A d-q MODEL FOR SIX PHASE INDUCTION MACHINES

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SUMMARY

Six phase induction machines are finding utility high horsepower in variable frequency drive systems. In this paper a d-q model for a six phase machine is derived. In particular, the slot leakage coupling between three phase groups is incorporated into the model. When the equations are specialized to sinusoidal steady-state, it is shown that the usual per phase equivalent circuit results. However, this term is shown to have a significant effect when the machine is driven from a current source inverter.

INTRODUCTION

The past decade has seen the current source inverter (CSI) induction motor system emerge as a strong contender in the adjustable speed drive market. Since regeneration and reversing are inherent, CSI drives are beginning to compete effectively with conventional DC motor drives in the high horsepower range. A limiting factor in pushing up the horsepower rating of a CSI drive indefinitely is the peak commutating voltage, since this quantity fixes the size of the commutating capacitors and the peak inverse voltage of the blocking diodes. At present, a practical limit to the peak commutating voltage is approximately 1500 V. As one means to raise the horsepower limit, attention is turning to multiple groups of inverters directly feeding separate machine windings as shown in Fig. 1.

At present, modeling techniques for such "six phase" machines are not well understood. Because the reactances of the machine essentially cause the high peak commutating voltage of the inverter, it is clear that through understanding of the machine model and its parameters is one key to a successful high horsepower application. A six phase machine can be easily constructed by "splitting" the 60° phase belt into two portions each spanning 30° [2]. This technique is illustrated in Fig. 2a and 2b for a simple machine having six slots per pole. Since the winding distribution factor changes due to the split phase belt, however, the parameters of each winding

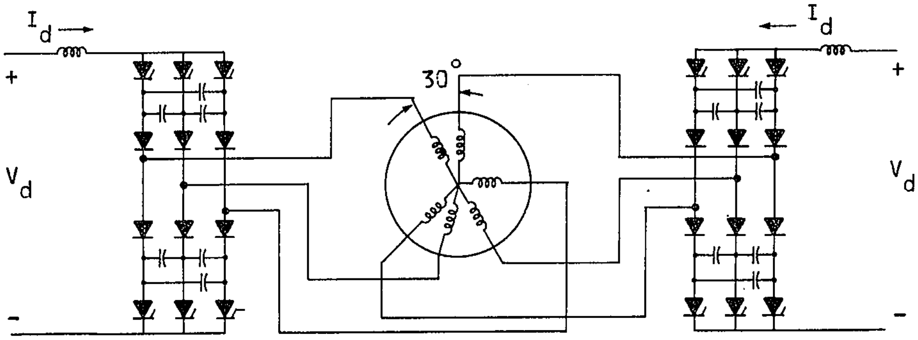


Fig. 1 12 Pulse CSI Induction Motor Drive System

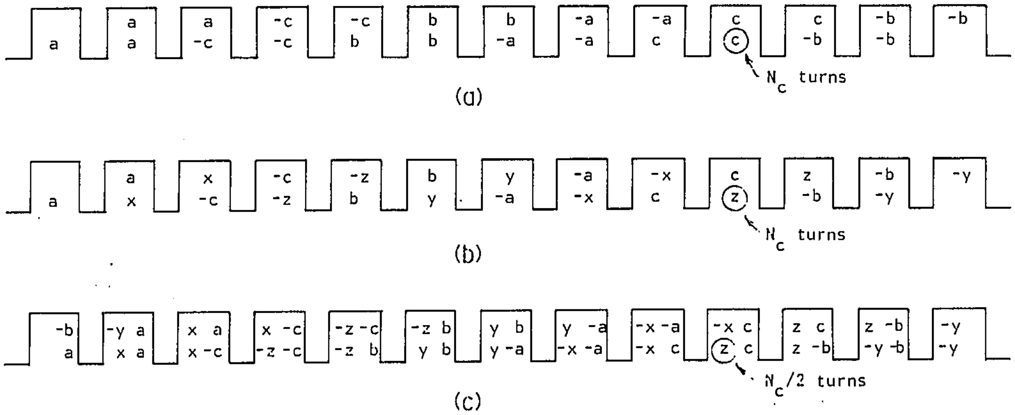


Fig. 2 Winding Distributions for 5/6 Pitch Machine a) Three Phase Machine, b) Six Phase Machine with Split Phase Belts, c) True Six Phase Winding Distribution

are not simply one-fourth that of the three phase machine. A "true" six phase distribution which retains the same winding pitch and distribution factors as the original three phase machine is shown in Fig. 2c. If the turns and copper cross-sectional area per slot is the same as Fig. 2a, then the machine inductances per phase will be exactly one-fourth the phase three case. However, due to the extra coil side insulation required this "true" six phase distribution is generally less desirable than the split phase belt connection.

