

Torsional Coordination of High Speed Synchronous Motors—Part II

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Abstract—The increasingly frequent use of larger four-pole and six-pole synchronous motors to drive high speed equipment, such as compressors, has focused much attention on coordination of the motor and driven equipment system. A relatively straight forward analytical method of calculating the motor-developed air-gap torque from the d and q axis equivalent circuits is presented. The problem of relating air-gap torque to shaft torque is discussed briefly, and methods of experimentally determining both quantities are presented. A method of assessing the capability of a shaft section to withstand a given torsional duty is also presented.

INTRODUCTION

THIS paper is the second of a series on torsional coordination of synchronous motors. It discusses several aspects of the work involved in determining that the components of a system will coordinate together in a satisfactory manner.

The first paper in this series [1] discussed how differences in the synchronous motor rotor-pole construction, laminated- or solid-iron, can influence the amount of effort required to achieve a satisfactory coordination between a synchronous motor and its driven load. The substantially greater pulsating torque produced by the solid-iron-pole construction during the starting period makes it difficult to avoid excessive shaft responses at speeds where the frequency of excitation coincides with torsional natural frequencies. The savings in the purchase cost of four-pole motors with solid-iron-pole construction over the six-pole motors with laminated-pole construction must be weighed against the additional expense of making more involved analytical studies and the possibly greater lead time necessary to make the coordination studies in advance of finalizing equipment designs.

Responsibility for the coordination work is ordinarily assigned to the manufacturer of the driven equipment. The analytical methods which have been developed and applied to laminated-pole synchronous machines require refinement and greater complexity when applied to solid-iron-pole machines. The development of an analytical model for the

solid-iron portions of the magnetic circuit is not sufficiently advanced at this time to permit the reliable calculation of the pulsating component of motor torque. The matter is confusing because it is possible to calculate the starting current and average starting torque with reasonable fidelity even with fairly crude modeling techniques. The absence of a convenient means of directly measuring air-gap torque has hampered the broad understanding of this important characteristic.

COORDINATION PROCESS

Industry experience with coordination efforts on laminated-pole machines has been acceptable. Only in rare instances has it been necessary to make extensive analytical calculations. The majority of synchronous motors are applied with nominal amounts or no analytical work being expended. The vast majority work quite successfully without experiencing broken shafts or damaged gears. When failures have occurred and an investigation made into the cause of the failure, some indication of the potential for such failure can frequently be identified. These experiences have been incorporated into the coordination process.

The complete coordination process consists of several phases which occur sequentially, and it is often necessary to return to earlier phases as a result of findings in the latter phases. Those phases are as follows.

1) Develop equipment designs—The basic specifications to which the equipment is to be designed are produced. Process requirements are converted to power ratings, operating speed, overload requirements, starting torques, load inertias, design life, operating cycles, etc. The electrical power distribution system to which the drive is to be connected will establish voltage and frequency rating, starting current limitations, etc.

2) Formulate study models—The system of rotors, shafts, couplings, and gears is used to define a torsional spring-mass model to represent the actual system. The degree of complexity of the model is often reduced to the least complex form that gives the essential features of the system. The operating conditions to be studied are defined in terms of electrical circuit conditions, which can be related to air-gap torque produced in the motor, and process flow conditions, which can be related to the load torque required by the driven load. When the torsional study is directed towards the starting condition, the power system voltage and system impedance as viewed from the motor terminals should be established. Normal and abnormal situations should be identified, as the abnormal condition may be so severe as to offset the fact that it will occur infrequently.

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3) Gather data and develop parameters for the study model—This requires that shaft sketches, rotor weights and geometries, coupling characteristics, and gear information on the tentative equipment designs be available. If the equipment component designs are not well-developed, the degree of approximation in parameters may make the data of little value except for some sort of screening purpose.

4) Determination of equipment responses—The spring-mass model is studied by means of calculations or simulation studies to estimate the responses of the actual equipment to the study conditions. This phase is what is often meant by use of the phrase “torsional study.” The result expected from this phase is an identification of the magnitude of peak torques and the number of cycles with which they occur in the various shafts during each starting event.

5) Evaluation of equipment to withstand expected duties—The manufacturer of each component of the system should be advised of the results of the torsional study and his comments solicited as to the effect this duty will have on the performance of the equipment which he is producing. More useful comments will be obtained if the information provided is clearly stated, is restricted only to those portions that apply, and does not alter the contractual arrangements by which the equipment was purchased. It is helpful to know in advance how each supplier will evaluate the information provided. For example, the motor supplier will be concerned about shaft responses which exceed the maximum or pullout torque capabilities of the machine, and about an excess number of torque peaks resulting from the required starting duty. The gear manufacturer will be greatly concerned about any possibility that torque reversals in the gear unit will load the back flank of the gear face, inasmuch as the localized stress in the gear mesh can exceed the corresponding external stress induced in the shaft extensions on the gear unit.

