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A New Approach to Flux and Torque Sensing in Induction Machines

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A New Approach to Flux and Torque-Sensing in Induction Machines

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Abstract—**A new method of air-gap flux sensing in induction machines is introduced which is based on sensing the voltage across individual motor coils. By subtracting the voltage across two properly located motor coils a signal is obtained which is independent of stator *ir* drop and nearly independent of motor leakage reactance drop. By combining the proposed flux sensing method with conventional current transducers, the electromagnetic torque is readily computed. The method has been implemented in the laboratory and good correlation has been obtained for both sinusoidal and inverter power supplies.**

Introduction

RECENT PROGRESS in the control of ac electric machines concerns to a great extent the evolution of the principle of field orientation. This control concept views the ac currents and flux linkages in the machine as rotating vectors in a two-dimensional space (i.e., a plane). Torque production is viewed as the interaction of the stator current vector and the rotor flux linkage vector. Proper alignment of the stator current vector with respect to the rotor flux linkage vector results in the required value of electromagnetic torque together with a desirable value of air-gap flux. Since the stator current must be oriented with respect to the rotor flux, a key requirement to this control scheme is the computation of the instantaneous location of the rotating flux linkage vector. While this measurement is easily accomplished in the case of a synchronous machine by means of a rotor position sensor, the problem is considerably more difficult with induction machines since the position of the rotor is not directly related to the rotor flux position due to the rotor slip.

The present state of the art utilizes the so-called indirect field orientation in which the rotor flux is located only indirectly by measuring the rotor speed and calculating the rotor slip frequency. This type of approach is inherently inaccurate since it relies on precise knowledge of the rotor resistance and inductance, quantities which continually change with temperature and saturation level in the iron of the machine. The problem associated with locating the rotor flux can be eliminated if the rotor flux, or, alternatively, the air-gap flux, is measured directly. The present state of the art, however, requires the use of either search coils or hall sensors [1], [2].

Search coils are small coils made of very thin wire which are placed around slots of the stator of the machine. The voltage induced in these coils are sensed and then integrated to produce a measure of the flux linking the coil. Flux coils require extra operations in installing the armature winding of the machine and therefore increase the cost. Also, since the coils are formed of very thin wire (approximately 40 gauge), they are subject to breakage due to vibration or continual flexing. Repair of the flux sensors involves removal of the tooth top insulator strip from the stator slots containing the flux coil. This procedure could contribute to insulation failure of the motor coils upon reassembly. Hall sensors are also unreliable since they are very temperature sensitive. Since the armature coils are one of the hottest parts within the machine, placement of Hall sensors in the stator armature has rarely, if ever, been attempted.

A low-cost and reliable means of sensing flux in an electrical machine is the key to solving many of the problems which have plagued the designers of variable frequency induction motor drives. This paper describes a new method which permits the measurement of air-gap flux in a low-cost reliable manner. The implementation of this flux sensing scheme does not involve placing additional search coils in the motor slots or inserting hall probes but uses the coils of the motor themselves as the sensing medium. Hence negligible additional cost is incurred by the introduction of flux sensing. Components used external to the motor involve only low-cost operational amplifiers and transconductance-type multipliers.

Flux Sensing Principle

To describe the flux sensing scheme being proposed, Fig. 1 shows a typical placement of coils around the stator of a typical 36-slot four-pole machine. The windings shown are associated with one of the three motor phases (windings of the other two phases are not shown). This type of winding is called a *double-layer lap winding* and is the most popular pattern for winding ac induction or synchronous machine armatures. Note that the winding which makes up a phase actually consists of a number of individual coils which are connected in series and/or parallel to form a given phase winding. The coils which are located in the same part of consecutive slots are called a *phase belt* and are always connected in series in order to result in phasor addition of the induced voltages.

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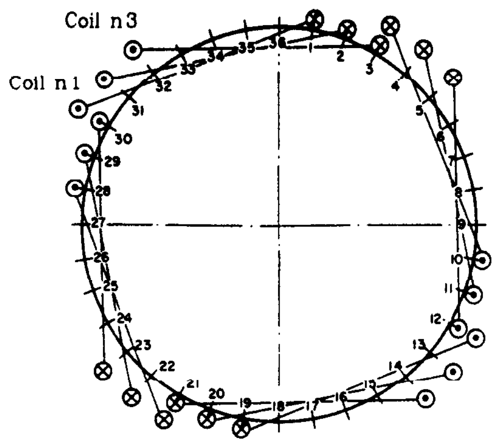

Fig. 1. Coil placement of one phase of double-layer lap winding having 7/9 slot pitch.

