

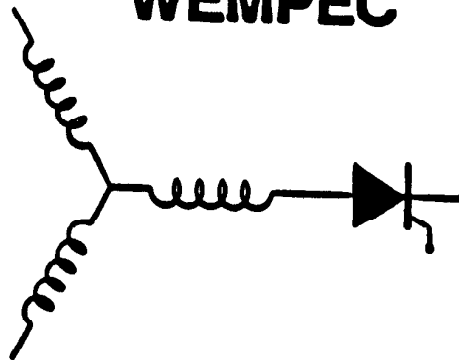
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Equivalent Circuit Model for Superconducting Machine Based on Three Dimensional
Field Solution

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Abstract In spite of the strong interest in field analysis of superconducting machines, no completely satisfactory simulation model has yet been developed for performance evaluation. In this paper a basic per phase equivalent circuit model for an air core superconducting machine with rotor conductive screen is presented. The evaluation of the circuit parameters are based on the knowledge of the three dimensional field distribution within the machine where end effects due to the finite stator axial length and the existence of the end winding are included. The results exhibit explicitly the relationships between the relative dimensions of the machine and these parameters. A significant change in the parameter values, consequently in the performance, is expected for machine lengths below certain defined length. Design guides are provided for quick estimates of the equivalent circuit parameters as a function of the machine dimensions.

1. Introduction

The most important part in the design of superconducting synchronous machines is to ensure satisfactory performance in both steady and transient states, with special attention paid to the stability problem. One of the basic simulation techniques to evaluate the performance is the equivalent circuit model. Unlike the conventional iron core machines, where the rotor and the stator windings have a defined current sheets, the parameters related to the superconducting machine are complex to determine. This is mainly due to their special structure [1], where the stator is an air core winding and the rotor has a continuous conductive cylindrical shell (Fig.1).

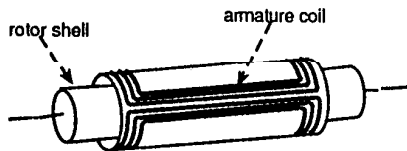


Fig.1 Model Basic Configuration

For a simplified model of an iron core winding of infinite axial length, and a negligible end winding, the model equivalent circuit can be represented as shown in Fig. 2. The inductances of this circuit are defined by considering the model magnetic circuit. However, it is clear that for the same model with an air-core winding, the magnetic circuit concept can not be applied [2]. To calculate the equivalent circuit parameters, a field analysis approach must now be utilized. In addition, since the field solution for an infinitely long model (two dimensional problem) differs substantially from that of the finite length model (three dimensional problem), the solution for the equivalent circuit of a finite length model is expected to be more challenging as well.

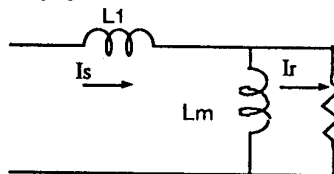


Fig.2 Equivalent Circuit.

In this paper a new strategy to define the equivalent circuit parameters is developed. By comparing the field solution for an approximate model of an infinite axial length with the actual model having a finite axial length and end windings, an equivalent circuit for the latter can be obtained. The field solution for the finite length model with end winding is based on a recently published work [3]. The comparison study provides information about the dimension limitation imposed by the assumption of an infinitely long excitation. It can be shown that the equivalent circuit parameters of the finite length model are highly dependant on the excitation length. Also the existence of the end windings affects the circuit leakage reactance. The steps of calculating the equivalent circuit parameters for a finite length model with end windings is explained in the following sections.

2. Analysis of the Infinitely Long Approximate Model: Field Solution

The simplest model capable of approximating the necessary field information within the machine is shown in Fig. 3. In this model the (2p) poles, 3-phase sinusoidally distributed stator winding is represented by an infinitely long current sheet of thickness Δr_s and located at radius r_s . The rotor screen is represented by a conductive layer of conductivity σ and thickness Δr_a , located at radius r_a . In this model, the end effects are neglected and the current in the sheet has only an axial component. The excitation current sheet is expressed in terms of the number of conductors per pole per phase (n), the fundamental winding factor k_w and the sinusoidal phase current of amplitude I_s and frequency ω_0 as:

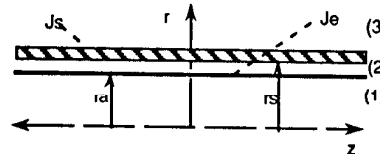


Fig.3 Infinite Length model

$$\vec{J}_s = J_{sm} e^{j(\omega_0 t - p\theta)} \frac{-}{az} \quad (1)$$

where

$$J_{sm} = \frac{3 n k_w I_s p}{\pi r_s \Delta r_s} \quad \text{Amp/m}^2 \quad (2)$$

For the above excitation, the corresponding magnetic vector potential (m.v.p.) A is also a sinusoidally distributed function and has only a z-component. From the solution of the general field differential equation, under the assumption that $\partial/\partial z = 0$, A is defined in the different regions as:

$$\begin{aligned} A_1(r) &= C_1(r)^p & 0 < r < r_a \\ A_2(r) &= C_2(r)^p + D_2(r)^{-p} & r_a < r < r_s \\ A_3(r) &= D_3(r)^{-p} & r_s < r < \infty \end{aligned} \quad (3)$$

where the solution constants C_1 , C_2 , D_2 and D_3 are obtained from the boundary conditions as described in Appendix I.

From Eqs. 3, the closed form expression for the field solution at the stator surface $A(r_s)$ and at the rotor surface $A(r_a)$ are defined as :

$$A(r_s) = \frac{\mu_0}{2} (J_{sm} \Delta r_s) \left(\frac{r_s}{p} \right) \frac{q_2}{q_1} \frac{1}{e} j(\omega_e t - p\theta - \alpha_1 + \alpha_2) \quad (4)$$

$$A(r_a) = \frac{\mu_0}{2} (J_{sm} \Delta r_s) \left(\frac{r_a}{p} \right) \left(\frac{r_a}{r_s} \right)^D \frac{1}{q_1} \frac{1}{e} j(\omega_e t - p\theta - \alpha_1) \quad (5)$$

where

$$\begin{aligned} s &= \text{slip} \\ \tau &= \mu_0 \Delta r_s \Delta r_a \frac{r_a}{2p} \\ (q_1)^2 &= 1 + (s\omega_e \tau)^2 \\ (q_2)^2 &= 1 + (a_s s\omega_e \tau)^2 \\ a_s &= 1 - \left(\frac{r_a}{r_s} \right)^{2p} \\ \alpha_1 &= \tan^{-1}(s\omega_e \tau) \\ \alpha_2 &= \tan^{-1}(a_s s\omega_e \tau) \end{aligned}$$

The form of Eqs. 4 and 5 and are helpful in defining all the associated physical quantities such as the shield induced current J_e , the radial component of the magnetic flux density B_r , the force density over rotor surface f_θ , and the electric torque T_e (Appendix II).

Airgap Power

Expressing the power in the gap between the stator surface and the rotor surface (P_g) in terms of the mechanical output power, one obtains:

$$P_g = \frac{T_e \omega_m}{(1-s)p} = T_e \frac{\omega_e}{p} \quad (6)$$

From the field solution (Eq. 5), and the associated physical quantities (Appendix II), a closed form expression for the airgap power per unit axial length is obtained as :

$$P_g = \frac{\mu_0 \pi}{2} (J_{sm} \Delta r_s)^2 (r_s)^2 \left(\frac{r_a}{r_s} \right)^{2p} \frac{(s\omega_e \tau) \omega_e}{1 + (s\omega_e \tau)^2} \quad (7)$$

Equivalent Circuit Parameters

In the equivalent circuit of Fig.2, the rotor resistance and the mutual inductance are related to the air gap power as:

$$P_g = \frac{\frac{3}{2} I_s^2 (R_r/s)}{1 + \left(\frac{R_r/s}{x_m} \right)^2} \quad (8)$$

Obviously Eqs. 7 and 8 can be rewritten in the general form:

$$P_g = \frac{c_1 \omega_e^2}{c_2 + \omega_e^2} \quad (9)$$

Where c_1 and c_2 are frequency independent constants. Comparing the constants of the two equations, expressions for R_r and L_m , per unit length, are obtained as:

$$\left(\frac{R_r}{s} \right) = \frac{2 \mu_0 \pi}{3} \frac{1}{p} (r_s)^2 \left(\frac{r_a}{r_s} \right)^{2p} \frac{1}{s\tau} N_e^2 \quad (10)$$

$$L_m = \frac{2 \mu_0 \pi}{3} \frac{1}{p} (r_s)^2 \left(\frac{r_a}{r_s} \right)^{2p} N_e^2 \quad (11)$$

