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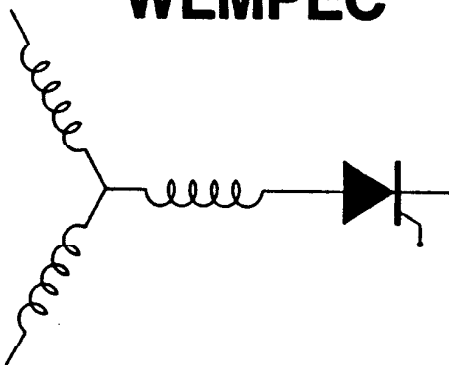
RESEARCH REPORT
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AN IMPROVED MODEL OF SATURATED INDUCTION MACHINES

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ABSTRACT Conventional models of induction machines represent instantaneous saturation of the magnetizing flux by using either saturation factors or by adjusting the tangential and slope reactances. These models are often inaccurate in predicting states of the machine during transient conditions such as on line starting. To remedy these inconsistencies, an improved equivalent circuit model is proposed which accounts not only for the saturation effects in the stator and rotor teeth but independently also accounts for saturation in both the rotor and stator cores. The new model is compared with test results as well as with more conventional models. The new model demonstrates an improvement over other known models showing that detailed representations of saturation effects could be important in induction machine analysis, particularly when the machine is subjected to large signal disturbances.

INTRODUCTION

The prediction of induction machine performance has traditionally been based on constant parameter models which have yielded good engineering results for both steady state and most transient conditions[1-6]. However, the model has frequently not been sufficiently accurate for certain large signal transient conditions such as on-line starting. A perplexing problem concerning these conventional models is that while they may be accurate for predicting certain states of the machine such as in-rush currents, they tend to give higher than measured electromagnetic torque. Since the electromagnetic torque is, in essence, the space vector product of stator current and air gap flux, this inconsistency casts doubt on the validity of conventional saturation models during such conditions.

To remedy these inconsistencies, an improved equivalent circuit model is proposed which accounts not only for saturation in the stator and rotor cores but also independently accounts for saturation effects in the stator and rotor teeth.

In the past, experimental difficulties encountered in the determination of the parameters of more detailed equivalent circuits have discouraged the use of a more detailed induction machine representation. However, this limitation has been removed with the advent of finite element methods which can now be used to calculate the necessary parameters for more advanced equivalent circuits. The modeling approach proposed in this paper which combines the advantage of finite element methods for parameter determination and the simplicity of the equivalent circuit approach makes it attractive in terms of computational time compared to other advanced methods that solve the complete magnetic circuits

at every time step[7].

With the aid of saturation factors obtained by means of finite element methods and confirmed by tests, the equations of the induction machine in the arbitrary reference frame accounting for saturation effects in the stator and rotor cores and the stator and rotor teeth are derived. The model is used to predict the transient performance during a direct on line start of an induction machine both with and without series compensation capacitors. The predicted results are compared with test results and with simulation results obtained from conventional models. The new model demonstrates an improvement over previous models showing that appropriate representation of the saturation effects in the stator and rotor cores and teeth is important in induction machine analysis particularly when the machine is subjected to large signal disturbances.

INDUCTION MACHINE EQUATIONS INCLUDING SATURATION EFFECTS

The magnetomotive forces (MMFs) due to the currents flowing in the stator and rotor circuits give rise to flux linkages the components of which can be aggregated as follows

- (1) The leakage flux linkages in the stator and rotor end winding regions.
- (2) The slot leakage flux linkages of the windings placed in the stator and rotor slots
- (3) The useful (air gap) flux linkages in the stator, rotor teeth and in the airgap of the machine
- (4) The stator and rotor core flux linkages which are algebraic combinations of flux linkages in Eqs. (2) and (3) above.

The magnetic paths through which the magnetomotive drops in the end winding leakage paths, cores, the slot leakage paths, the stator and rotor teeth and the airgap can be represented by reluctances as shown in Fig. 1(a). In this figure the reluctances corresponding to the stator and rotor core and stator and rotor teeth can be considered as saturable. Figure 1(b) is the electrical equivalent of Figure 1(a) including the transformer coupling between the stator and rotor circuits.