Very often this evaluation identifies a need for modification of the equipment design which returns the study process to phase 1) for a second or third iteration. If more than one independent investigator has been involved in assessing the equipment design, it is usually at this point that differences in conclusions and recommendations from each of the reporting parties must be reconciled to the satisfaction of the equipment owner/operator who will bear the consequences of any failures which may occur. In some instances, the principal benefit of the analytical work will be an aid in planning for maintenance or operating contingencies rather than in providing an iron-clad assurance against failure.

6) Experimental confirmation of analytical methods and results—In the past 25 years, numerous reports have been published in the technical literature of torsional analytical studies made on equipment which was subsequently instrumented to measure actual shaft torque. Invariably, the investigator has reported satisfaction with the degree of correlation between the analytical and experimental results obtained. There are a few published references which document work undertaken as a result of failures that were experienced in service. For example, Misage [2] reported on the analysis of a series of shaft failures in motor generator (MG) sets driven by synchronous motors. He attributed the failures to exces-

sive torsional response during starting as a result of the torque pulsation characteristics of a synchronous motor designed for low starting current (450 percent of rated). His solution was to redesign the motor for a more normal starting current (600 percent rated). More often the experience goes unreported and any insight that might have been useful to the industry is retained only by the investigator.

AIR-GAP TORQUE DETERMINATION

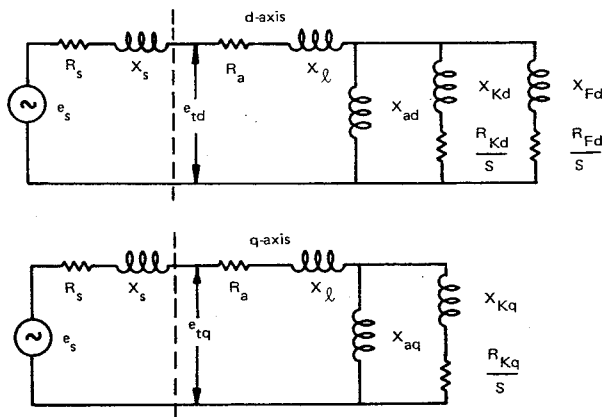
Electric machines are designed to convert energy from electrical to mechanical form and vice versa. The electro-mechanical conversion process occurs in the coupling fields which exist in the air gap between the stator and rotor. The air gap is the interface between the electromagnetic and mechanical portions of the system but it is not readily accessible for measurements of all variables of interest. Discrepancies between calculated and actual values of variables are more likely to be recognized when the variables are measured directly than if their values are inferred by observing the effect of a related variable on the variable of interest.

The lack of ability to make simple measurements has hampered a broader understanding of the actual nature of the torque, which is developed in the air gap of the machine. Motor characteristics, such as starting current and average torque, are not often subject to large discrepancies between calculation and test. Both variables can be calculated in an approximate fashion by solving the conventional synchronous motor two-axis equivalent circuits of Fig. 1 separately and taking the average of the two independently determined axis quantities. Concordia discusses this technique in [3], and (A43) and (A48) in the Appendix prove the validity of his technique.

Test results tend to confirm calculations made in this manner, as the error between the approximate and the more theoretically correct methods is generally small. By contrast, the pulsating torque characteristic has been the subject of greater disagreement and controversy. When the independent axis approach, which gives good results in calculating average torque, is extended to the calculation of pulsating torque, errors have arisen which substantially underestimate the amount of pulsating torque. When the pulsating torque is equated to the difference between the torques calculated from independent d and q axis equivalent circuits, the result will understate the amount of pulsating torque (See Appendix, (A43)).

DETERMINATION BY CALCULATION

When the motor designer is requested to provide information about the average and pulsating torque developed by a synchronous motor, it has been the industry practice for many years to provide that information in the form of curves, which are plotted as functions of speeds. In addition, the effect of motor terminal voltage on each characteristic has been indicated in some manner. In the future, it is likely this practice will continue to be satisfactory for the routine applications, particularly where there have been similar successful prior applications.



$HP_B = 4500$	$X_l = .0932$	$R_a = .0051$	per unit
$kVA_B = 3435$	$X_{ad} = 1.28$	$R_{Kd} = .085$	
$kV_B = 4.00$	$X_{aq} = .770$	$R_{Fd} = .001$	
$N_B = 1800$	$X_{Kd} = .096$	$R_{Kq} = .032$	
	$X_{Kq} = .115$		
	$X_{Fd} = .1838$		

$e_s = 1.00$ per unit

e_t	R_s	X_s
1.00	0.0	0.0
.80	.003	.0458
.50	.003	.1835

Fig. 1. Direct and quadrature axis equivalent circuits for typical synchronous motor with external source impedance.

The special or high risk applications can now be analyzed by means of a computer-based simulation study. A computer simulation that models electrical currents and magnetic fluxes can more nearly represent actual transient torques than more simple methods. Simulation techniques, although extremely important analytical tools, are not necessary to this discussion and so will not be pursued further at this time.