where (N_e) is the amplitude of the conductors distribution function (N), defined as :

$$N = N_e e^{-j(p\theta)} \quad (12)$$

$$N_e = \frac{3 \pi k_w p}{\pi r_s} \text{ cond/m}$$

To define the leakage inductance L_1 (Fig. 2) the open circuit condition is considered. For $I_r = 0$, i.e. $s = 0$, the inductances L_1 and L_m are related to the stator flux linkage as:

$$2p\lambda_s = \frac{3}{2} I_s \{ L_1 + L_m \} \quad (13)$$

Defining the elementary stator flux linkage ($d\lambda_s$) in terms of the m.v.p. $A(r_s)$, Eq. 4, when $s=0$, and the conductor distribution function (N), as:

$$d\lambda_s = 0.5 \text{ Real} \left\{ \int \bar{A} \cdot \bar{dl} \right\} \{ N \} \quad (14)$$

Integrating Eq. 14 the total stator flux linkage is then obtained as:

$$2p\lambda_s = 0.5 \left[2 \int_{-L_s}^{+L_s} A_z dz \right] (N_e) \pi r_s \quad (15)$$

Using Eqs. 11 and 13, the resultant closed form expression for the leakage inductance per unit length is :

$$L_1 = \frac{2 \mu_0 \pi}{3} \frac{1}{p} r_s^2 N_e^2 \left\{ 1 - \left(\frac{r_a}{r_s} \right)^{2p} \right\} \quad (16)$$

3. Analysis of the Finite Length Model - Field Solution:

For the actual winding configuration (Fig. 1), the stator excitation can be split into two current sheets: an axial component current sheet J_{sz} and a circumferential component $J_{s\theta}$. Both current sheets have a finite axial length and are governed by the divergence equation for a continuous current density vector :

$$\nabla \cdot \bar{J}_s = 0 \quad (17)$$

i.e.

$$\frac{\partial}{\partial z} J_{sz} = -\frac{p}{r_s} J_{s\theta}$$

For a machine with a finite axial length $2L_s$ and an end winding of width w_R (Fig. 4), the current source vector will be:

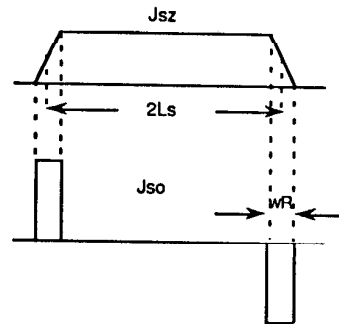


Fig.4 stator current sheets

$$\bar{J}_s = \bar{J}_{sz} + \bar{J}_{s\theta} \quad (18)$$

$$\bar{J}_{sz} = J_{zm} \frac{1}{e} j(\omega_e t - p\theta) \bar{a}_z \quad (19)$$

where

$$J_{zm} = \frac{3n k_w p I_s}{\pi r_s} \quad -L_s < z < L_s \quad (20)$$

The corresponding end winding current source will be :

$$J_{s\theta} = J_{\theta m e} \left[(\omega_e t - p\theta - \frac{\pi}{2}) - a_\theta \right] \quad (21)$$

where

$$J_{\theta m} = \frac{r_s}{pWR} \quad J_{zmWR/2} < z < WR/2 \quad (22)$$

Using the Fourier transform technique, the field solution due to a finite excitation current sheet is obtained [3]. Considering the two components of the current source, the distribution of the axial, $A_z(r, \theta, z)$, and the circumferential, $A_\theta(r, \theta, z)$, components of the m.v.p. are numerically computed.

Airgap Power

Following the same procedure as in Section 2, the airgap power for the finite length model is evaluated. At this point, it is worth mentioning that the end winding also contributes to the radial magnetic flux density. The average of the circumferential force density due to the interaction of this flux and the shield induced current is zero, which means that the airgap power is calculated from the distribution of the axial component of the magnetic vector potential only.

To allow analogy between the finite length model and the infinite length one, the air gap power variation with excitation frequency (ω_e) is plotted. For each supply frequency, the field solution is obtained, then the corresponding airgap power is evaluated. By comparing the airgap power variation $P_g(\omega_e)$ for the infinite length model to that for the finite length model, it is found that $P_g(\omega_e)$ behaves in the same manner, i.e. the variation of P_g with ω_e is consistent with the relation of Eq. 9. It can also be shown that the maximum value of P_g depends on the excitation length ($2L_s$) for constant values of r_a and r_s .