Since the machine parameters for the split phase belt winding changes from the ideal "one-fourth", it is of interest to determine how the parameters change from the ideal and if they tend to improve or degrade performance. It can be noted from Fig. 2b that the distribution factor increases for the split phase belt connection so that it might be expected that the rating will increase slightly. However, it will be shown that quite the reverse is true due to the effect of slot leakage coupling.

CALCULATION OF SLOT LEAKAGE COUPLING

Let N_c be the turns per coil of a three phase machine with P poles, S_1 stator slots and c circuits. Let P_T be the specific permeance [3] associated with the N_c turns in the top half of the slot, P_B be the specific permeance of the N_c turns in the bottom of the slot and P_{TB} the specific permeance corresponding to mutual coupling between coils in the top and bottom of the slot. Consider first the "ideal" six phase winding configuration shown in Fig. 2c. The slot leakage inductance associated with coils of a given phase in the top half of the slot is

$$L_{\ell T} = \mu_o \left(\frac{N_c}{2}\right)^2 P_T L \left(\frac{S_1}{3c}\right) \tag{1}$$

where L is the stack length, μ_o is the free space permeability, and where P_T includes the tooth top leakage permeance.

Similarly, the leakage inductance associated with coils in the bottom of the slot is

$$L_{\ell B} = \mu_o \left(\frac{N_c}{2}\right)^2 P_B L \left(\frac{S_1}{3c}\right) \tag{2}$$

When the windings are not pitched, the mutual coupling between coils in the top and the bottom of the slot is

$$L_{\ell TB} = \mu_o \left(\frac{N_c}{2}\right)^2 P_{TB} L \left(\frac{S_1}{3c}\right) \tag{3}$$

It is assumed that the phases are pitched so that $2/3 \leq p \leq 1$. The slot leakage inductance of phase a is

$$\begin{aligned} \lambda_{\ell a} = {} & [L_{\ell T} + L_{\ell B} + 2L_{\ell TB}(3p - 2)]i_a - L_{\ell TB}(3 - 3p)(i_b + i_c) \\ & + [L_{\ell T}/2 + L_{\ell B}/2 + L_{\ell TB}\ (3p - 2)]\ (i_x - i_y) \end{aligned} \tag{4}$$

Similar equations apply for the remaining five phases and can be found by symmetry. If the machine is connected without a neutral return it can be shown that Eq. 4 reduces to

$$\begin{aligned} \lambda_{\ell a} = {} & [L_{\ell T} + L_{\ell B} + L_{\ell TB}(3p - 1)]\ i_a \\ & + [L_{\ell T}/2 + L_{\ell B}/2 + L_{\ell TB}(3p - 2)](i_x - i_y) \end{aligned} \tag{5}$$

For the purposes of comparison consider the slot leakage inductance of a six phase machine in which the phase belts have been "split". Again, two three wire, three phase groups are assumed. Since the turns per coil are doubled and the slots per coil are halved then when $5/6 \leq p \leq 1$

$$\lambda_{\ell a} = [2L_{\ell T} + 2L_{\ell B} + 4L_{\ell TB}(6p - 5)]i_a + 2L_{\ell TB}(6 - 6p)(i_x - i_y) \tag{6}$$

When $2/3 \leq p \leq 5/6$

$$\lambda_{\ell a} = [2L_{\ell T} + 2L_{\ell B} + 4L_{\ell TB}(5 - 6p)]i_a + 2L_{\ell TB}(6p - 4)(i_x - i_y) \tag{7}$$

In order to compare the two winding arrangements the inductances are plotted in Fig. 3 as a per unit of $L_{\ell T} + L_{\ell B}$. That is

$$\lambda_{\ell A} = K_s(L_{\ell T} + L_{\ell B}) \tag{8}$$

where K_s is obtained from Fig. 3. Also included in Fig. 3 is the normalized mutual component of leakage to the adjacent 3 phase group expressed as

$$\lambda_{\ell M} = K_m(L_{\ell T} + L_{\ell B}) \tag{9}$$

The quantity K_m contains a factor $\sqrt{3}$ not explicitly shown in Eqs. 7-9. The justification for this factor is described in the next section. In general, the ratio $L_{\ell TB}/(L_{\ell T} + L_{\ell B})$ varies from 0.25 to 0.35. Fig. 3 is drawn for a ratio of 0.3 which is the value for a straight rectangular slot without toothtop fringing.