The equations developed in the Appendix should be studied to gain some insight into the effect that the power system is likely to have on the manner in which the torsional system is excited. Reduced voltage starting is sometimes considered as a means to reduce the amount of pulsating torque produced by the motor. Unfortunately, the average torque is also reduced in direct proportion to the reduction to pulsating torque—both being proportional to the square of the motor voltage.

There are two commonly used reduced voltage starters: the reactor type (line or neutral), and the auto transformer type. If one approach could be shown to reduce the ratio of the pulsating torque to the average torque as the motor voltage is reduced, that type of starter would have a definite advantage over the other approach. Inspection of (A43) and (A51) shows that if only the applied voltage is changed, such as occurs with an auto transformer starter, both the pulsating and average torques are reduced in proportion to the square of the ratio of the applied motor voltage to the motor rated voltage. If there is an impedance external to the motor in the power system, it will most likely be a balanced three-phase impedance and will tend to dilute some of the asymmetry of the rotor portions of the motor circuit. The reactor starter, which

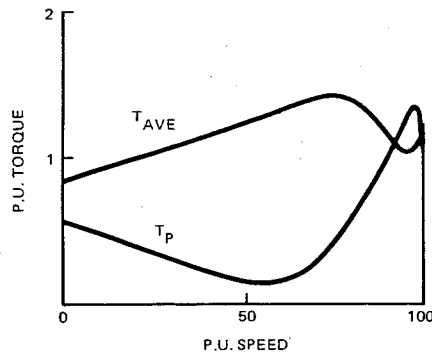


Fig. 2. Calculated speed-torque characteristics, full voltage, of typical synchronous motor.

intentionally adds impedance external to the motor to obtain the reduction in voltage, might result in some dilution effect, but unfortunately, the effect is small for any reasonable starting voltage, especially when the motor is operating below half speed.

To illustrate the actual magnitudes, calculations of pulsating and average torque for a typical motor were made in accordance with the method shown in the Appendix. Fig. 1 shows the d and q axis equivalent circuit and the values of parameters for a typical motor connected to a power system with external impedance. The full voltage motor characteristics during starting are plotted in Fig. 2. Table I tabulates a few of the results as a function of motor speed and external impedance.

Parametric studies on the equivalent circuit will reveal the importance of the amortisseur winding to the amount of pulsating torque developed by the synchronous motor in the speed range which is usually of interest. Salient-pole synchronous motors inherently have torque pulsations that can not be eliminated from the motor design. The laminated-pole construction gives the motor designer considerable freedom to tailor the resistances and reactances in the amortisseur winding to reduce pulsating torques. With solid-iron-pole construction, the designer must accept the relatively fixed values of amortisseur characteristics. Therefore, where large four-pole synchronous motors with solid-iron poles are selected as drives, the systems must be designed to accept considerably larger pulsating torques than where six-pole motors with laminated poles are selected.

DETERMINATION BY MEASUREMENT

The average torque can be determined by conducting a free acceleration test in accordance with IEEE Standard 115, *Test Code for Synchronous Machines* [10]. This is a routine test, which depending on the care taken in making the test, gives quite acceptable accuracy.

The pulsating torque is not so easily measured. There are three methods available that are proposed when the need arises.

- 1) Instantaneous kilowatts measured at the stator terminals is the simplest method to implement. Unfortunately, this method is subject to operating ranges, where it is badly in error. There are starting speed ranges where the instantaneous stator kilowatts correlate quite well with instantaneous air-

TABLE I
COMPARISON OF TYPICAL SYNCHRONOUS MOTOR CHARACTERISTICS WITH VARIOUS REACTOR-TYPE
REDUCED VOLTAGE STARTER SETTINGS

Speed	100 Percent V				80 Percent V				50 Percent V			
	I_L	T_{AVE}	T_p	T_p/T_{AVE}	I_L	T_{AVE}	T_p	T_p/T_{AVE}	I_L	T_{AVE}	T_p	T_p/T_{AVE}
0	5.56	0.894	0.492	0.550	4.43	0.565	0.313	0.554	2.76	0.217	0.121	0.559
50	5.07	1.271	0.175	0.138	4.13	0.842	0.116	0.138	2.65	0.347	0.048	0.138
60	4.88	1.359	0.138	0.101	4.00	0.916	0.093	0.101	2.44	0.387	0.039	0.100
75	4.42	1.461	0.457	0.313	3.70	1.027	0.319	0.311	2.48	0.463	0.144	0.311
90	3.51	1.210	1.081	0.894	3.03	0.954	0.821	0.861	1.76	0.534	0.427	0.800

gap torque and can provide useful data for a modest test effort. There are other speed ranges where the two variables are not correlated. The two most obvious examples are rather extreme cases, but exist during almost every start. The first example occurs when the breaker is first closed. There is a current and power flow into the stator at the moment the breaker is closed, as the magnetic field is established, but since the air-gap field is not at or near its full strength, the current can not react to produce the same torques. The second example occurs in the vicinity of half speed. At half speed, the backward flux wave which arises out of the asymmetric rotor geometry becomes stationary in the stator. Being stationary, the stator resistance becomes infinitely large at synchronous speed. The power pulsation is consequently much smaller than the torque pulsation and tends to confirm calculations, which understates the amount of torque pulsation.