By plotting $P_g(\omega_e)$ for different values of $2L_s$, it is discovered that the power $P_g(\omega_e)$ calculated from the field solution of the finite length model approaches the value obtained from the field solution of the infinitely long model when the ratio $L_s/(r_s - r_a)$ exceeds a certain limit. In other words, by increasing the ratio $L_s/(r_s - r_a)$, the approximation of infinite excitation length is quite acceptable. However, beyond certain dimension limitations the error in computing airgap power can be extremely high.

Equivalent Circuit Parameters

For a motor model of a specific dimensions (L_s, r_s, r_a), numerical values for P_g at different excitation frequencies are obtained as described before. From the curve fitting of $P_g(\omega_e)$ plot to the relation in Eq. 9, the constants c_1 and c_2 have been numerically calculated. Relating the constants values to the equivalent circuit parameters, Eq. 8, the rotor resistance and the mutual inductance are calculated as :

$$(R_r/s) = \frac{2c_1}{3I_s^2} \quad (23)$$

$$L_m = \frac{(R_r/s)}{c_2} \quad (24)$$

In the leakage inductance calculation, the major difference between the infinite length model and the finite length model is the existence of the end windings in the latter. To compute the stator flux linkage in the finite length model, the two components of the m.v.p. A_z (due to the main winding) and A_θ (due to the end winding) has to be considered. In this case the stator flux linkage will be :

$$2p\lambda_s = 0.5 \left\{ 2 \int_{-L_s}^{+L_s} [A_z(z) dz] \left[\frac{3n k_w p}{\pi r_s} \right] \right. \\ \left. + 2 \int_{-WR/2}^{WR/2} [A_\theta(z) dz] \left[\frac{r_s}{pWR} \frac{3n k_w p}{\pi r_s} \right] \right\} \pi r_s \quad (25)$$

From the field solution of the finite length model, with $s=0$, the numerical values for the integrations in Eq. 25 is evaluated. Consequently the total flux linkage is obtained. Using Eqs. 13 and 24 the stator self inductance ($L_1 + L_m$) and the leakage inductance L_1 are calculated.

Numerical Results

For a motor model of the dimensions in Table I, the field solution for a finite length model with end windings is shown in Fig. 5, where $A_z(r_a, z)$ and $A_\theta(r_a, z)$ are the distributions of the two components of the m.v.p. at rotor surface [3]. For the sake of comparison, the axial component $A_z(r_a)$ obtained from the infinite length model is also plotted in the same Figure. The corresponding force density distribution obtained for both the finite and the infinite length models are shown in Fig. 6.

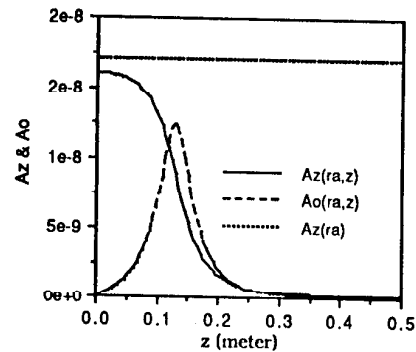


Fig.5 Distribution of the Magnetic Vector Potential Components.

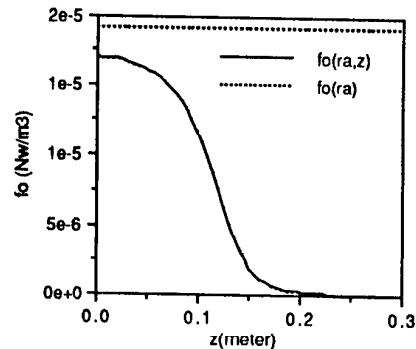


Fig.6 Force Density over Rotor Surface

Computing the airgap power at different supply frequencies, the resultant $P_g(\omega_e)$ obtained for the finite length model of $2L_s=0.26$ m and the infinite length model are shown in Fig. 7. It is worth mentioning that the difference between the two curves diminishes to zero by increasing the excitation length $2L_s$ of the finite length model. In Fig. 8 the error percentage defined as $[P_g(\text{infinite}) - P_g(\text{finite})] / P_g(\text{finite})$ is plotted against the ratio $[2L_s / (r_s - r_a)]$.