Using the principles of topological duality, and the q-d transformation of winding distributions, the q-d electric equivalent circuits of the machine can be derived as shown in Fig. 2 for the motoring convention. It is assumed here that the inductances representing the stator and rotor cores in addition to the magnetizing branch (representing the stator and rotor teeth as well as the airgap) are saturable. The possible saturation effects in the slot iron leakage paths will

i_{qs} and i_{ds} are the q and d currents flowing into the magnetizing branch from the stator,

i_{qr} and i_{dr} represent the q and d currents flowing into the rotor circuit from the rotor terminal,

and i_{qrr} and i_{drr} are the q and d current flowing into the magnetizing branch from the rotor.

The unsaturated magnetizing flux in the d and q axis is given as:

$$\lambda_{mq}(\text{unsat}) = L_m(i_{qrr} + i_{qss}) \quad (21)$$

$$\lambda_{md}(\text{unsat}) = L_m(i_{drr} + i_{dss}) \quad (22)$$

The equations for current, Eqs. (13)-(16) are substituted into Eqs. (1) to (4) to obtain the following flux linkage equations:

$$\frac{p}{\omega_b} \Psi_{qs} = v_{qs} + \frac{r_s}{X_{sa}} (\Psi_{qsc}(\text{sat}) - \Psi_{qs}) - \frac{\omega}{\omega_b} \Psi_{ds} \quad (23)$$

$$\frac{p}{\omega_b} \Psi_{ds} = v_{ds} + \frac{r_s}{X_{sa}} (\Psi_{dsc}(\text{sat}) - \Psi_{ds}) + \frac{\omega}{\omega_b} \Psi_{qs} \quad (24)$$

$$\frac{p}{\omega_b} \Psi_{qr} = v_{dr} + \frac{r_r}{X_{ra}} (\Psi_{qrc}(\text{sat}) - \Psi_{qr}) - \frac{1}{\omega_b} (\omega - \omega_r) \Psi_{dr} \quad (25)$$

$$\frac{p}{\omega_b} \Psi_{dr} = v_{qr} + \frac{r_r}{X_{ra}} (\Psi_{drc}(\text{sat}) - \Psi_{dr}) + \frac{1}{\omega_b} (\omega - \omega_r) \Psi_{qr} \quad (26)$$

The computation of the saturated flux linkages must now be considered. For this purpose the method of saturation factors can be employed [1,3]. The saturated q-d fluxes linkages in the magnetizing branch can be expressed:

$$\Psi_{mq}(\text{sat}) = \Psi_{mq}(\text{unsat}) - \Delta \Psi_{mq} \quad (27)$$

$$\Psi_{md}(\text{sat}) = \Psi_{md}(\text{unsat}) - \Delta \Psi_{md} \quad (28)$$

The changes in magnetizing flux linkages due to saturation are now given as follows:

$$\Delta \Psi_{mq} = (1 - K_m) \Psi_{mq}(\text{unsat}) \quad (29)$$

$$\Delta \Psi_{md} = (1 - K_m) \Psi_{md}(\text{unsat}) \quad (30)$$

where, K_m is the saturation factor which is a function of the resultant unsaturated magnetizing flux given as:

$$\Psi_m(\text{unsat}) = \sqrt{[\Psi_{mq}^2(\text{unsat}) + \Psi_{md}^2(\text{unsat})]} \quad (31)$$

and, $\Delta \Psi_{mq}$ and $\Delta \Psi_{md}$ are the changes in the flux linkage levels from the unsaturated value in the q and d axis respectively.