It is useful to consider the voltage induced, specifically in coils n_1 and n_3 in Fig. 1. Note that these two coils are, in general, displaced spatially by an electrical angle, say 2ϵ. The voltages induced in these two coils are therefore out of phase by the same angle 2ϵ. The voltage induced in the two coils can be diagrammed with respect to the three normal phase voltages as shown in Fig. 2. In general, the voltage induced in these two coils are normally summed together with the "middle" coil of the phase belt in Fig. 1 to produce a voltage which is in phase with the voltage of phase a on Fig. 2, as shown in Fig. 3. It is useful to consider, however, the result obtained when the voltages induced in these two coils are subtracted rather than added. In this case a voltage is measured which is displaced 90° with respect to phase a. This component is sometimes called the d component in electrical machine analysis. The process of subtracting the voltages enables the measurement of the flux in a particular direction which can be integrated in much the same manner as with search coils to measure the instantaneous air-gap flux in a particular direction.

It is very important to observe that the subtracting process has also eliminated certain parasitic terms which would normally produce error if the flux measurement were taken across the entire phase belt. In general, the voltage induced in a given coil is composed of three components: 1) the desired air-gap flux component, 2) a voltage drop due to leakage flux, and 3) a resistive ir drop due to the fact that the coil has some resistance. However, terms 2) and 3) are dependent only on the current that flows in the coil itself and are independent of the current in some other coil which is mutually coupled to the coil in question. Since the same identical current flows through coils n_1 and n_3 as a result of the fact that the coils are connected in series, the ir drop and the leakage reactance drop cancel and the voltage component which remains is a direct measure of air-gap flux.

Coupled Magnetic Circuit Analysis

Although the discussion of the previous section clearly shows how air flux may be sensed in the sinusoidal steady state, a more rigorous approach is required to demonstrate that the technique remains valid, even under transient conditions.

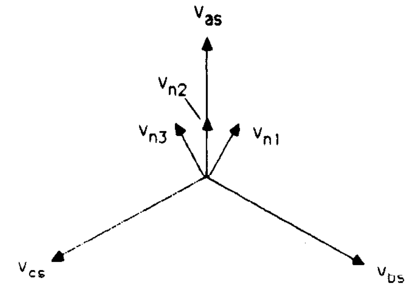

Fig. 2. Phasor diagram of voltage across coils n_1 and n_3 relative to voltages induced in three stator phases.

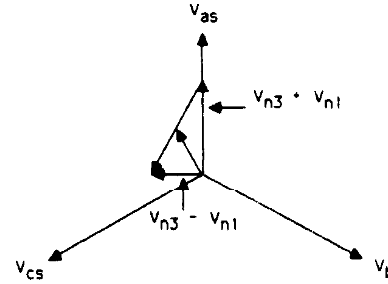

Fig. 3. Sum and difference voltages of coils n_1 and n_3 relative to voltages induced in three stator phases.

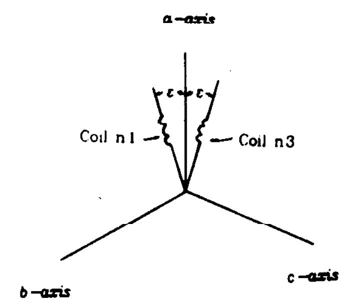

Fig. 4. Orientation of sensed coils relative to three-phase magnetic axes.