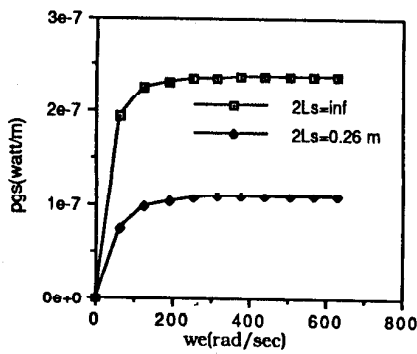


Fig.7 Airgap Power Density

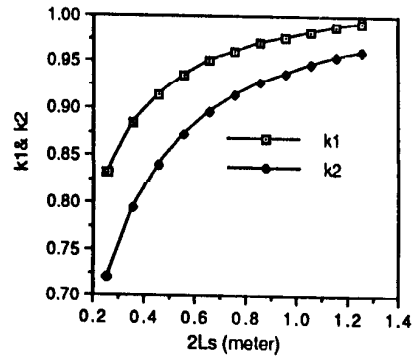


Fig.10 Correction Factors.

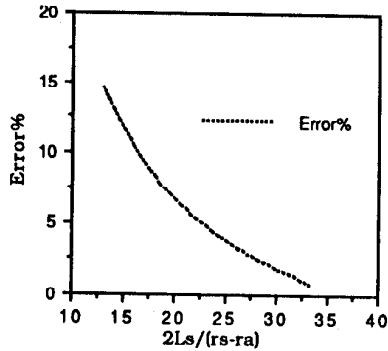


Fig.8 Error Percentage in the Airgap Power

Defining the critical length of the machine as the minimum winding axial length where both infinite and finite length models leads to the same value or the airgap power (within 1% deviation), the critical length L_{sc} is plotted for different values of r_a and r_s in Fig. 9. This curve represents the relationship between the machine main dimensions (L_s, r_s, r_a) and the validity of the infinite length assumption.

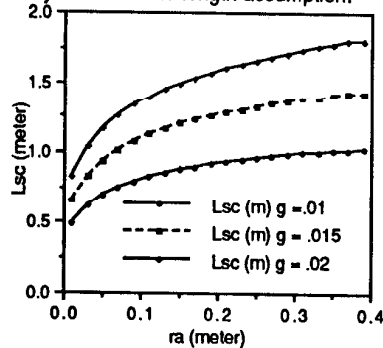


Fig.9 Critical Excitation Length.

As the main purpose of this paper is to define the equivalent circuit parameters, values for the equivalent rotor resistance, mutual inductance and leakage inductance are given in Table I. To make use of the parameters given by the closed form expressions (Eqs.10&11), correction factor curves have been introduced. In Fig.10, the correction factor $k_1 = R_r / R_r^*$ (where R_r corresponds to the actual finite length model and R_r^* to the infinite length model) is plotted for different values of $2L_s$. Similarly a correction factor $k_2 = L_m / L_m^*$ is plotted. Both curves are computed for specific radii corresponding to the working dimensions in Table I.

Table I

Dimensions of the motor model:

Main winding average length	$2L_s =$	0.26 m
Stator radius	$r_s =$	0.08334 m
Rotor radius	$r_a =$	0.063505 m
End winding width	$w_R =$	0.0635 m

with excitation conditions as :

Number of poles	$2p =$	2
Amplitude of conductors distribution function	$N_e =$	1.5 (cond / m)
Main winding current sheet	$(J_{sm} \Delta r_s) =$	1.0 (Amp/m ²)
Excitation frequency	$f =$	10 Hz

Equivalent circuit Parameters :

$R_r / r_s = 450.91 (n k_w)^2 (10)^{-8}$	Ω
$L_m = 15.382226 (n k_w)^2 (10)^{-8}$	H
$L_l = 17.442774 ((n k_w)^2 (10)^{-8}$	H

6. Conclusions

This paper has presented a new method for determining the parameters of a superconducting synchronous machine specifically including the effect of the rotor conducting shield. The new solution is compared to a previous solution which assumes an approximate infinitely long model. From the field solution of the infinitely long model, analytical expressions for the equivalent circuit parameters are obtained which are practically suitable for bringing out the effects of the model radii, the screen conductivity and the operating conditions. The comparison study between the approximate infinitely long model and the finite length model permits the determination of the effect of the stator length and the existence of the end turns on the value of key circuit parameters.