When the expressions for current in Eqs. (17) to (20) are substituted in Eqs. (21) and (22), the following equations are obtained:

$$\Psi_{mq}(\text{unsat}) = \frac{X_{mm}}{X_{sb}} \Psi_{qsc}(\text{sat}) + \frac{X_{mm}}{X_{rb}} \Psi_{qrc}(\text{sat}) + \quad (32)$$

$$X_{mm} \left[\frac{1}{X_{rb}} + \frac{1}{X_{sb}} \right] \Delta \Psi_{mq}$$

$$\Psi_{md}(\text{unsat}) = \frac{X_{mm}}{X_{sb}} \Psi_{dsc}(\text{sat}) + \frac{X_{mm}}{X_{rb}} \Psi_{drc}(\text{sat}) + \quad (33)$$

$$X_{mm} \left[\frac{1}{X_{rb}} + \frac{1}{X_{sb}} \right] \Delta \Psi_{md}$$

where,

$$X_{mm} = 1 / \left(\frac{1}{X_m} + \frac{1}{X_{sb}} + \frac{1}{X_{rb}} \right) \quad (34)$$

The unsaturated core fluxes linkages in volts are related to currents by:

$$\Psi_{qsc}(\text{unsat}) = X_{sc}(i_{qs} - i_{qss}) \quad (35)$$

$$\Psi_{dsc}(\text{unsat}) = X_{sc}(i_{ds} - i_{dss}) \quad (36)$$

$$\Psi_{qrc}(\text{unsat}) = X_{rc}(i_{qr} - i_{qrr}) \quad (37)$$

$$\Psi_{drc}(\text{unsat}) = X_{rc}(i_{dr} - i_{drr}) \quad (38)$$

In a similar manner as for the magnetizing flux linkage component, the changes in the core flux linkages due to saturation are:

$$\Delta \Psi_{qsc} = (1 - K_s) \Psi_{qsc}(\text{unsat}) \quad (39)$$

$$\Delta \Psi_{dsc} = (1 - K_s) \Psi_{dsc}(\text{unsat}) \quad (40)$$

$$\Delta \Psi_{qrc} = (1 - K_r) \Psi_{qrc}(\text{unsat}) \quad (41)$$

$$\Delta \Psi_{drc} = (1 - K_r) \Psi_{drc}(\text{unsat}) \quad (42)$$

The saturated q-d core fluxes are given as:

$$\Psi_{qsc}(\text{sat}) = \Psi_{qsc}(\text{unsat}) - \Delta \Psi_{qsc} \quad (43)$$

$$\Psi_{dsc}(\text{sat}) = \Psi_{dsc}(\text{unsat}) - \Delta \Psi_{dsc} \quad (44)$$

$$\Psi_{qrc}(\text{sat}) = \Psi_{qrc}(\text{unsat}) - \Delta \Psi_{qrc} \quad (45)$$

$$\Psi_{drc}(\text{sat}) = \Psi_{drc}(\text{unsat}) - \Delta \Psi_{drc} \quad (46)$$

where,

$\Delta \Psi_{qsc}$ and $\Delta \Psi_{dsc}$ are the changes in the d and q components of stator core fluxes due to saturation of the stator core.

$\Delta \Psi_{qrc}$ and $\Delta \Psi_{drc}$ are the changes in the rotor fluxes in the q and d axes due to saturation in the rotor core.

The quantities K_s and K_r are the saturation factors for the stator and rotor cores as functions of the unsaturated stator and rotor fluxes respectively. The unsaturated stator and rotor core fluxes are given respectively as:

$$\Psi_{sc}(\text{unsat}) = \sqrt{[\Psi_{qsc}^2(\text{unsat}) + \Psi_{dsc}^2(\text{unsat})]} \quad (47)$$

$$\Psi_{rc}(\text{unsat}) = \sqrt{[\Psi_{qrc}^2(\text{unsat}) + \Psi_{drc}^2(\text{unsat})]} \quad (48)$$

Substituting for the currents, the core flux linkages in volts can be written as:

$$\Psi_{qsc}(\text{unsat}) = X_{scc} \left[\frac{\Psi_{qs}}{X_{sa}} + \frac{\Psi_{mq}}{X_{sb}}(\text{sat}) \right] + X_{scc} \left[\frac{1}{X_{sa}} + \frac{1}{X_{sb}} \right] \Delta \Psi_{qsc} \quad (49)$$

$$\Psi_{dsc}(\text{unsat}) = X_{scc} \left[\frac{\Psi_{ds}}{X_{sa}} + \frac{\Psi_{md}}{X_{sb}}(\text{sat}) \right] + X_{scc} \left[\frac{1}{X_{sa}} + \frac{1}{X_{sb}} \right] \Delta \Psi_{dsc} \quad (50)$$

