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SINGLE PHASE INDUCTION MOTOR WITH AN ELECTRONICALLY CONTROLLED CAPACITOR

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Abstract - A single phase induction motor using a new electronically controlled capacitor is described. The system uses a dc capacitor switched by a transistor H bridge. By proper control of the transistor switching the circuit synthesizes a continuously variable capacitance in series with the auxiliary winding. The system could be used to replace standard single phase motor capacitor configurations to provide improved machine performance. Basic system operation, a comparison with conventional motor operation and illustrations of some of the design flexibility inherent in the new system are included.

INTRODUCTION

Standard capacitor-run single phase induction motors use a capacitor in series with an auxiliary winding to produce starting torque and to enhance running performance. These systems have the disadvantage that the capacitor size required for proper system operation is large at locked rotor and much smaller at full speed. To overcome this problem a large capacitor is often centrifugally switched open before the operating speed is reached to give the desired performance. While this switching provides different capacitor values in two speed ranges, improved machine performance requires the capacitance to be continuously varied. An alternative approach, which uses electronic switching to replace the centrifugal switch and to supply a continuously variable capacitance to the machine, is described in this paper.

The intent of the paper is to demonstrate the technical feasibility of the proposed system and to illustrate some of the design trade offs which are possible. After a description of the basic system a comparison with a normal capacitor run motor is presented. The effects of independent adjustments in several system parameters are then considered and the circuit device requirements are described. Both analog and digital simulations are used to carry out the analysis.

SYSTEM DESCRIPTION

The major distinguishing feature of this system is a dc charged capacitor switched by a transistor H bridge. A schematic of the electronically switched system, including the switching modulation scheme, is given in Fig. 1. The H bridge is pulse width modulated using the conventional sine-triangle modulation algorithm using a switching frequency of 1 kHz. The capacitor is charged with a dc voltage, obtained from the motor ac supply, and may be of the electrolytic type.

The transistor switching is controlled by using the machine source voltage V_s as a reference for the modulation. The modulation voltage V_{ref} is synchronized and is phase-adjustable in relation to V_s. With a large enough capacitor the voltage V_{cap} is dc with no appreciable ripple, so the bridge output V_{br} is a pulse width modulated approximation to a sine wave with a fundamental component phase adjustable in relation to V_s. The phase angle between V_{br} and V_s is called the "bridge phase" and is the primary adjustment used to control the transistor switching. By controlling the switching in this manner the bridge output synthesizes a capacitance that may be varied by adjustment of the bridge phase. The size of the dc capacitor in the bridge need only be large enough to limit the ac ripple at the capacitor terminals to an acceptable value.

A secondary adjustment used to control the transistor switching is the magnitude of the modulation reference voltage V_{ref} in relation to the triangle wave peak V_{tr}. This ratio is called the modulation scale factor and symbolized by "a". As will be shown, the effect of this adjustment is to change the dc voltage of the capacitor V_{cap} in relation to the fundamental voltage at the bridge output.