Fig. 4 depicts the two coils n_1 and n_3 mutually displaced with respect to the phase a axis by ϵ. In general, the voltages induced into these two coils are, for coil n_1 and n_3, respectively,

$$v_{n1} = r_1 i_{as} + \frac{d\lambda_{1s}}{dt} + \frac{d\lambda_{1r}}{dt} \tag{1}$$

$$v_{n3} = r_1 i_{as} + \frac{d\lambda_{3s}}{dt} + \frac{d\lambda_{3r}}{dt} \tag{2}$$

where λ_{1s} denotes the total flux linking the coil due to the three stator phase currents and λ_{1r} represents the flux linking coil 1 due to rotor currents. Note that since the coils are identical, the resistances of both coils are equal. The voltage difference between coil 3 and coil 1 is

$$v_{n3} - v_{n1} = \left(\frac{d\lambda_{3s}}{dt} - \frac{d\lambda_{1s}}{dt}\right) + \left(\frac{d\lambda_{3r}}{dt} - \frac{d\lambda_{1r}}{dt}\right). \tag{3}$$

It can be noted immediately that the ir term drops out of (3), and therefore a measure of the voltage difference is independent of stator resistance.

It is useful to consider the flux linking the coil from the stator currents and from the rotor currents separately. The flux linking coil n_1 due to stator currents can be separated into leakage components corresponding to flux lines which do not link the rotor and air-gap components resulting from flux which crosses the gap and therefore links the rotor. That is, let

$$\lambda_{1s} = \lambda_{1ls} + \lambda_{1ms} \tag{4}$$

$$\lambda_{3s} = \lambda_{3ls} + \lambda_{3ms}. \tag{5}$$

When the pitch is not unity, the double-layer lap winding results in slots containing coils associated with different phases. Fig. 5 shows the coil placement for the 7/9 slot pitch case of Fig. 3. In this case it can be noted that a "mutual coupling" exists which is associated only with leakage flux components. If L_{lT} and L_{lB} denote the leakage inductances associated with coil sides in the top and bottom of the slot, and if L_{lTB} denotes the "mutual coupling" of coils in the top and the bottom of the slot, the leakage portion of the two coil flux linkages can be written [3]

$$\lambda_{1ls} = (L_{lT} + L_{lB}) i_{as} + L_{lTB} i_{as} - i_{cs} L_{lTB} + L_{le} i_{as} \tag{6}$$

$$\lambda_{3ls} = (L_{lT} + L_{lB}) i_{as} + L_{lTB} i_{as} - i_{bs} L_{lTB} + L_{le} i_{as}. \tag{7}$$

In (6) and (7) the additional term $L_{le} i_{as}$ has been introduced to represent end-winding, belt, and zig-zag leakage components which are assumed not to have "mutual leakage" components.

When (6) and (7) are subtracted, the difference between the leakage flux in the two coils is

$$\lambda_{3ls} - \lambda_{1ls} = (L_{lTB})(i_{cs} - i_{bs}). \tag{8}$$

However, the direct axis voltage in the stationary reference frame is defined by [4]

$$i_{ds}^{s} = \frac{1}{\sqrt{3}} (i_{cs} - i_{bs}). \tag{9}$$

Hence

$$\lambda_{3ls} - \lambda_{1ls} = \sqrt{3} L_{lTB} \sin \epsilon\, i_{ds}^{s}. \tag{10}$$

Ideally, subtraction of the two leakage flux components should result in complete cancellation. The right side of (10) can therefore be considered as a parasitic term. It can be observed from Fig. 5 that this term can be completely cancelled if the winding has a full pitch.

The remaining component of flux contributed by the stator currents is the air-gap component. Since coils n_1 and n_3 are oriented symmetrically about the magnetic axis of phase a by the angle ϵ, the air-gap flux linking the two windings is [5]

$$\lambda_{1ms} = i_{as} L_{m1s} \cos \epsilon + i_{bs} L_{m1s} \cos (\epsilon + 120°) + i_{cs} \cos (\epsilon - 120°) \tag{11}$$

$$\lambda_{3ms} = i_{as} L_{m1s} \cos (-\epsilon) + i_{bs} L_{m1s} \cos (-\epsilon + 120°) + i_{cs} \cos (-\epsilon - 120°) \tag{12}$$

where L_{m1s} denotes the mutual inductance between a single coil and the entire stator phase a winding. Here, it has been assumed that only the fundamental component of the winding distribution produces useful air-gap flux.