It can be observed that the leakage inductance of the machine with split phase belts will always be greater than the machine with ideal six phase windings. The penalty ranges for a minimum of 38% at $p = 5/6$ to 250% at $p = 2/3$. Clearly, the pitch should be selected as 5/6 if it is desired to minimize the commutating reactance of the machine.

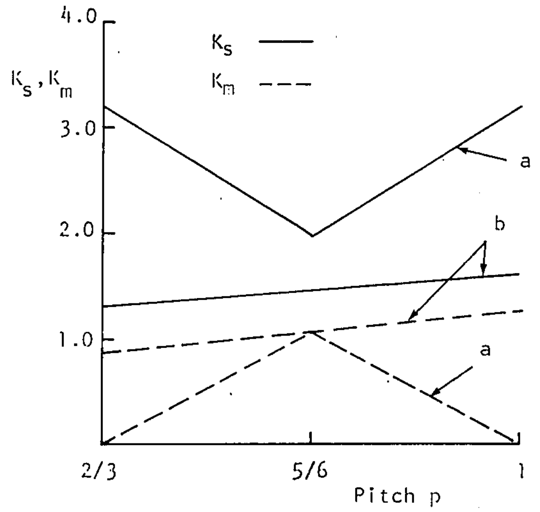


Fig. 3 Normalized Self and Mutual Leakage Inductance Components versus Pitch for $L_{\ell TB}/(L_{\ell T}+L_{\ell B}) = 0.3$,

a) Split Phase Belts, b) Ideal Six Phase Winding

TRANSFORMATION TO A d-q REFERENCE FRAME

It will be assumed here for simplicity that p = 5/6 since values of p do not affect the development. It has been shown that the flux linkage equation for a six phase machine with split phase belts are of the form

$$\lambda_{\ell a} = 2(L_{\ell T} + L_{\ell B})i_a + 2L_{\ell TB}(i_x - i_y)$$
$$\lambda_{\ell b} = 2(L_{\ell T} + L_{\ell B})i_b + 2L_{\ell TB}(i_y - i_z)$$
$$\lambda_{\ell c} = 2(L_{\ell T} + L_{\ell B})i_c + 2L_{\ell TB}(i_z - i_x)$$

and so forth for $\lambda_{\ell x}$, $\lambda_{\ell y}$ and $\lambda_{\ell z}$.

Consider a transformation of variables to a d-q axis fixed in the stator as indicated in Fig. 4. If the neutral currents of the three phase groups are zero, the corresponding equations of transformation are

$$f_{q1} = \frac{2}{3} f_a - \frac{1}{3} f_b - \frac{1}{3} f_c \tag{13}$$

$$f_{d1} = \frac{1}{\sqrt{3}} (f_c - f_b) \tag{14}$$

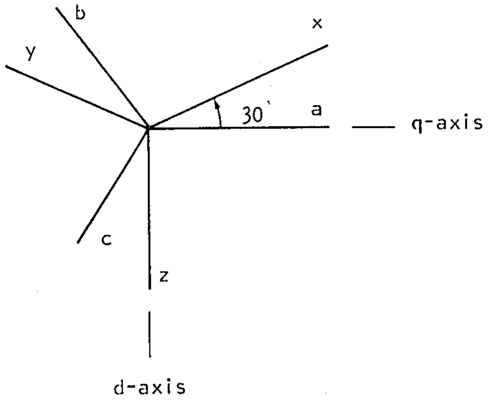


Fig. 4 Orientation of Six Phase Windings to d-q Axes

$$f_{q2} = \frac{1}{\sqrt{3}} (f_x - f_y) \tag{15}$$

$$f_{d2} = \frac{2}{3} f_z - \frac{1}{3} f_x - \frac{1}{3} f_y \tag{16}$$

where 'f' denotes the variables 'v', 'i' or 'λ'. The leakage flux linkage equations reduce to the form