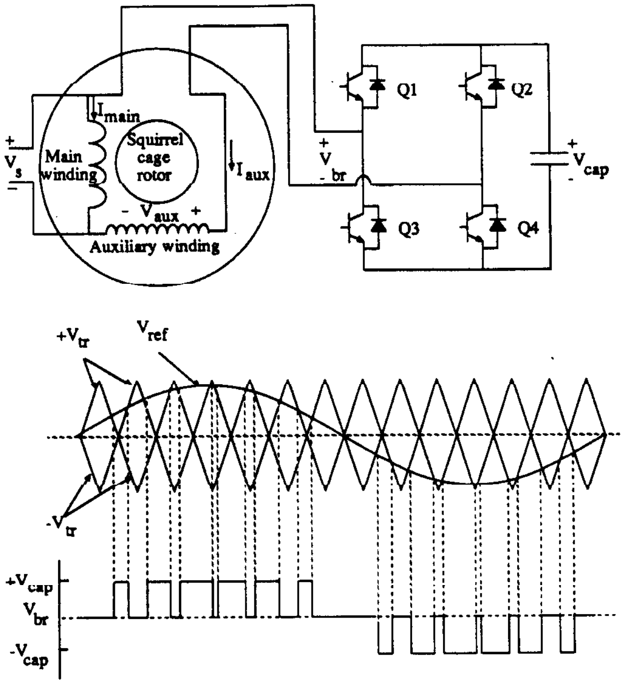

Fig. 1 Capacitor Bridge Run Motor and Switching Modulation Scheme

Theory of Operation

Under steady-state conditions the capacitor bridge run machine operates in the same manner as a capacitor run motor with a variable capacitance. The capacitor bridge circuit supplies an "effective" ac capacitance to the machine which may be changed by adjusting the bridge phase. In order to appreciate the significance of phase adjustment first consider the machine running in a steady state condition. The capacitor bridge circuit only has switches and an energy storage element, the capacitor, and can neither absorb nor generate average power. For this reason, the auxiliary current I_{aux} must lead or lag the bridge voltage V_{br} by ninety degrees. Because the energy storage element in the bridge is a capacitor, I_{aux} leads V_{br} and the bridge acts as a capacitor. Because the capacitor bridge output is capacitive and the phase of the bridge output voltage may be set in relation to the main winding voltage V_s, steady state operation can be related to that of a standard capacitor-run motor.

A capacitor run motor in any steady-state condition has a unique capacitor voltage magnitude and phase angle. A change in capacitor size will give a different voltage magnitude and phase at

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the same motor speed. From a conceptual point of view, the two capacitor constraints (i.e. the fixed ratio of capacitor voltage and current and the 90° phase relationship) combined with the motor constraint equations determine the unique operating point.

In the capacitor bridge run motor there is also a capacitive element in the circuit as a result of the 90° relation between the bridge voltage and the auxiliary current. However, the second constraint is on the bridge phase angle, as set by the modulation phase angle, rather than on the ratio of amplitudes as in a capacitor. These two constraints (i.e. the phase of the bridge voltage with respect to the source and the 90° phase relationship between bridge voltage and auxiliary current) are sufficient to uniquely determine the operating point. In effect, there is only one possible voltage magnitude and "effective capacitance" at the bridge output and the dc capacitor voltage must adjust itself to provide the required bridge voltage.

Thus, the basic mode of operation is to use the phase of the voltage reference for the switching modulation to control the effective capacitance of the bridge. Because the bridge voltage is fixed by machine constraints for any particular bridge phase, changes in the modulation ratio "a" cannot change V_{br}. Instead changing "a" only changes the ratio of V_{cap} to V_{br}, which in turn changes V_{cap}. The modulation ratio can therefore be used to control the dc capacitor voltage but will have no direct influence on steady state machine performance.

Analyzing the capacitor bridge run machine as a capacitor run machine with a variable capacitor, optimal operation of the system can be explored. Under any start or run condition there is some value of capacitance that will yield the highest torque output, running efficiency, or other desirable property. If this value of capacitance is known for all machine conditions, the phase of V_{ref} can be adjusted to give the proper effective capacitance at all times. With this adjustment the motor will run optimally under every condition.

Under a transient condition where either the machine operating condition or bridge switching modulation is changed, the dc capacitor must charge or discharge to a new voltage. Under this transient condition the bridge circuit either absorbs or delivers average power and is thus no longer completely capacitive. Therefore, the auxiliary current does not lead the bridge voltage by 90°. The duration of this transient condition depends on the size of dc capacitor used. However, the capacitor voltage can be controlled by the modulation scaling factor "a". With proper control of "a", V_{cap} can be kept constant through any transient operation of the motor, and the bridge always acts as a capacitive element in the system.

SIMULATION METHODS

The basis for all simulation was the 1/3 hp capacitor run motor with ratings and parameters shown in Table 1. Simulations were performed in which the ac run capacitor was replaced by the capacitor bridge circuit. Initially an analog computer was used for exploratory evaluation of the circuit. Subsequently, the capacitor bridge run motor was verified as equivalent in the steady state to a standard capacitor run motor with an adjustable capacitance. A digital computer simulation of the capacitor run motor was used for steady state analysis by using the run capacitor size as a variable. The results of the analog and digital simulations matched closely and both are used here.