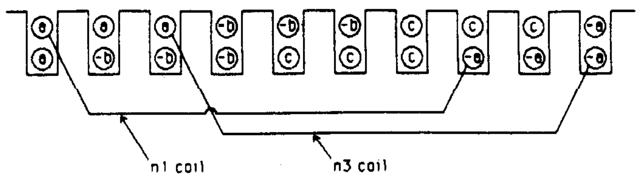

Fig. 5. Coil placement of double-layer lap winding having nine slots per pole and 7/9 slot pitch.

Upon subtracting (12) from (11), the air-gap component of sensed stator flux reduces to

$$\lambda_{3ms} - \lambda_{1ms} = \sqrt{3} L_{m1s} \sin \epsilon (i_{cs} - i_{bs}). \tag{13}$$

However, from (9), this expression reduces to

$$\lambda_{3ms} - \lambda_{1ms} = 3 L_{m1s} \sin \epsilon_{ds}^{s}. \tag{14}$$

The flux which links the two coils due to rotor current components is computed in a similar manner. However, in this case the rotation of the rotor with respect to the stationary coils requires a change in variables. For this purpose it is conventional to replace the actual currents in the rotor bars by equivalent two-phase currents which produce the same fundamental MMF distribution. The location of the magnetic axes of these two-phase currents with respect to phase as is shown in Fig. 6. The flux which links the n_1 and n_3 coils due to currents in the rotor is [5]

$$\lambda_{1r} = L_{m1r} i_{qr}^{\prime r} \cos (\theta_r + \epsilon) - L_{m1r} i_{dr}^{\prime r} \sin (\theta_r + \epsilon) \tag{15}$$

$$\lambda_{3r} = L_{m1r} i_{qr}^{\prime r} \cos (\theta_r - \epsilon) - L_{m1r} i_{dr}^{\prime r} \sin (\theta_r - \epsilon) \tag{16}$$

where L_{m1r} represents the maximum mutual coupling between one of the two sensing coils and one of the two equivalent d–q currents. Upon subtracting (16) from (15) the following result can be obtained

$$\lambda_{3r} - \lambda_{1r} = -2 L_{m1r} \sin \epsilon (i_{qr}^{\prime r} \sin \theta_r + i_{dr}^{\prime r} \cos \theta_r). \tag{17}$$

However, from the d–q equations of transformation, it is possible to relate the rotor d-axis current in the rotor reference frame to the stator stationary reference frame by the transformation equation

$$i_{dr}^{\prime s} = (i_{qr}^{\prime r} \sin \theta_r + i_{dr}^{\prime r} \cos \theta_r). \tag{18}$$

Therefore, (17) reduces to

$$\lambda_{3r} - \lambda_{1r} = 2 L_{m1r} \sin \epsilon\, i_{dr}^{\prime s}. \tag{19}$$

Finally, it can be shown that

$$L_{m1r} = \frac{3}{2} L_{m1s} \tag{20}$$

where the 3/2 term appears because of the conversion of the rotor current from three-phase to two-phase variables. By

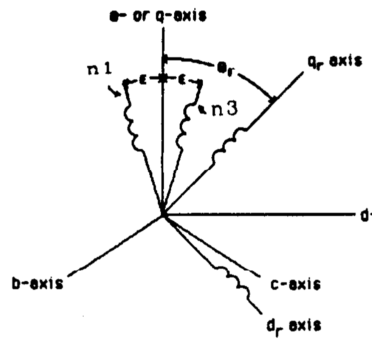

Fig. 6. Orientation of rotor two-phase axes with respect to magnetic axis of phase *as*.

utilizing (10), (14), and (19), the voltage difference between coils 3 and 1 can now be expressed in the form

$$v_{n3}-v_{n1}=\frac{d}{dt}\left[\sqrt{3}L_{lTB}i^{s}_{ds}+3L_{m1s}\sin\epsilon\left(i^{s}_{ds}+i'^{s}_{dr}\right)\right]. \quad (21)$$

It can be shown that the actual air-gap flux linking the d-axis winding can be written as [4]

$$\lambda_{md}=\frac{3}{2}L_{ms}\left(i^{s}_{ds}+i'^{s}_{dr}\right). \quad (22)$$

where L_{ms} is the magnetizing inductance of one stator phase. The air-gap flux linking the magnetic *ds*-axis can therefore be written approximately in terms of the coil voltages as

$$\lambda^{s}_{md}\approx\frac{4}{3\sin\epsilon}\left(\frac{3}{2}\frac{L_{ms}}{L_{m1s}}\right)\int\left[v_{n3}-v_{n2}\right]\,dt. \quad (23)$$

The ratio $(3/2)L_{ms}/L_{m1s}$ effectively corresponds to the ratio of the number of effective stator turns to the number of turns of one coil and is readily calculated from machine data or by simple laboratory test.