2) The second method requires that the motor be uncoupled from any load and that angular accelerometers be mounted on the rotor to be responsive only to angular acceleration. The basis for the method is fundamental—the angular acceleration equals the applied torque if there are no other torques acting on the rotor. In practice, this method has three rather serious limitations: it is subject to additional signals not related to the applied torque as a result of rotor structural resonances; it is sensitive to very small errors in alignment, causing centrifugal acceleration to appear as rotational acceleration; and it can only be used to test a motor uncoupled from its load.

3) An instrument winding placed in the air gap of the motor to sense air-gap flux in combination with stator current can be used to develop a good signal proportional to air-gap torque. The idea of an instrument winding has been discussed in the literature mainly in the context of control of ac adjustable-speed drives [4], but it has also been discussed as a way to measure air-gap torque in an induction motor [5]. The instrument winding approach comes the closest to sensing actual air-gap torque, but the winding is not easily installed in large motors.

On occasion, shaft response will be looked at as an indication of air-gap torque. There are so many unknown factors which affect the shaft response that this method is of little value.

SHAFT CAPABILITY EVALUATION

The design of the shaft for an electric motor has several aspects to be balanced in arriving at final proportions. In

addition to transmitting torque to the load, it must carry the weight of the rotor, resist magnetic pull, accommodate bearings and fans, provide stable transverse motion within the operating range, and provide a means for coupling to driven shafts and resist short circuits. With motors of large power ratings which operate at high speeds, these design requirements place severe constraints on the proportions available to the designer. The majority of motor shafts are designed to a "rigid shaft" criterion rather than "flexible shaft" criteria. The rotor diameter may be limited mechanically by centrifugal stresses at the maximum speed condition. The type of field-pole construction, method of pole attachment, and type of main field winding give some design latitude in maximum diameters, but for four-pole 60-Hz synchronous machines, limits of 40–50 in are generally observed. Diameters of 60–75 in for six-pole 60-Hz machines give rise to comparable stresses. Limits for 50-Hz machines are about 20 percent larger than for a 60-Hz machine of the same type of construction and number of poles. The rotor span between bearings is limited by shaft flexure (normally expressed in terms of revolutions per minute—rigid bearing critical speed) if a rigid shaft criterion is applicable. These dimensional limits correspond with limitations in available power ratings for a given number of poles at a specified frequency. Four-pole 60-Hz machines with ratings of 8000 hp at 1.0-power factor (PF) or 6000 hp at 0.8-PF operation have been designed and built with laminated-iron-pole construction to meet a rigid shaft criterion. Synchronous machines with solid-iron-pole construction and strip-wound main field windings have been designed for somewhat larger diameters than are presently available when laminated-iron-pole construction is employed. This increased rotor diameter permits the designer to extend available ratings by at least 50 percent while still observing rigid shaft requirements. Six-pole 60-Hz synchronous machines have been built at ratings in excess of 30 000 hp, which meet a rigid shaft criterion. The overhung weight of a resilient coupling, which may be necessary to limit torsional response in the shafts, gears, and couplings, will be increasingly restrictive to design options as the rigid shaft limits are approached. When the more complex flexible shaft design concepts are employed, synchronous machine designs, whether laminated-pole or solid-iron-pole construction, can be extended to larger ratings than would otherwise be possible.

The maximum transient torque that occurs in the shaft and coupling during starting is generally in excess of the rated torque of the machine. It is also necessary to assess

the ability of shafts, gears, and couplings to transmit the oscillating torques that occur during each start. If the oscillating torques impose local alternating stresses on system components that are greater than the fatigue endurance limit of the material (and they very frequently will be greater), it is necessary to determine that the duty to be imposed is within the design life of the equipment. The design life is related to the frequency of starts during that life in addition to elapsed time (i.e., the total number of starts for which the equipment is to be designed should be a defined quantity for all large high speed synchronous or induction motors). Motors with smaller power and lower speed ratings are inherently capable of more frequent starting duty than are larger high speed motors.

STARTING DUTY

The total number of starts can be reliably estimated from a knowledge of how frequently the drive must be stopped for routine maintenance or process shutdowns. Many equipment operators have equipped motor starters with operations counters and/or elapsed time meters. Analysis of maintenance records on similar or related equipments and processes can be of value in this important phase of equipment specification. Experience has shown that an allowance should also be made for additional starting duty during the first year of operation, as errors in process sequencing and interlocks result in unexpected equipment stoppages. As pointed out in Step 5) of the coordination process, information on the total number of starting events is used in conjunction with the results from torsional studies conducted for the specific equipment under consideration to arrive at the magnitude and total number of cycles of alternating stress that must be withstood. Of additional concern are any occurrences of torque reversals at bolted joints, gear meshes, or similar connections.

