Analysis and Simulation of Five-Phase Synchronous Reluctance Machines Including Third Harmonic of Airgap MMF

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***Abstract*—In this paper, winding function method in conjunction with the coupled magnetic circuit approach are used to develop the dynamic equations of the five-phase synchronous reluctance machine in the natural frame of reference $(a–b–c–d–e)$. The effect of third harmonic of airgap MMF is included in this development. A five-phase transformation from the $(a–b–c–d–e)$ system to a $(d_1–q_1–d_3–q_3–n)$ system is introduced. This transformation removes the angular dependency of the inductances. Using this transformation, equations for a five-phase synchronous reluctance machine including the third harmonic of the airgap MMF were obtained. The resulting equations are compared with the equations in the natural $(a–b–c–d–e)$ system. Simulation traces are used to confirm the validity of the model. Finally, the developed equations are compared to the experimental results and the similarities and discrepancies identified.**

***Index Terms*—Concentrated winding, synchronous reluctance motor, third harmonic.**

I. Introduction

In traditional machine applications, three phases are normally selected, since this typically results in the lowest transmission cost. However, when the machine is connected to an inverter supply, the need for a specific number of phases, such as three, disappears, and other phase numbers can be chosen. For example, in large machines, two pairs of three-phase windings shifted by 30 electrical degrees have found favor [1]. High phase number drives possess several advantages over conventional three-phase drives, such as reducing the amplitude and increasing the frequency of torque pulsations, reducing the rotor harmonic currents, reducing the current per phase without increasing the voltage per phase, lowering the dc-link current harmonics, and higher reliability [2]–[10]. It is also shown that increasing the number of phases can result in increased torque per rms ampere for the same volume machine [11]–[13].

In a recent paper [14], the benefits of five phases have been recognized. In particular, it was determined that, whereas three-phase machines can utilize only the fundamental component to develop torque, torque can be developed by both the first and the third harmonic in a five-phase machine. By extension, seven-phase machines can be controlled to utilize the first, third, and fifth harmonics for torque production, and so forth. In this paper, it is shown that the classical $d–q$ analysis can be extended to such machines. In particular, the case of a five-phase reluctance machine has been chosen for analysis, but the approach can be extended to higher order phase numbers. The set of voltage equations developed are sufficiently detailed to describe both the transient and steady-state behavior of a five-phase synchronous reluctance machine. Section II describes the system equations for a five-phase synchronous reluctance machine. Inductances for such a machine are calculated in Section III. Stator terminal voltage equations are given in Section IV. Synchronous reluctance machine equations are transformed into the $(d_1–q_1–d_3–q_3–n)$ reference frame in Section V. Section VI presents the electromagnetic torque calculation, and Section VII illustrates the computer simulation results. Experimental results are presented in Section VIII, which is followed by a conclusion and necessary comments in Section IX.

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II. System Equations

Let us consider the idealized representation of a five-phase two-pole reluctance motor, as shown in Fig. 1. The winding axes of the five stator windings are displaced by 72°. Saturation of iron will be neglected in this analysis. Although the machine is assumed to have only two poles, the effects of multiple number of pole pairs can be readily included, when it becomes necessary, by traditional methods.

It is convenient to denote each physical winding by an equivalent coil aligned along the point of maximum MMF produced by each winding. It is assumed initially that none of the circuits are physically interconnected. In general, the equation which describes the electrical behavior of this machine in matrix form is