It has been shown that, within certain range of the model dimensions, a three-dimension representation is quite essential for accurate prediction of the equivalent circuit parameters. In other words, the mostly commonly used infinitely long approximate model leads to considerable parameter errors. In this study, a definition for the critical stator length is introduced, which justifies the validity of using the simple approximate model. For stator lengths below this critical length, use of the approximate model is not recommended. As a generalization of this study, a graphical tool to decide the appropriate model for representing specific machine dimensions is given by plotting the relationship between the critical machine length and the stator and the rotor radii.

Since the relation between the stator length and the parameter values are nonlinear (for machine lengths below the critical length), the effect of the stator length on the values of the equivalent rotor resistance and the mutual inductance is provided. The rotor resistance and the mutual inductance can be obtained directly for different model lengths by multiplying the analytical expressions for the parameters (corresponding to the infinitely long model) with a correction factor corresponding to the desired stator length.

Acknowledgments

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Appendix I General Field Solution

With the assumptions of Section 1, the electromagnetic fields of the three regions in Fig. 3 can be obtained in terms of the m.v.p. A, which is governed by Maxwell's equations. These equations can be combined to yield the general field differential equation in cylindrical coordinates as:

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{1}{r^2} \frac{\partial^2 A}{\partial \theta^2} + \frac{\partial^2 A}{\partial z^2} = -\mu_0 (J_s + J_e)$$

where J_s and J_e are the excitation and the induced current density vectors respectively.

For the assumed excitation, Eq. 1, the m.v.p. will be in the form :

$$\bar{A} = A_m e^{j(\omega t - p\theta)} \frac{-}{a_z}$$

In each of the three regions, Fig. 3, solution for the amplitude of A is in the form of Eq. 3. To find the arbitrary constants the following boundary conditions are applied:

B_r is continuous at $r = r_a$ and $r = r_s$

$$(H_\theta)_a^+ - (H_\theta)_a^- = J_e \Delta r_a$$

$$(H_\theta)_s^+ - (H_\theta)_s^- = J_s \Delta r_s$$

The solution constants obtained from the four boundary conditions are :

$$C_1 = \frac{\mu_0}{2} (J_s m \Delta r_s) \frac{r_s}{p} (r_s)^{-p} \frac{1}{(1+j\omega\epsilon r)}$$

$$C_2 = \frac{\mu_0}{2} (J_s m \Delta r_s) \frac{r_s}{p} (r_s)^{-p}$$

$$D_2 = -\frac{\mu_0}{2} (J_s m \Delta r_s) \frac{r_s}{p} (r_a)^p \left(\frac{r_a}{r_s}\right)^p \frac{j\omega\epsilon r}{1+j\omega\epsilon r}$$

$$D_3 = \frac{\mu_0}{2} (J_s m \Delta r_s) \frac{r_s}{p} (r_s)^p \left(1 - \frac{j\omega\epsilon r}{1+j\omega\epsilon r}\right) \left(\frac{r_a}{r_s}\right)^{2p}$$

Appendix II Field Physical Quantities

From the closed form expression for the m.v.p. the two components of the magnetic flux density can be derive immediately as:

$$B_r = \frac{1}{r} \frac{\partial A}{\partial \theta} \quad \text{and} \quad B_\theta = -\frac{\partial A}{\partial r}$$

Similarly, the shield induced current is obtained from the field constitutive relation as:

$$J_{ze} = \sigma \left[-\frac{\partial A}{\partial t} + \omega_m \frac{p}{r_a} \frac{\partial A}{\partial \theta} \right]$$

Substituting for the m.v.p. at rotor surface $r = r_a$, Eq. 5, the axial component of the shield induced current is obtained as:

$$J_{ze} = -j s \omega_e \sigma A(r_a)$$

Using the Lorentz force expression, the circumferential force density over rotor volume is:

$$f_\theta = 0.5 \text{ Real} \{ J_{ze} B_r^*(r_a) \}$$

where $(B_r)^*$ is conjugate of the radial component of magnetic flux density.

Expressing the induced current J_{ze} and the radial component of the magnetic flux density B_r in terms of the m.v.p. at rotor surface, $A(r_a)$, and integrating over rotor volume, the total force per unit axial length of the rotor is:

$$F = \frac{2\pi r_a}{\mu_0} (s\omega_e r) \left(\frac{p}{r_a}\right)^2 |A(r_a)|^2$$

The corresponding torque and air gap power per unit axial length are:

$$T_e = F r_a$$

$$P_g = T_e \frac{\omega_e}{p}$$