The state of the art is such that different investigators looking at the same information may make substantially different assessments of the amount of fatigue damage incurred by the various components [6]-[8]. If all the variances involved were in a direction to predict failure when in fact no such failure would occur, the result would be conservative design rather than failure risk. Failure to account for stress raisers or the use of fatigue failure curves in place of design fatigue curves are not conservative practices and are difficult to justify. The following procedure is proposed as being straightforward, well-supported by available published references, and reasonably conservative.

FATIGUE EVALUATION

The shaft duty is expressed as a specific number of cycles (N_a) of alternating stress (S_a). The value of cycles is

$$N_a = N_c \times N_s \times N_y \quad (1)$$

where

- N_a number of cycles of alternating stress,
- N_c number of cycles of high oscillating torque per starting event,

- N_s number of starting events per year,
- N_y number of years of service for which the equipment is designed.

The alternating stress, as computed on the basis of a nominal section, must be increased by a "fatigue weakening factor" to evaluate the possibility of fatigue damage. Actual fatigue weakening factors are generally not available, but elastic, geometrical stress concentration factors, which are available, are recognized to equal or exceed the corresponding fatigue weakening factors.

For a uniform circular cross-section shaft of a specified diameter, the nominal shear stress due to rated torque is

$$\tau = \frac{16T}{\pi d^3} \quad (2)$$

where

- τ nominal maximum shear stress for rated torque (lbf/in²),
- T transmitted torque—rated (lbf/in²),
- d shaft diameter (in).

Step changes in shaft diameter and keyways are stress raisers, which increase the localized stresses. Therefore, the appropriate stress concentration factors (SCF) that apply at these points must be determined. At the bearing journal, the shaft steps down from a larger diameter to a smaller diameter with a small radius or fillet at the step. The torsional handbook published by BICERA [9] suggests using (3) for the SCF in this case.

$$SCF_1 = 1 + \left(\frac{d}{12r}\right) \left[1 - \frac{\left(1 + \frac{2r}{d}\right)}{\left(1 + \frac{6r}{d}\right) \frac{D}{d}} \right] \quad (3)$$

where

- SCF_1 stress concentration factor for stepped shafts in torsion,
- D larger shaft diameter (in),
- d smaller shaft diameter (in),
- r radius of fillet transition (in).

The alternating component of oscillating torque in per unit (pu) of rated torque is

$$T_a = T_r/2T \quad (4)$$

where

- T_a alternating component of oscillating torque (pu),
- T_r range of oscillating torque (lbf/in²),
- T rated torque (lbf/in²).

An alternating shear stress due to pure torsion (i.e., no bending in the shaft) can be converted to the corresponding stress intensity (based on maximum shear theory-Mohr circle approach) by (5):

$$S_a = \tau \times SCF_1 \times T_a \times 2 \quad (5)$$

where S_a is the alternating stress intensity (lbf/in²), and τ , SCF_1 , and T_a are given above.

A reliable design life can then be determined using a design fatigue curve such as contained in [11]. The distinction between a *design fatigue curve* and a *fatigue failure curve* is that a *design fatigue curve* is a design standard which takes into account design and application considerations, while a *fatigue failure curve* represents test data on samples which have been tested to failure (usually in a laboratory). There are environmental and size effects, surface conditions, and the scatter of test results which must be considered in applying test data to design work. The design curve in the reference cited also includes the maximum effect of mean stress. This simplifies the procedure, but also adds some additional conservatism. Reliable design procedures for removing this conservatism are available if the mean stress is accurately known. These are not presented here, in the interests of brevity.

At the shaft extension, a coupling is generally installed by means of a shrink fit and further secured by a rectangular key fit into the shaft and coupling. On occasion, the coupling is installed on the shaft extension without any keys, depending entirely on the shrink fit to carry the torque. A separate evaluation of the shaft extension similar to that made for the shaft journal is required, as this area may be more limiting even though somewhat larger diameters are involved. Experience has shown that motor shafts that fail from torsional loading are generally the result of low cycle fatigue damage, with the initial crack occurring in the shaft extension where the keyway initially enters the shaft. See Fig. 3.

The procedures here have assumed that the fatigue life can be satisfactorily characterized by a single oscillating torque amplitude. This may be unduly conservative. A reliable design procedure for extending the evaluation to a number of combinations of stress ranges and cycles is also included in [11].

CONCLUSION

This paper as well as Part I [1] has discussed the two basic types of synchronous motor rotor construction: the laminated-field-pole and the solid-iron-field-pole. The material presented shows that the solid-iron-pole can greatly increase the complexity of coordination with the driven equipment.