$$\Psi_{qrc}(\text{unsat}) = X_{rcc} \left[\frac{\Psi_{qr}}{X_{ra}} + \frac{\Psi_{mq}}{X_{rb}}(\text{sat}) \right] + X_{rcc} \left[\frac{1}{X_{ra}} + \frac{1}{X_{rb}} \right] \Delta \Psi_{qrc} \quad (51)$$

$$\Psi_{drc}(\text{unsat}) = X_{rcc} \left[\frac{\Psi_{dr}}{X_{ra}} + \frac{\Psi_{md}}{X_{rb}}(\text{sat}) \right] + X_{rcc} \left[\frac{1}{X_{ra}} + \frac{1}{X_{rb}} \right] \Delta \Psi_{drc} \quad (52)$$

where,

$$X_{scc} = 1 / \left(\frac{1}{X_{sc}} + \frac{1}{X_{sa}} + \frac{1}{X_{sb}} \right) \quad (53)$$

$$X_{rcc} = 1 / \left(\frac{1}{X_{rc}} + \frac{1}{X_{ra}} + \frac{1}{X_{rb}} \right) \quad (54)$$

Finally, the equation for the electromagnetic torque for the saturated induction machine and the equation of motion can be written in the conventional form:

$$T_e = \frac{3}{2} \frac{P}{2} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \quad (55)$$