Both simulations were based upon a standard d-q axis model of a symmetrical two phase induction machine in the stationary reference frame [2]. The symmetrical two phase machine was modified by a step-up transformer on the q-axis to give a non-unity auxiliary to main winding turn ratio. The motor parameters were modified to give both windings of the symmetrical machine identical leakage reactance and resistance values for simplicity, corresponding to equal wire bulks for both windings of the capacitor-run motor.

The simulations were run in either an adjustable torque mode to simulate steady state machine operation or an adjustable speed mode to analyze starting conditions. Although the analog computer was capable of transient simulation for the starting condition, detailed analyses were made with the motor speed set to different points in the starting curve to analyze changes in the transistor switching algorithm easily. This approach assumes that the capacitor voltage is controlled to be constant during machine transients.

TABLE 1
Motor Ratings and Parameters

Marathon Electric capacitor run motor

Motor Ratings:	**Motor Parameters:**
Power: 1/3 Hp.	$R_{1m} = 2.89\ \Omega$
Voltage: 115 v	$R_{1a} = 77.0\ \Omega$
Speed: 1100 RPM	$R_2 = 4.02\ \Omega$
Capacitor: 5 µF, 370 v	$X_{1m} = 3.28\ \Omega$
	$X_{1a} = 43.0\ \Omega$
	$X_2 = 3.28\ \Omega$
	$X_m = 47.1\ \Omega$
	$R_m = 600\ \Omega$

Auxiliary : Primary turns ratio = 3.39 : 1

SYSTEM PERFORMANCE

To illustrate the type of control which can be achieved the performance of the capacitor bridge machine is now compared to the the performance of the standard 5 µF capacitor run machine. The capacitor bridge machine was simulated with a large dc capacitor of 100 µF and the bridge phase adjusted for "optimal" performance at each operating point. The maximization of the average torque is considered as optimal for the start conditions and the minimization of the pulsating torque optimal for the run conditions.

Speed-Torque Curve

Figure 2 shows the average torque versus speed plots for the bridge machine and capacitor run machine. The bridge phase angle of the capacitor bridge machine was adjusted at each speed to maximize the average torque output. It is apparent that the capacitor bridge machine has a greater torque output at all speeds. As expected, the difference is large at low speeds where the run capacitor is far too small; the locked rotor torque for the capacitor bridge machine is 3.5 Nm compared to 0.5 Nm for the capacitor run machine. The breakdown torque is about 10% greater for the bridge run machine. At speeds near rated conditions however, the torque outputs of the two machines are nearly the same.

Pulsating Torque

Figure 3 shows the pulsating torque of the capacitor bridge run machine and 5 µF capacitor run machine for all load torques from zero to twice the 2.0 Nm rated load. The capacitor bridge machine operation was simulated with the bridge phase adjusted at each load torque to minimize the torque pulsations. For loads less than 2.0 Nm, the capacitor bridge machine has significantly lower torque pulsations. For larger loads however, the two curves are similar.

THE EFFECT OF CHANGES IN SYSTEM PARAMETERS

The system of the previous section had a properly adjusted bridge phase for "optimal" operation, a large capacitor of 100 µF to avoid any complications produced by ripple in the dc voltage and a fixed auxiliary to main winding turn ratio of 3.4:1. To illustrate the range of options and trade offs inherent in the capacitor bridge machine, a number variations from this nominal system can be considered.

Bridge Phase

The adjustment of the capacitor bridge voltage phase angle is the fundamental control variable for system operation. The effects of adjustments in the phase relationship between V_{ref} and the source voltage on machine operation are considered for one run and one start condition. The capacitor voltage is kept constant to eliminate any transients.