The horsepower requirement for today's geared compressor drives are often well beyond the range where four-pole synchronous motors can be built with laminated-pole iron. It is the authors' experience that the use of the six-pole synchronous motor, which at these larger ratings can still be built with laminated-iron-poles, will, even with the expense of increased gear ratio, generally result in a simpler, more expeditious system torsional coordination and possibly a more economical drive.

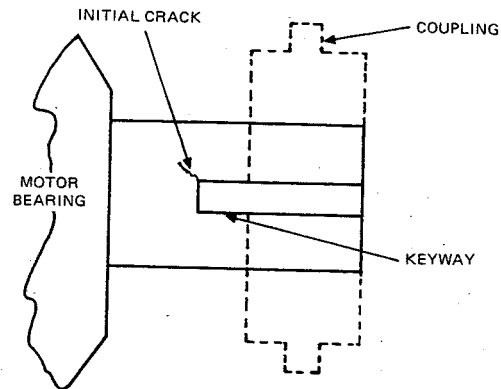


Fig. 3. Sketch of motor shaft extension showing location of initial crack.

APPENDIX

METHOD FOR CALCULATING PULSATING AND AVERAGE TORQUE DURING STARTING OF SYNCHRONOUS MOTORS

An alternate derivation of average and pulsating starting torques of synchronous machines under a balanced supply is presented. The standard reference for this calculation is generally taken to be that of Concordia [3]. Concordia's results, however, are not presented in a form convenient for machine design or systems analysis purposes, nor do they permit a ready physical interpretation. It is the purpose of this Appendix, therefore, to present an alternate, straightforward derivation which yields somewhat simpler expressions for both pulsating and average torque.

Using motor convention and the notation of Concordia [3], Park's equations may be written in per unit as

$$e_d = p\psi_d - \psi_q p\theta + r_i d \quad (A1)$$

$$e_q = p\psi_q + \psi_d p\theta + r_i q \quad (A2)$$

$$0 = p\psi_{kd} + r_{kd} i_{kd} \quad (A3)$$

$$e_{fd} = p\psi_{fd} + r_{fd} i_{fd} \quad (A4)$$

$$0 = p\psi_{kq} + r_{kq} i_{kq} \quad (A5)$$

$$T = \psi_d i_q - \psi_q i_d \quad (A6)$$

where θ is the angular position of the rotor in electrical radians, p is the normalized time operator:

$$p = \frac{1}{\omega_0} \frac{d}{dt},$$

and ω_0 is the angular frequency of the applied stator voltages. The flux linkages are defined by

$$\psi_d = x_{\pi d} i_d + x_{ad}(i_d + i_{kd} + i_{fd}) \quad (A7)$$

$$\psi_q = x_{\pi q} i_q + x_{aq}(i_q + i_{kq}) \quad (A8)$$

$$\psi_{kd} = x_{kd} i_{kd} + x_{ad}(i_d + i_{kd} + i_{fd}) \quad (A9)$$

$$\psi_{fd} = x_{fd} i_{fd} + x_{ad}(i_d + i_{kd} + i_{fd}) \quad (A10)$$

$$\psi_{kq} = x_{kq} i_{kq} + x_{aq}(i_q + i_{kq}) \quad (A11)$$

In (A1)–(A11), the following definitions for the per unit machine parameters apply [3]:

r	armature resistance per phase,
x_l	armature leakage reactance per phase,
x_{ad}	direct axis mutual reactance,
r_{kd}	direct axis amortisseur resistance,
x_{kd}	direct axis amortisseur leakage reactance,
r_{fd}	field resistance (including discharge resistance during starting),
x_{fd}	field leakage reactance,
x_{aq}	quadrature axis mutual reactance,
x_{kq}	quadrature axis amortisseur leakage reactance,
r_{kq}	quadrature axis amortisseur resistance.

It can be observed that, for simplicity, only one amortisseur circuit has been assumed in each axis. If desired, more accurate representations of the damper circuits can be incorporated by adding extra amortisseur circuits. Alternatively, a single amortisseur circuit in each axis can be retained and the parameters r_{kd} , x_{kd} , r_{kq} and x_{kq} considered as variables. In order to maintain the simplicity of the equivalent circuit, this second approach is recommended. For example, the effects of deep bars, discrete bars, and solid iron can be modeled by this technique. In such cases, it is suggested that these equivalent circuit parameters be supplied in tabular form by the manufacturer.