$$J \frac{d\omega_r}{dt} = T_e - T_l \quad (56)$$

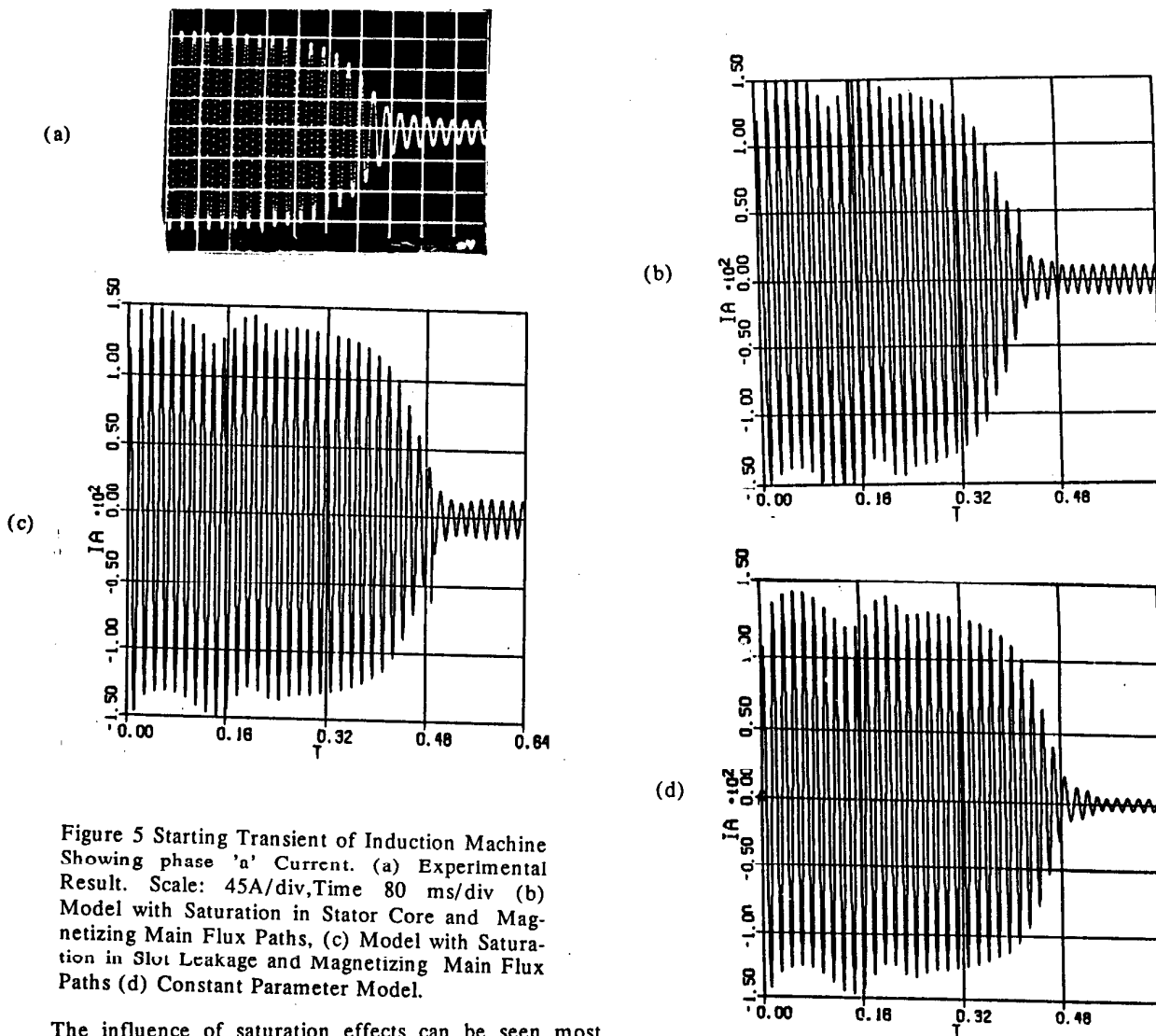


Figure 5 Starting Transient of Induction Machine Showing phase 'a' Current. (a) Experimental Result. Scale: 45A/div, Time 80 ms/div (b) Model with Saturation in Stator Core and Magnetizing Main Flux Paths, (c) Model with Saturation in Slot Leakage and Magnetizing Main Flux Paths (d) Constant Parameter Model.

The influence of saturation effects can be seen most convincingly in the damping of electrical transients. In particular, it appears that saturation is particularly important in situations where the machine has nearly zero damping since the added damping introduced by saturation has a major influence on whether the machine is ultimately stable or unstable. Since small differences in saturation have a noticeable effect on the stability margin, the starting of an induction machine connected to the supply compensated with series capacitors. The condition was chosen to further verify the accuracy of the improved model. Series capacitance values corresponding to successful starting were chosen for the simulation and experiment.

Figures 8 through 11 show the performance of the machine under this operating condition. It is seen that while the machine had a successful start and attained rated steady state speed, the constant parameter model and the model that accounts for saturation in the magnetizing branch and slot leakage path predicted that the machine lapses into sub-synchronous resonance. The improved model, however, gave a prediction that is very close to test results. The improved model predicts that the machine will achieve rated speed operation and has some oscillations in the steady state currents and developed torque which are also evident in the test results. In this case it is quite apparent that the model accounting for saturation in the magnetizing branch and stator leakage paths yields better predictions than the constant

parameter model.

CONCLUSIONS

The saturation of both the stator and rotor core and teeth play an important role in the transient as well as dynamic performance of the induction machine. It has been shown that including both stator core and main flux (i.e. tooth) saturation significantly improves the prediction of starting torque and damping of electrical transients. It is clear that this model requires advanced measurement and/or finite element computations for calculation of the equivalent circuit parameters. Hence, use of this new model on a routine basis is inappropriate. However, the model should be seriously considered for abnormal conditions in which conventional models fail. The model proposed in this paper should also prove to be useful in improved computation of the iron losses of induction machines, a component of loss which plays an important role in the design optimization of high efficiency machines.

