairman in 1979. He is a member of the Steering Committee for the International Conference on Electrical Machines, an Associate Editor of the journal *Electric Machines and Power Systems*, and Editor of the forthcoming *IEEE Journal of Power Electronics*. He is a member of Pi Mu Epsilon, Eta Kappa Nu, Tau Beta Pi, and Sigma Xi.