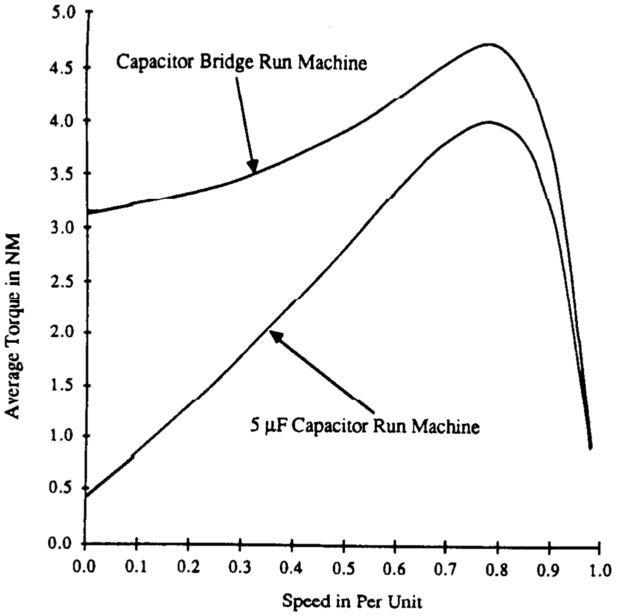


Fig. 2 Average Torque of Capacitor Run and Capacitor Bridge Run Machines - Bridge Phase Adjusted for Maximum Torque

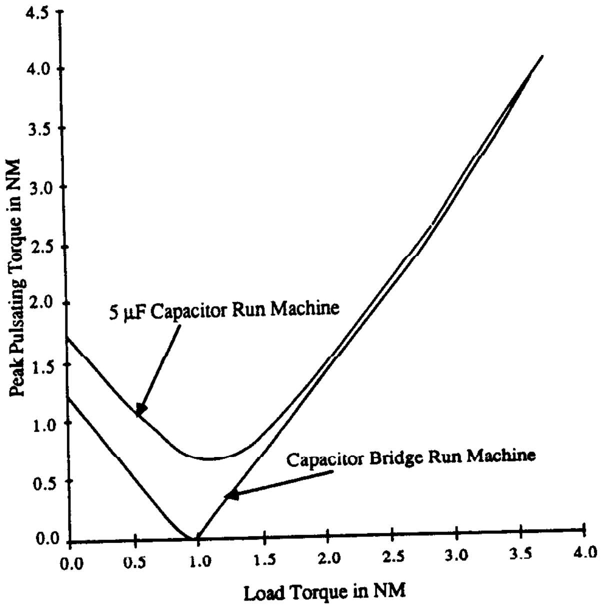


Fig. 3 Pulsating Torque of Capacitor Run and Capacitor Bridge Run Machines - Bridge Phase Adjusted for Minimum Torque

Run Conditions - Figure 4 shows phasor diagrams of the currents and fundamental voltages from an analog simulation of machine operation with the rated load of 2.0 Nm for three different bridge phase angles. Diagram 1 shows the machine voltages and currents for an arbitrarily large bridge phase angle of 90°. The bridge voltage for this case is large, and a large auxiliary voltage and auxiliary current results. The main and auxiliary winding currents are far out of quadrature and the machine is poorly balanced.

At the other extreme, Diagram 3 of Fig. 4 shows simulated machine voltages and currents with an arbitrarily small bridge phase of 60°. In this case, the bridge voltage magnitude is small and the auxiliary voltage and current are also small. While the two currents are nearly in quadrature, the main winding current is much larger than the auxiliary current and the machine is not well balanced.

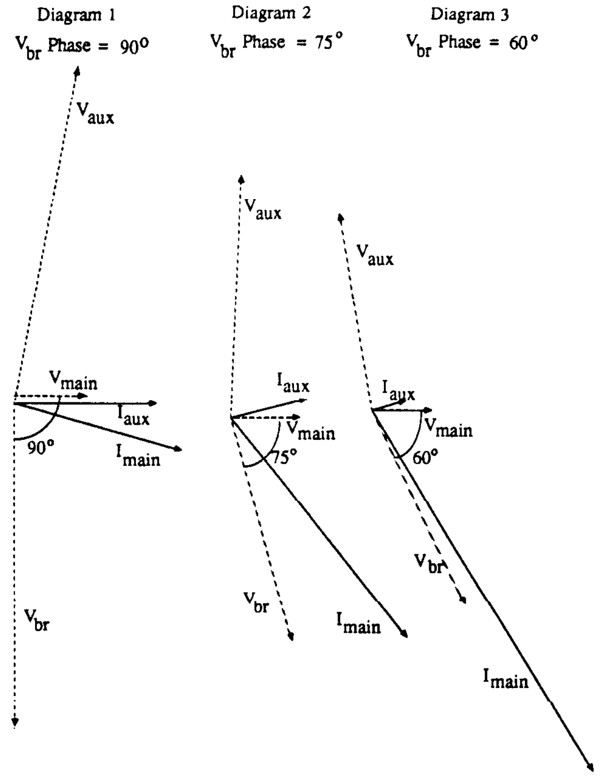


Fig. 4 Phasor Diagrams for Capacitor Bridge Run Machine at Rated Torque for Bridge Phase Angles of 90°, 75° and 60°