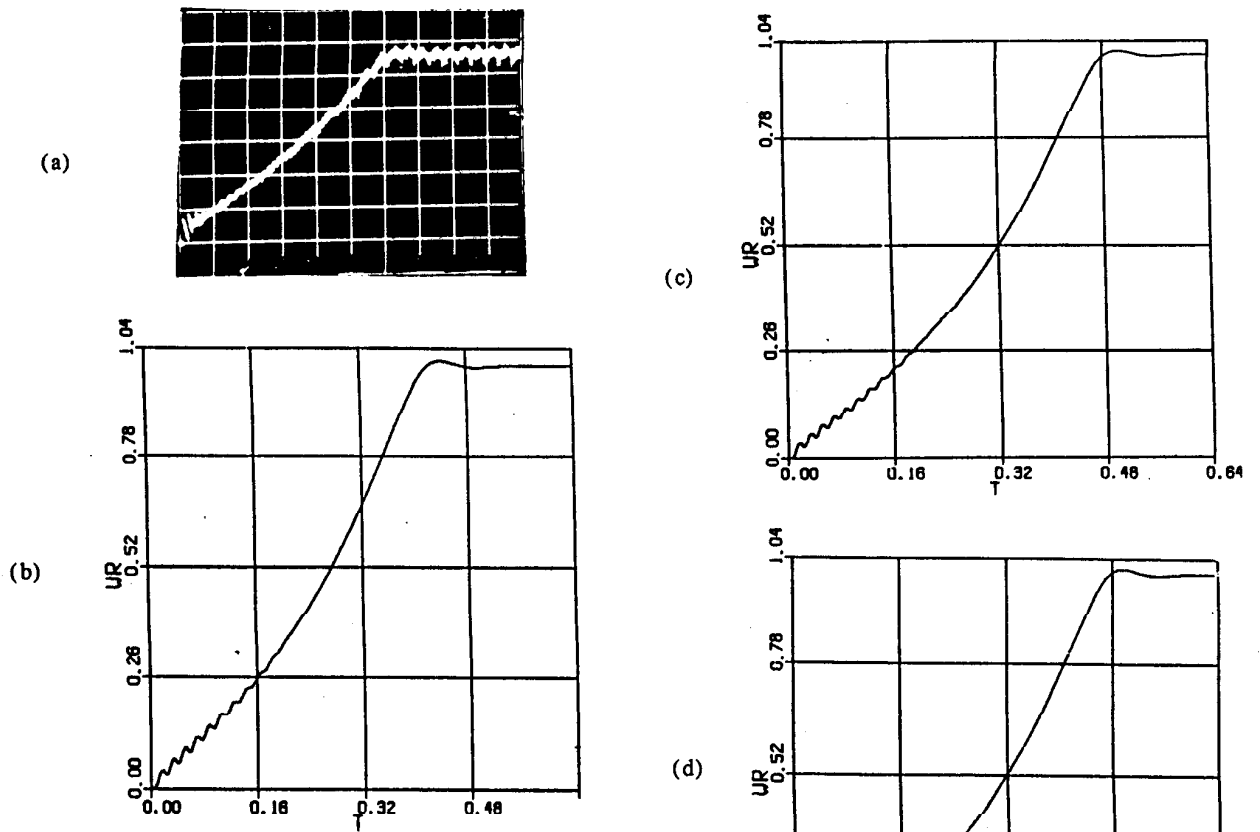


Figure 7 Starting Transient of Induction Machine Showing Speed Run Up. (a) Experimental Result. Scale: Time 80 ms/div (b) Model with Saturation in Stator Core and Magnetizing Main Flux Paths, (c) Model with Saturation in Slot Leakage and Magnetizing Main Flux Paths, (d) Constant Parameter Model.

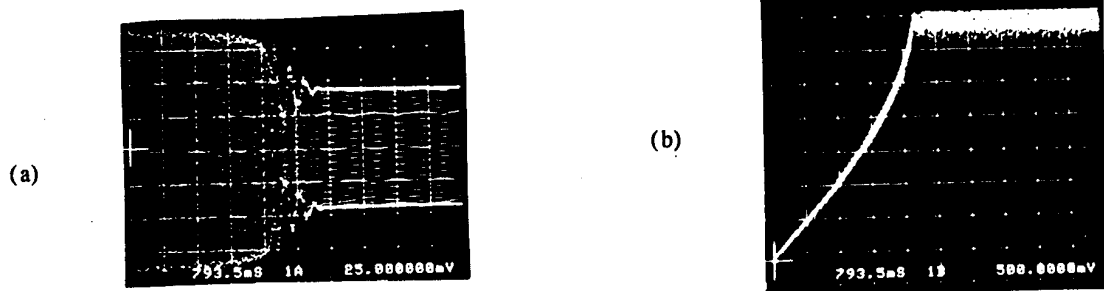


Figure 8 Starting Performance of Induction Machine Fed by Series Compensated lines. Experimental Results. (a) Stator Phase 'a' Current, Scale 25A/div, (b) Run Up Speed in Per Unit.