$$\lambda_{\ell q1} = L_{\ell A} i_{q1} + L_{\ell M} i_{q2} \tag{17}$$

$$\lambda_{\ell d1} = L_{\ell A} i_{d1} + L_{\ell M} i_{d2} \tag{18}$$

$$\lambda_{\ell q2} = L_{\ell A} i_{q2} + L_{\ell M} i_{q1} \tag{19}$$

$$\lambda_{\ell d2} = L_{\ell A} i_{d2} + L_{\ell M} i_{d1} \tag{20}$$

where $L_{\ell A}$ is a "self leakage" defined by

$$L_{\ell A} = 2(L_{\ell T} + L_{\ell B}) \qquad (p = 5/6)$$

and $L_{\ell M}$ is the mutual leakage

$$L_{\ell M} = 2 \sqrt{3}\, L_{\ell TB} \qquad (p = 5/6)$$

Although the analysis has thusfar dealt only with slot leakage fluxes the remaining flux linkages of each winding can be handled in a similar manner. In general other leakage components arise from end winding, belt and zig-zag components. With reasonable approximation the end winding leakage components can be assumed to vary as the slot leakage. However, more study is needed to calculate self and mutual components of belt and zig-zag leakage. It is assumed in this paper that these terms contribute only to the self leakage.

When expressed in d-q form the total q-axis flux linkages for the six-phase machine can be expressed

$$\lambda_{q1} = L_{\ell\ell} i_{q1} + L_{\ell m}(i_{q1} + i_{q2}) + L_m(i_{q1} + i_{q2} + i'_{qr}) \tag{21}$$

$$\lambda_{q2} = L_{\ell\ell} i_{q2} + L_{\ell m}(i_{q1} + i_{q2}) + L_m(i_{q1} + i_{q2} + i'_{qr}) \tag{22}$$

$$\lambda'_{qr} = L'_{\ell r} i'_{qr} + L_m(i_{q1} + i_{q2} + i'_{qr}) \tag{23}$$

where

$$L_{\ell\ell} + L_{\ell m} = L_{\ell A} + \text{other "self leakage" components}$$

$$L_{\ell m} = L_{\ell M} + \text{other "mutual leakage" components}$$

Analogous equations apply for the d-axis flux linkages. The transformation of machine voltage equations are handled in much the same manner as conventional three-phase machines. The resulting equations for the q-axis are

$$v_{q1} = r_{s1} i_{q1} + \frac{d\lambda_{q1}}{dt} \tag{24}$$

$$v_{q2} = r_{s1} i_{q2} + \frac{d\lambda_{q2}}{dt} \tag{25}$$

$$0 = r'_r i'_{qr} - \omega_r \lambda'_{dr} + \frac{d\lambda'_{qr}}{dt} \tag{26}$$

Similar equations apply for the d-axis. These equations generate the d-q equivalent circuit shown in Fig. 5. Here, the machine is represented in the stationary reference frame. However it is apparent that the six phase machine model retains the symmetry needed to permit representation in any

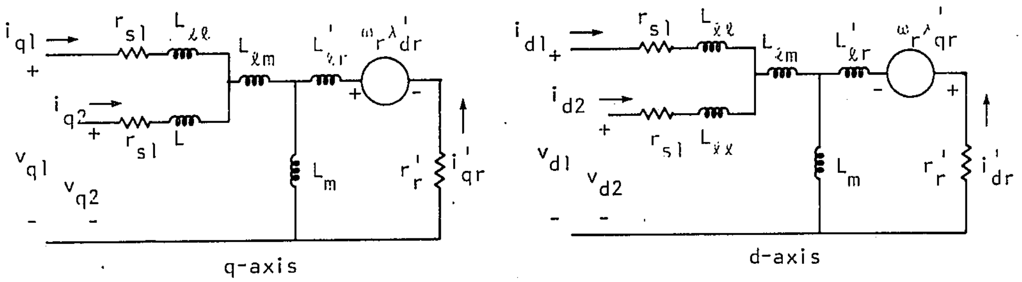


Fig. 5 d-q Axis Equivalent Circuit in the Stationary Reference Frame

real, rotating (i.e. arbitrary) reference frame.