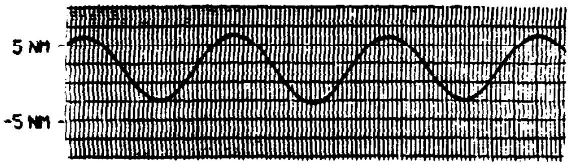
The conditions of Diagrams 1 and 3 are the extremes and the "optimal" condition is shown in Diagram 2. The bridge voltage phase angle is 75°, and the two voltages along with the two currents are nearly balanced. The auxiliary voltage is close to 3.4 (equal to the machine auxiliary to main winding turn ratio) times the size of the main winding voltage, and the two voltages are close to being in quadrature. The currents are related in the same manner. This bridge phase angle was found experimentally to give the most balanced machine operation.

The machine torque waveforms for the three conditions of Fig. 4 are shown in Fig. 5. An unbalanced motor has a double frequency torque pulsation and it can be seen that of the three conditions shown, the bridge phase of 75° gave much lower torque pulsations. The machine slip was significantly different for the three conditions of Fig. 4. The machine run with bridge phases of 90°, 75°, and 60° had slips of 0.05, 0.03, and 0.05 respectively.

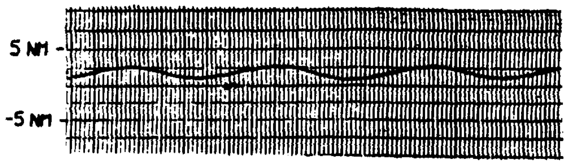
Start Condition - Two phasor diagrams are shown for the motor simulated under the start condition of 10% speed in Fig. 6. Diagrams 1 and 2 show the motor with arbitrarily large and small bridge phases of 83° and 66°. The magnitudes of the voltages and currents are nearly the same for both cases. The main and auxiliary winding currents are closer to being in quadrature for the condition of Diagram 2, and the machine runs more balanced with a slightly greater average torque output. However, the effect of this large change in bridge phase on the machine currents at start up is much less than for the run condition previously described. This suggests that the effect of bridge phase on effective capacitance and machine performance at start up is much less than for the run condition.

Effective Capacitance

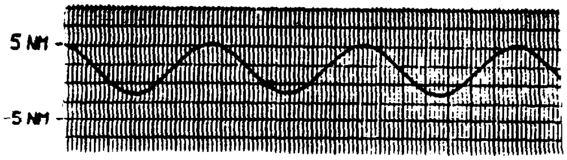
The results above illustrate that the capacitor bridge circuit acts as a capacitive element in the complete motor system. The phasor diagrams of Figures 4 and 6 show that the current into the bridge circuit I_{aux} always leads the bridge output voltage V_{br} by 90°. The effective capacitance at the output of the bridge circuit can be calculated from:



Torque - Bridge Phase = 90°



Torque - Bridge Phase = 75°



Torque - Bridge Phase = 60°

Fig. 5 Pulsating Torque of Capacitor Bridge Run Machine for Bridge Phase Angles of 90°, 75° and 60°

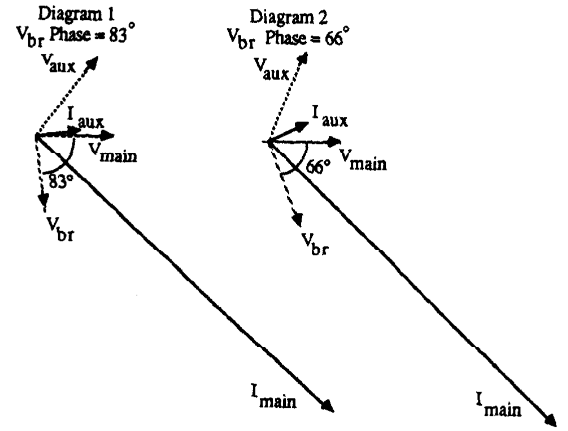


Fig. 6 Phasor Diagrams for Capacitor Bridge Run Machine at 10% Speed for Bridge Phase Angles of 83° and 66°