The measurement of air-gap flux linkage is apparently in error by the ratio $L_{lTB} \blacktriangleleft L_{m1s}$. Compensation of the term is, of course, possible since stator current is also measured as well as stator flux. However, since saturation probably affects L_{lTB} differently than L_{m1s}, the compensation procedure would become complicated if the saturation level in the machine varied with operating conditions.

Equation (23) is, in effect, a measure of the air-gap flux in the magnetic axis normal to the magnetic axis of the coils which comprise the phase belt. That is, when the voltages of coils 1 and 3 are added (together with coil 2) the voltages induced along the magnetic axis of phase *as* (*as*- or *qs*-axis) is obtained. However, when the voltages of coils 1 and 3 are subtracted, a measure of the time rate of change of air-gap flux in the d-axis (axis normal to the *as*-axis) is obtained. It is important to note that, since L_{m1s} is proportional to the air-gap inductance L_{ms} of an entire phase, saturation of the magnetic circuit will affect L_{m1s} and L_{ms} equally. Hence (23) remains an accurate measure of flux when saturation occurs.

Similar measurements are readily derived for the time rate of change of air-gap voltage along axes normal to the *bs*- and *cs*-axes. In practice, only one additional measurement is necessary since from Gauss' law the sum of the flux lines crossing the air gap must be identically zero. The air-gap voltage of the third axes can therefore be obtained as the negative of the sum of the other two.

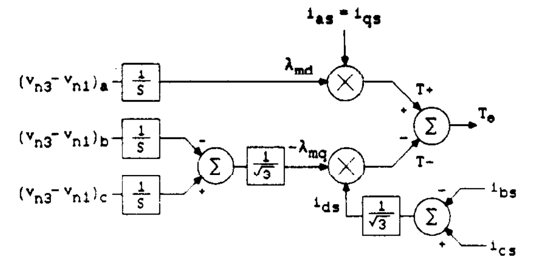

Fig. 7. Implementation of electromagnetic torque from air-gap voltage and stator current measurements.

While the status of the flux in the air gap is an important concern, the most important consequence of implementing a reliable, accurate measurement of air-gap flux is that it opens up the possibility of a direct calculation of the electromagnetic torque. In general, the electromagnetic torque is expressed in terms of the d-q air-gap flux and stator currents by [4]

$$T_e=\frac{3}{2}\frac{P}{2}\left(\lambda_{md}i_{qs}-\lambda_{mq}i_{ds}\right). \quad (24)$$

Fig. 7 shows an implementation scheme for electromagnetic torque employing air-gap voltage and stator current measurements. When (24) is evaluated in terms of the measured flux (23), it is important to observe that the parasitic flux error term represented by $\sqrt{3}L_{lTB}$ cancels completely from the torque expression. Hence, if the motor leakage inductances are constant, the computation of electromagnetic torque is totally independent of leakage inductance as well as stator resistance. In practice, the inductance represented by L_{lTB} may not be identical in each of the three phases at every instant due to saturation, so that a small error term certainly remains. While small, the severity of the error would be of interest, but is, unfortunately, beyond the scope of this study.

Implementation of the Flux Sensing Scheme

The measurement of the voltage across the coils can be easily accomplished by simply bringing out extra wires so that the voltages induced in individual coils can be conveniently accessed with the aid of isolating transformers. It appears that the minimum number of extra leads that must be provided from the machine is three per phase. (Although two leads are required for each of the two coils, one of the leads will have already been brought out as one of the phase terminals.) Installation of these leads must be provided in at least two of the three phases in order to have a complete measurement of the flux location at all instants resulting in a minimum of six additional leads exiting from the machine. While the profusion of leads can be considered as a disadvantage, it is important to mention that the process of tapping at the desired points requires little if any extra time on the part of the motor assembler. In contrast to conventional flux coils, relatively heavy insulated wire can be used so that placement, working of the wires, insulation damage, etc., do not present a problem.