During steady asynchronous operation at a slip s , the dq axes quantities pulsate at slip frequency. Equations (A1)–(A5) by be written in phasor form:

$$jV = js\Psi_d - (1-s)\Psi_q + rI_d \quad (A12)$$

$$V = (1-s)\Psi_d + js\Psi_q + rI_q \quad (A13)$$

$$0 = js\Psi_{kd} + r_{kd}I_{kd} \quad (A14)$$

$$0 = js\Psi_{fd} + r_{fd}I_{fd} \quad (A15)$$

$$0 = js\Psi_{kq} + r_{kq}I_{kq} \quad (A16)$$

If r is neglected in (A12) and (A13), then the flux linkages Ψ_d and Ψ_q are approximately

$$\Psi_d = V \quad (A17)$$

$$\Psi_q = jV. \quad (A18)$$

Equations (A12) and (A13) reduce to

$$jV = j\Psi_d \quad (A19)$$

$$V = j\Psi_q. \quad (A20)$$

Note that (A14)–(A16) contain inductive voltage drops proportional to slip s , whereas the simplified stator equations (A19) and (A20) have inductive voltage drops proportional to line frequency ω_0 . This difficulty can be eliminated if (A14)–(A16) are divided by slip s . Park's equations then reduce to the form

$$jV = jx_l I_d + jx_{ad}(I_d + I_{kd} + I_{fd}) \quad (A21)$$

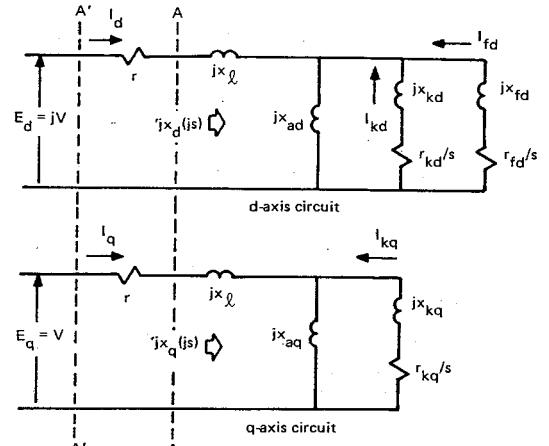


Fig. 4. Equivalent circuit valid for steady-state asynchronous operation.

$$V = jx_l I_q + jx_{aq}(I_q + I_{kq}) \quad (A22)$$

$$0 = jx_{ka}I_{ka} + jx_{ad}(I_d + I_{kd} + I_{fd}) + r_{kd}I_{kd}/s \quad (A23)$$

$$0 = jx_{fa}I_{fa} + jx_{ad}(I_d + I_{kd} + I_{fd}) + r_{fd}I_{fd}/s \quad (A24)$$

$$0 = jx_{kq}I_{kq} + jx_{aq}(I_q + I_{kq}) + r_{kq}I_{kq}/s. \quad (A25)$$

Equations (A21)–(A25) can be used to construct the equivalent circuit to the right of $A-A$ shown in Fig. 4. The impedances seen looking into these two equivalent circuits are defined as $jx_d(js)$ and $jx_q(js)$, respectively. Although the stator resistance has been neglected in the development of this circuit, its effect can be approximated by simply reinserting this resistance in series with the impedances $jx_d(js)$ and $jx_q(js)$ as if they existed external to a machine having zero stator resistance. The terminal voltages V and jV now must appear at terminals $A'-A'$ rather than at $A-A$. It is important to note that this manipulation is valid only if I_d and I_q as well as Ψ_d and Ψ_q are in time quadrature. It is clear that this will be true only if the rotor is symmetrical. Although only approximately correct for salient-pole machines, the error is quite small and is considerably more accurate, particularly around half speed, than if the stator resistance were neglected entirely.

The computation of torque during asynchronous operation can now be calculated in a manner following that of Concordia [3]. Let the direct and quadrature axis currents be defined in complex notation as

$$I_d = a + jb \quad (A26)$$

$$I_q = c + jd. \quad (A27)$$

Let the operational impedances be denoted as the complex quantities

$$x_d(js) = D_x - jD_r \quad (A28)$$

$$x_q(js) = Q_x - jQ_r. \quad (A29)$$

These impedances can be calculated from the usual direct and quadrature axis equivalent circuits. For example, $jx_d(js)$ is

the impedance calculated for frequency $s\omega_0$ when looking into the direct axis circuit of Fig. 4 at $A-A$.

In complex notation, the flux linkages are found as

$$\Psi_d = x_d(js)I_d = aD_x + bD_r + j(bD_x - aD_r) \quad (\text{A30})$$

$$\Psi_q = x_q(js)I_q = cQ_x + dQ_r + j(dQ_x - cQ_r). \quad (\text{A31})$$

Note that motor sign conventions are used here in contrast to the generator sign convention of Concordia [3].