When sinusoidal stator voltages are applied to the terminals of the machine and steady-state is assumed, phasor analysis can be employed on the d-q machine equations. The d-q equivalent circuit reduces to the phasor equivalent circuit of Fig. 6. Note that although the circuit is termed "per phase", in reality the circuit is drawn with two stator circuits, one per three phase group. In the case of balanced, sinusoidal steady-state it is readily seen that the equivalent circuit can be reduced by Thevenin's Theorem to the usual per phase equivalent circuit.

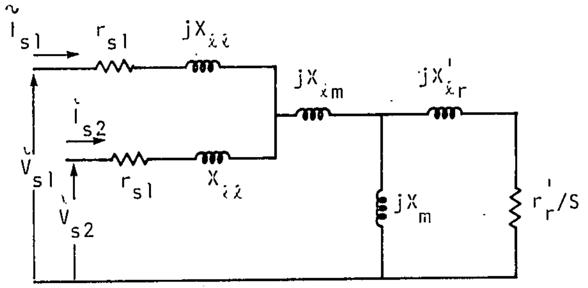


Fig. 6 Equivalent Circuit for Sinusoidal Steady State

Three Phase	Six Phase
$r_s = 0.0035\ \Omega$	$r_{s1} = .0070\ \Omega$
$x_{\ell s} = 0.0110\ \Omega$	$x_{\ell\ell} = 0.00728\ \Omega$
$r'_r = 0.0019\ \Omega$	$x_{\ell m} = 0.00768\ \Omega$
$x'_{\ell r} = 0.0065\ \Omega$	$r'_r = 0.00204\ \Omega$
$x_m = 0.4310\ \Omega$	$x'_{\ell r} = 0.00697\ \Omega$
	$x_m = 0.4620\ \Omega$

Table 1

Parameters of a 920 HP, 460 V, 6 Pole,
45 Hz, 5/6 pitch Induction Machine Wound
for Three and Six Phase Operation

EFFECT OF SLOT LEAKAGE COUPLING ON MACHINE PERFORMANCE

In order to illustrate the effect that six phase operation has on machine performance, the winding configuration of a conventional 920 HP machine with 60° phase belts was modified for six phase, split phase belt operation. The parameters of the identical machine configured for 3 phase at 460 V and 6 phase operation at 460 V is summarized in Table 1. the