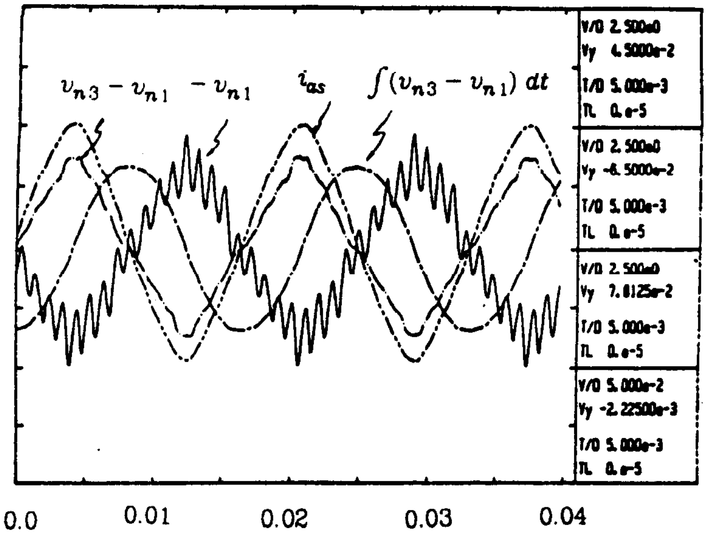

Fig. 8. Flux coil voltage, flux, and current for sinusoidal operation.

Note that the analysis as well as the implementation of the flux-sensing scheme has been carried out for a two-layer four-pole lap-wound machine having 36 slots. The slot pitch of the machine is two slot pitches (7/9 pitch). Clearly, the same principle can be used for any lap-wound machine of any number of layers having a distribution of at least two slots. Machines wound with "fractional-slot" windings, that is, windings with unequal numbers of coils per group, can also be utilized with suitable selection of coils used for flux sensing. It does not appear that the technique is practical for concentric coil configurations since the voltages induced in each coil of the group are in time phase. In general, the principle in applying the flux-sensing scheme is to select the outer two coils of one phase belt as the flux-sensing coils such that the angle ϵ in Fig. 6 is as large as possible.

Implementation of the flux signal and calculation of the electromagnetic torque, according to the flow diagram of Fig. 7, in analog circuitry is straightforward. In practice, it has not been found necessary to filter or further manipulate the air-gap voltage signal in any way. Noise has not been observed to be a problem. All of the analog components needed to compute air-gap flux, and electromagnetic torque can be readily installed on a small rack-mounted printed circuit card.

Correlation with Experimental Results

The flux coil method described has been experimentally tested and verified. In particular, a 10-hp machine has been built to specifications. Fig. 8 shows the signals that are obtained from the coils for a particular case of sine-wave excitation. Note the high harmonic content in the voltage signal of a single coil caused by the rotor slots as they rotate past the stator coil. However, subtraction of the two coil voltages clearly eliminates much of the harmonic content. Integration of the voltage $v_{n3} - v_{n1}$ yields a nearly sinusoidal waveform.

In Fig. 9 is shown a correlation of the calculated torque as computed from the motor coils and stator current with tested results. The measured points noted on Fig. 9 were taken from a spring scale measurement. Similar correlation (not shown) has been obtained with an in-line torque transducer. Note that the correlation is very satisfactory, even at no load when the motor is in saturation. In Fig. 10 the torque–speed measurement is repeated, but in this case a six-step voltage inverter was used, operating at a line frequency of 9.8 Hz. At this point the *ir* drop in the stator coils is roughly twice the leakage inductance drop, and if the measurement were not independent of *ir* drop, an appreciable error would occur. Note that the correlation between calculation and test remains essentially the same as Fig. 9, even in the presence of substantial harmonic voltage content produced by the inverter.