The real components (actual physical values) of current and flux can be calculated in the manner typified by

$$i_d = \text{real of } (I_d e^{js\omega_0 t}). \quad (\text{A32})$$

The resulting real values are

$$\begin{aligned} i_d &= a \cos s\omega_0 t - b \sin s\omega_0 t \\ &= |I_d| \cos(s\omega_0 t + \phi_d) \end{aligned} \quad (\text{A33})$$

$$\begin{aligned} i_q &= c \cos s\omega_0 t - d \sin s\omega_0 t \\ &= |I_d| \cos(s\omega_0 t + \phi_q) \end{aligned} \quad (\text{A34})$$

$$\begin{aligned} \psi_d &= (aD_x + bD_r) \cos s\omega_0 t \\ &\quad + (aD_r - bD_x) \sin s\omega_0 t \end{aligned} \quad (\text{A35})$$

$$\begin{aligned} \psi_q &= (cQ_x + dQ_r) \cos s\omega_0 t \\ &\quad + (cQ_r - dQ_x) \sin s\omega_0 t. \end{aligned} \quad (\text{A36})$$

The instantaneous torque can now be computed from (A6), which is identical to (62) of Concordia. The result is

$$\begin{aligned} T &= \frac{1}{2} \{(ac + bd)(D_x - Q_x) \\ &\quad + (bc - ad)(D_r + Q_r)\} \\ &\quad + \frac{1}{2} \{(ac - bd)(D_x - Q_x) \\ &\quad + (ad + bc)(D_r - Q_r)\} \cos 2s\omega_0 t \\ &\quad + \frac{1}{2} \{(ac - bd)(D_r - Q_r) - (ad + bc) \\ &\quad + (D_x - Q_x)\} \sin 2s\omega_0 t. \end{aligned} \quad (\text{A37})$$

The first term of (A37) is the average torque. The remainder is the pulsating torque at twice slip frequency.

The amplitude of the pulsating torque T_{PULS} is found by taking the square root of the sum of the squares of the amplitudes of the $\cos(2s\omega_0 t)$ and $\sin(2s\omega_0 t)$ terms. The similarity of terms permits one to write almost by inspection that

$$\begin{aligned} 4T_{\text{PULS}}^2 &= \{(ad - bd)^2 + (ad + bc)^2\} \\ &\quad \cdot \{(D_x - Q_x)^2 + (D_r - Q_r)^2\} \end{aligned} \quad (\text{A38})$$

which is still further reducible to

$$\begin{aligned} 4T_{\text{PULS}}^2 &= \{(a^2 + b^2)(c^2 + d^2)\} \\ &\quad \cdot \{(D_x - Q_x)^2 + (D_r - Q_r)^2\}. \end{aligned} \quad (\text{A39})$$

By inspection of (A28), (A29), (A33), and (A34), it is seen that

$$a^2 + b^2 = |I_d|^2 \quad (\text{A40})$$

$$c^2 + d^2 = |I_q|^2 \quad (\text{A41})$$

$$(D_x - Q_x)^2 + (D_r - Q_r)^2 = |x_d(js) - x_q(js)|^2 \quad (\text{A42})$$

where $|\dots|$ means "absolute value of." Thus the pulsating torque amplitude at any time is compactly expressed as

$$T_{\text{PULS}} = \frac{1}{2} |I_d| |I_q| |x_d(js) - x_q(js)|. \quad (\text{A43})$$

Note that the last member of the product is the absolute value of the difference between the complex impedances. It is *not* the difference between the absolute values of the impedances.

In a similar manner the average torque T_{AVE} is

$$\begin{aligned} T_{\text{AVE}} &= \frac{1}{2} \{(ac + bd)(D_x - Q_x) \\ &\quad + (bc - ad)(D_r + Q_r)\}. \end{aligned} \quad (\text{A44})$$

Equation (A44) can be reduced to the form

$$T_{\text{AVE}} = \frac{1}{2} \text{real of } \{I_d x_d(js) I_q^* - I_q x_q(js) I_d^*\} \quad (\text{A45})$$

where the superscript means "conjugate of," and "real of" means the real part of the adjoining bracket complex quantity. Physical insight into the significance of (A45) can be realized with the following approximation. From Fig. 4 it is apparent that if stator resistance is neglected, then approximately

$$E_d = jx_d(js)I_d \quad (\text{A46})$$

$$E_q = jx_q(js)I_q. \quad (\text{A47})$$

Substituting into (A45)

$$T_{\text{AVE}} = \frac{1}{2} \text{imag of } \{E_q I_d^* - E_d I_q^*\} \quad (\text{A48})$$

which can be interpreted as the cross product between the axes stator voltages and currents. Equation (A45) can be reduced still further if it is recalled that the dq axes stator voltages are in time quadrature. That is

$$E_d = jV - jE_q \quad (\text{A49})$$

$$E_q = V = jE_d. \quad (\text{A50})$$

Substituting (A49) and (A50) into (A48), the average torque can be expressed approximately as

$$T_{\text{AVE}} = \frac{1}{2} \text{real of } \{E_d I_d^* + E_q I_q^*\} \quad (\text{A51})$$

which can be interpreted as the average value of the d and q axes air-gap power.

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Edward L. Owen (S'61-M'65), for a photograph and biography, please see page 571 of this TRANSACTIONS.



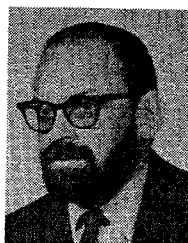
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