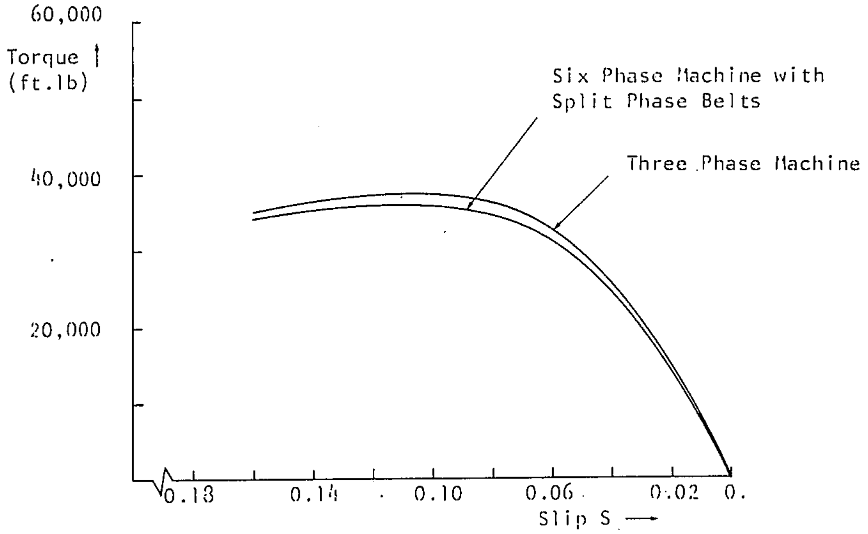


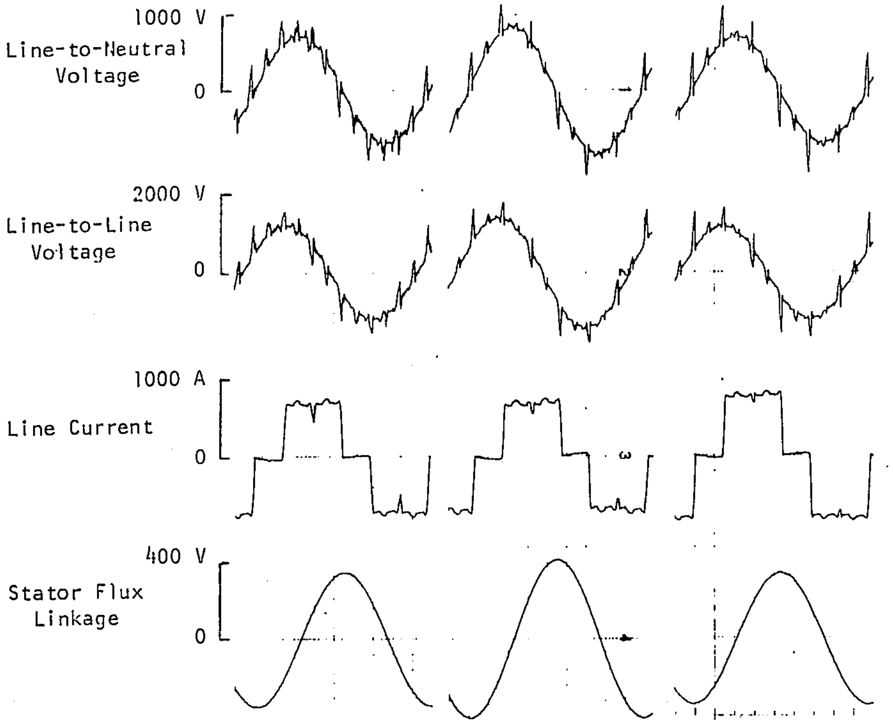
Fig. 7 Comparison of 920 HP Machine for 3 Phase and 6 Phase Operation with Split Phase Belts

torque-speed curves obtained for the same machine with a three and a six phase connection is given in Fig. 7. It can be observed that the torque is slightly reduced over the entire speed range making the six-phase connection slightly inferior to the three phase connection. Although the deficiency in torque could be compensated for by either increasing the voltage or decreasing the number of turns, the torque/KVA ratio would remain the same.

Although the use of split phase belts increases leakage inductance and hence the commutation voltage, an even more pessimistic result will be obtained if the slot mutual coupling of the two three-phase groups is neglected completely. Fig. 8 shows the results of an analog computer study in which the six phase machine and dual three phase converter was modeled in detail [4]. In Fig. 8a the effect of slot flux coupling was incorporated into the simulation whereas in Figs. 8b and 8c this component was neglected. In Fig. 8b the link current was adjusted to match the current obtained in Fig. 8a whereas in Fig. 8c the air gap flux was matched to the air gap flux. It can be noted that the peak commutating voltage obtained from either Fig. 8b or 8c when mutual slot coupling is neglected it is greater than that obtained in Fig. 8a which shows the result when this term is included.

CONCLUSION

Continuing advancement in the state-of-the-art of a CSI converter design will inevitably lead to higher and higher horsepower applications. With today's devices inverter drives in the MW range can be readily accomplished by use of machines with multiple three phase groups. In this paper it is shown that the simplest practical six phase connection using split phase belts results in a penalty of increased stator leakage inductance and consequently increased inverter commutation voltage.



(a) (b) (c)

Fig. 8 Comparison of Six Phase CSI Driven Induction Machine (a) Including Slot Leakage Coupling, (b) Neglecting Slot Coupling, Current same as (a), (c) Neglecting Slot Coupling, Flux same as (a).

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REFERENCES

1. T. A. Lipo and E. P. Cornell, "State-Variable Steady-State Analysis of a Controlled Current Induction Motor Drive," IEEE Trans. on Industry Applications, Vol. IA-11, No. 6, Nov./Dec. 1975, pp. 704-712.
2. R. H. Nelson and P. C. Krause, "Induction Machine Analysis for Arbitrary Displacements Between Multiple Winding Sets," IEEE Trans. on Power Apparatus and Systems, Vol. PAS-93, May/June 1974, pp. 841-848.
3. P. L. Alger, "Induction Machines," Gordon and Breach Science Publ., 2nd ed., New York, NY, 1970.
4. T. A. Lipo, "Simulation of a Current Source Induction Motor Drive," IEEE Trans. on Industrial Electronics and Control Instrumentation, Vol. IECI-24, May 1979, pp. 98-103.