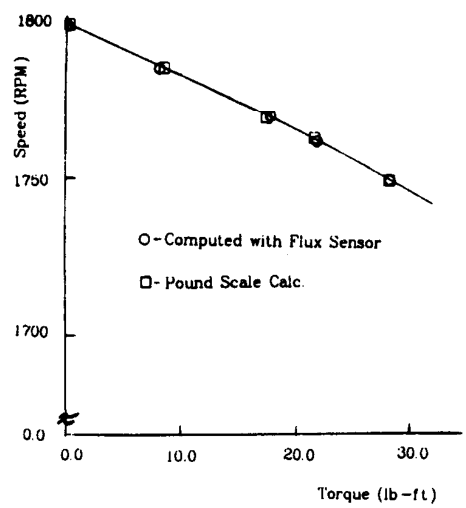

Fig. 9. Comparison of measured and computed torque–speed curves for sinusoidal 60-Hz supply.

Although Figs. 9 and 10 show steady-state correlation, an important feature of this measuring scheme is that it is equally valid for instantaneous flux and torque measurement, i.e.,

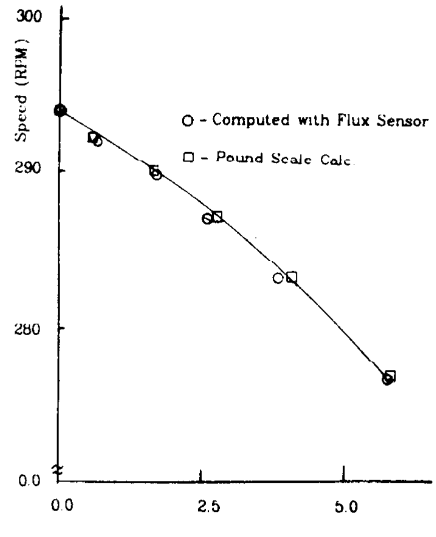

Fig. 10. Comparison of measured and computed torque-speed curves with square wave inverter supply operating at 9.8 Hz.

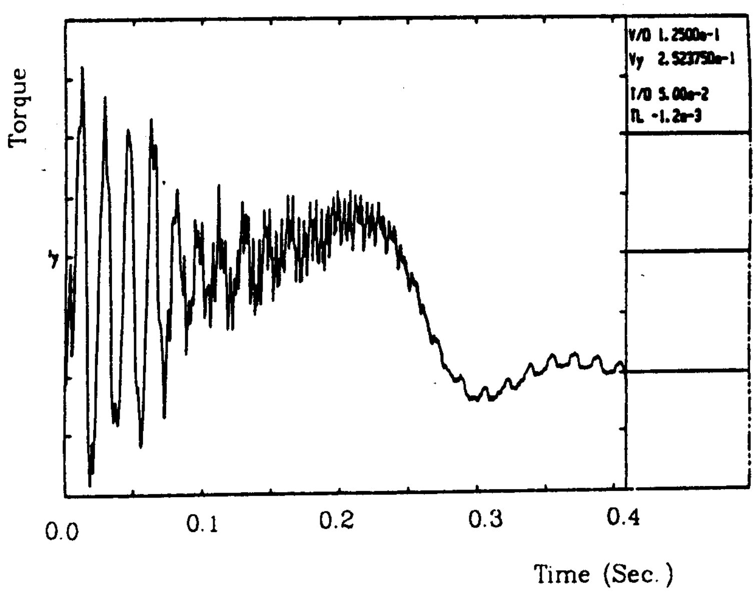

Fig. 11. Measured free-acceleration torque-speed characteristics.

transient. Fig. 11 shows the measured electromagnetic torque for a starting transient when the voltages are suddenly applied to the motor. Note the 60-Hz pulsations of torque which occur. These pulsations are extremely difficult to measure by any conventional means, i.e., with strain gauges or pressure transducers. This result suggests that the torque measurement method described in this paper could also find application as a general-purpose torque transducer.

Conclusion

This paper has presented a new method for measuring air-gap flux which utilizes the coils of the motor itself. The scheme is not affected by *ir* drop in the motor coils and therefore remains a reliable measurement even when the ohmic drop is relatively large. It has been shown that the flux measurement, combined with current transducers, can be used as a high-quality torque sensor. Since the measurement is independent of ohmic drop, the scheme should prove particularly valuable as a replacement for search coils as a flux sensor in a fast-response adjustable-speed drive for which torque sensing at low frequency is critical. Although flux sensing in an induction motor has been the subject of this paper, the approach is equally applicable to synchronous machines and could, for example, also find favor in applications such as LCI synchronous motor drives.

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