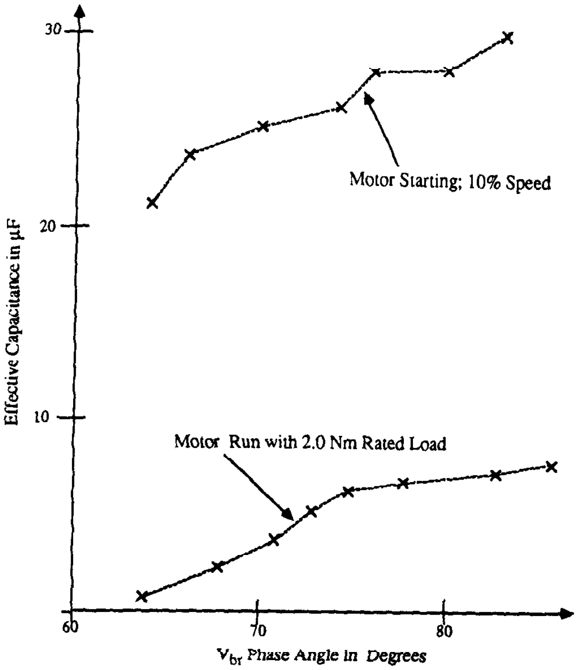


Fig. 7 Effective Capacitance of Capacitor Bridge Circuit

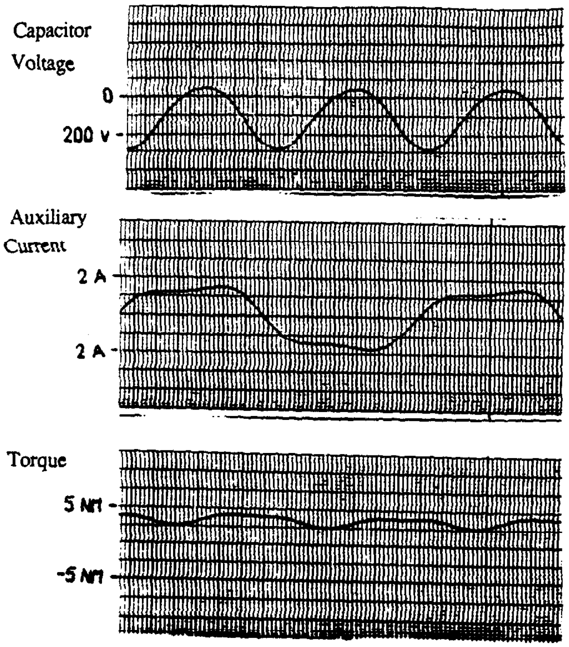
$$C_{eff} = |I_{aux}|/(\omega|V_{br}|)$$

The value of this effective capacitance changes when the bridge phase is adjusted for any machine operating condition. To show the relationship between the bridge phase and effective capacitance, plots of effective capacitance versus the bridge voltage phase angle for simulated machine operation are shown in Figure 7. Plots are shown for both motor operation with rated load and for a starting case at 10% speed.

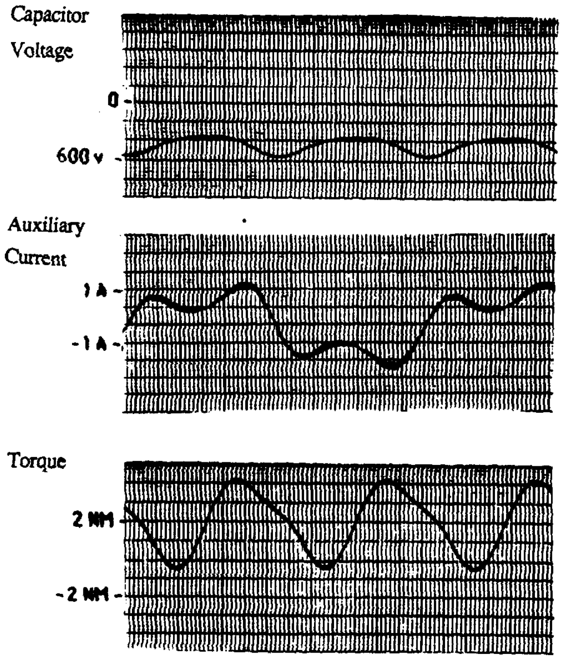
Both plots show that the effective capacitance increases with increases in the bridge phase. However, the effective capacitance was always much larger for the start condition than for the run condition. The plots for all run conditions are nearly identical to that shown for rated load.