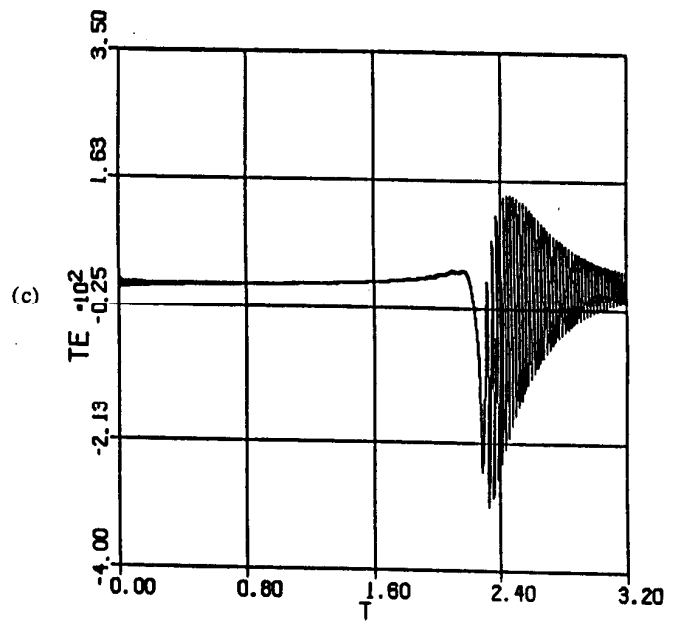
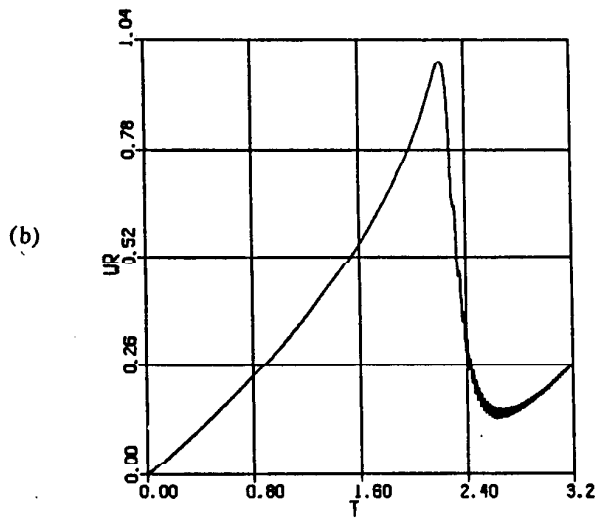
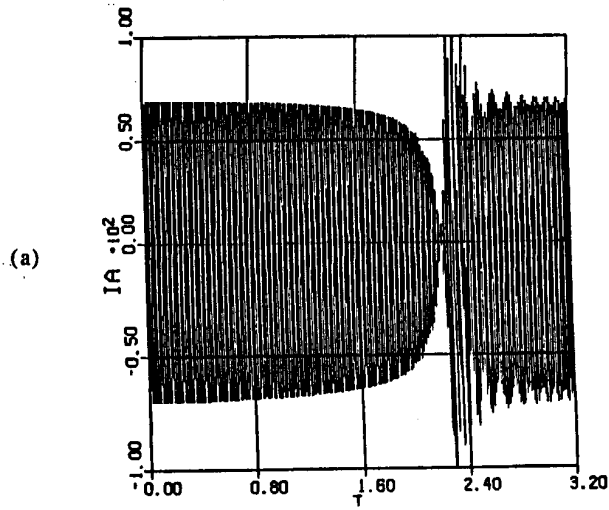


Figure 11 Starting Performance of Induction Machine Fed by Series Compensated lines. Model with Constant Parameter. (a) Stator Phase 'a' Current, (b) Run Up Speed in Per Unit, (c) Developed Electromagnetic Torque.