DC Capacitor Size

The results shown thusfar were simulated for operation with a large dc capacitor of 100 μF to eliminate any significant voltage ripple. In practice, it is clearly desirable to use the smallest possible dc link capacitor. As the capacitor size is reduced a voltage ripple at twice the supply frequency is superimposed on the dc voltage. This causes the auxiliary current and torque pulsations to become non sinusoidal. Figure 8 shows the capacitor voltage, auxiliary current, and torque for the machine run at rated load and starting at 10% speed with a dc capacitor size of 5 μF. The capacitor voltage ripple is much larger for the starting condition than the run condition. However, the distortion of the auxiliary current and torque waveforms is much greater for the run than the start condition.



a) 10% speed, dc capacitance = 5 μF, bridge phase = 66°



b) rated torque, dc capacitance = 5 μF, bridge phase = 75°

Fig. 8 Effect of DC Ripple on Waveforms

Modulation Scale Factor

Adjustments in the modulation scale factor "a" were confirmed by simulation to have no effect on machine operation. However, they do have important effects on the capacitor bridge operation and the sizing of its components. It is possible to show these effects analytically.

Based on the switching algorithm and the requirement that the instantaneous power flow at the bridge output must must equal the power delivered to the capacitor, the relationships showing the effects of "a" are:

$$V_{br} = a\, V_{cap0} \qquad (1)$$

$$I_{cap} = (a/2)\, I_{aux} \qquad (2)$$

where V_{cap0} is the average value of the dc capacitor voltage, V_{br} is the peak fundamental bridge output voltage and I_{cap} and I_{aux} are the peak values of sinusoids at twice the source frequency and at source frequency respectively. If the peak voltage ripple is now constrained, it can be shown that

$$C_{dc} > a^2\, C_{eff}/(4\, k) \qquad (3)$$

where C_{dc} is the required capacitor value to give a peak voltage ripple that is less than the fraction k of the capacitor dc voltage V_{cap0}. These equations take only the fundamental components into account and are inaccurate when distortion in the winding currents occur with large values of k. "a" is the modulation scale factor and may be calculated as:

$$a = V_{ref} / V_{tr} \qquad a < 0.9 \qquad (4)$$

where V_{ref} and V_{tr} are peak values of the modulation voltages as shown in Fig. 1. These relationships indicate that the value of "a" is critical in determining the dc capacitor requirements.

An equivalent circuit representing the effect of "a" is shown in Fig. 9. This circuit uses a transformer of ratio a:1 at the output of a bridge circuit with 100% modulation (a=1). It may be seen that a reduction in "a" increases the capacitor voltage but decreases the capacitor current and required size.

The transistor - diode switches must have a voltage rating equal to the highest possible capacitor voltage and a current rating equal to the highest possible auxiliary winding current. Lowering the value of "a" increases the required blocking voltage of the switches but does not reduce the current requirements since there is circulating current in the diodes.

In order to eliminate any transient condition that occurs when the capacitor voltage changes, it is advantageous to control "a" so as to keep the voltage V_{cap} constant for all conditions. The equivalent circuit of Fig. 9 shows the advantages in minimizing the capacitor requirements by properly adjusting "a" to charge the capacitor to its highest allowable voltage. For these reasons all calculations for capacitor requirements are made assuming "a" is adjusted for a constant capacitor voltage equal to its maximum allowable value.

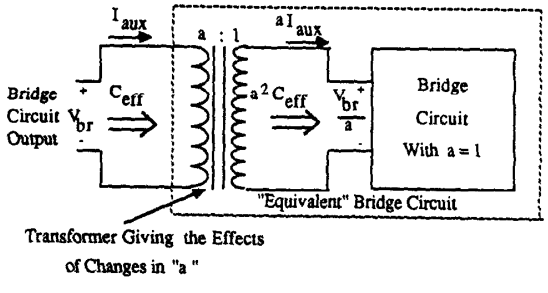


Fig. 9 Equivalent Circuit Illustrating the Effect of the Modulation Scaling Factor "a"

COMPONENT REQUIREMENTS AND MACHINE PERFORMANCE FOR DIFFERENT WINDING RATIOS

In conventional capacitor-run motors, the auxiliary winding has a much larger number of winding turns than the main winding. This large turns ratio allows the capacitor to operate at a higher voltage thus decreasing its current and size requirement. However, the large winding ratio may be shown to decrease the starting torque and increase the torque pulsations at rated load.

In the capacitor bridge system, the effect of the modulation ratio "a" is equivalent to a variable turn ratio transformer at the bridge output. The value of "a" may be reduced to deliver a high voltage level to the capacitor and reduce its size requirement. For this reason, the conventional criterion for the selection of a winding ratio is no longer valid. This suggests that a lower winding ratio will give better machine performance without a significant increase in component cost.

Illustration of Winding Ratio Changes

The required size of the circuit components is strongly dependent on the winding ratio and desired performance of the system. The required dc capacitance is largely determined by the allowable capacitor voltage ripple. This ripple may be allowed to be larger at starting than during running conditions for the same distortion in machine currents. Voltage ripples of 20 percent for starting and 10 percent for running were found experimentally to be tolerable. The transistor switches are required to block the capacitor voltage and conduct the machine auxiliary current.

Table 2 summarizes the simulated machine performance and circuit requirements for both the locked rotor and rated load condition with three winding ratios. The system was simulated in a manner to maximize the locked rotor torque and minimize the pulsating torque under run conditions. The capacitor voltage was maintained at 600 volts for all cases. To maintain this voltage the value of 'a' is much lower at locked rotor than at full speed. The lower winding ratios are characterized by a larger starting torque and lower torque pulsations at full load. However, the component requirements are greater with the lower winding ratios.

CONCLUSIONS

This paper has demonstrated that pwm controlled capacitor bridge circuit is a workable replacement for standard single phase induction motor capacitor systems. The new system has been simulated and results show it to be capable of better performance than an ac capacitor-run motor. The simulations also have shown that:

-The bridge voltage phase angle is the most important adjustment in the system and gives direct control of the bridge circuit effective capacitance.

-The effects of the modulation scale factor "a" are equivalent to the effects of a variable turn ratio transformer at the bridge circuit output.

-There are further advantages in system operation possible by changing the machine auxiliary to main winding turn ratio or other design parameters.

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Table 2

Effect of Winding Turn Ratio Changes on Capacitor Bridge Component Requirements

Bridge Phase Adjusted to Maximize the Locked Rotor Torque

Winding Ratio	"a" for $V_{cap} = 600v$	Vbr (peak)	I_{cap} (rms)	Isw (peak)	Iaux (rms)	Capacitor Size for 20% rip.	Ceff	Average Torque
2.0 : 1	.36	218 V	.76 A	5.92 A	4.20 A	11.7 µF	72.2 µF	5.37 Nm
2.8 : 1	.36	218 V	.39 A	3.02 A	2.14 A	5.96 µF	36.8 µF	3.84 Nm
3.4 : 1	.36	218 V	.26 A	2.06 A	1.46 A	4.05 µF	25.0 µF	3.16 Nm

Bridge Phase Adjusted to Minimize the Pulsating Torque
Load Torque = 2.0 Nm

Winding Ratio	"a" for $V_{cap} = 600v$	Vbr (peak)	I_{cap} (rms)	Isw (peak)	Iaux (rms)	Capacitor Size for 10% rip.	Ceff	Pulsating Torque
2.0 : 1	.59	354 V	.38 A	1.84 A	1.30 A	11.9 µF	13.7 µF	0.66 Nm
2.8 : 1	.76	453 V	.34 A	1.24 A	0.88 A	10.5 µF	7.3 µF	1.44 Nm
3.4 : 1	.88	527 V	.30 A	0.97 A	0.69 A	9.5 µF	4.9 µF	1.80 Nm