$$V_s = R_s I_s + \frac{d\Lambda_s}{dt} \tag{1}$$

where

$$\Lambda_s = L_{ss} I_s \tag{2}$$

and corresponds to the flux linkages of the windings and where

$$I_s = [i_a^s \quad i_b^s \quad \cdots \quad i_e^s] \tag{3}$$

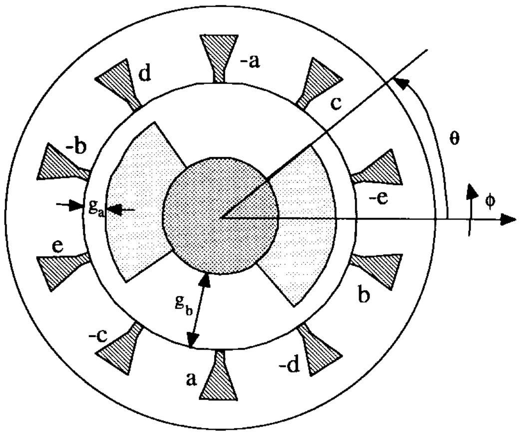

Fig. 1. Synchronous reluctance motor with two-pole five-phase concentrated windings.

$$V_s = [v_a^s \quad v_b^s \quad \cdots \quad v_e^s] \tag{4}$$
$$\Lambda_s = [\lambda_a^s \quad \lambda_b^s \quad \cdots \quad \lambda_e^s]. \tag{5}$$

The matrix R_s is a diagonal 5 × 5 matrix given by

$$R_s = r_s I \tag{6}$$

where the matrix I is a 5 × 5 identity matrix and r_s is the resistance of each coil, assuming that all coils are similar.

Due to conservation of energy, the matrix L_{ss} is a symmetric 5 × 5 matrix of the form

$$L_{ss} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{ad} & L_{ae} \\ L_{ab} & L_{bb} & L_{bc} & L_{bd} & L_{be} \\ L_{ac} & L_{bc} & L_{cc} & L_{cd} & L_{ce} \\ L_{ad} & L_{bd} & L_{cd} & L_{dd} & L_{de} \\ L_{ae} & L_{be} & L_{cd} & L_{de} & L_{dd} \end{bmatrix} \tag{7}$$

where the diagonal elements are the self inductances of each of the phases, including both leakage and magnetizing components. The off-diagonal entries are the mutual inductances between each pair of phases.

III. Stator Inductances

The axis of phase is used as the reference point for the circumferential angle ϕ used to define the winding function $N(\phi)$ in Fig. 2 [15]. In general, the winding function is a stepped-like function due to the discrete nature of the winding slots. In the so-called sinusoidally wound machines, the low-order harmonic components of the winding function for each phase are small relative to the fundamental component. However, in the case of a concentrated winding machine, the third harmonic of the winding function for each phase is specifically not negligible and, therefore, the effect of third harmonic must be considered in the winding functions.

Notice that the incorporation of the third harmonic for the stator winding function is a necessary condition for existence of linkage of the airgap flux third harmonic component with the stator phase windings. Without this term, the third harmonic of airgap flux would not link the stator winding. Assuming that only the fundamental and third harmonic components are significant, the winding functions of the five stator

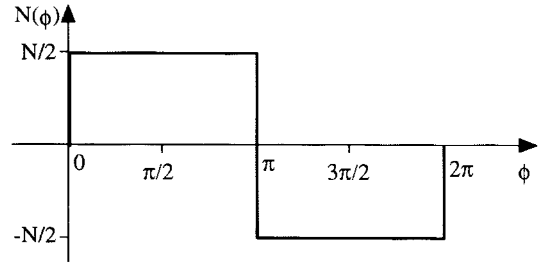

Fig. 2. Winding function for each phase of the five-phase synchronous reluctance machine.

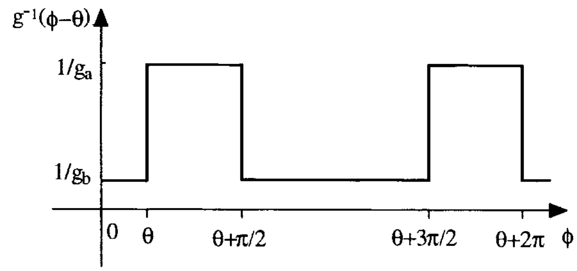

Fig. 3. Inverse airgap function for a synchronous reluctance machine.

windings can then be expressed as

$$\begin{aligned} N_a(\phi) &= \frac{4}{\pi}\frac{N}{2}\left[\cos\phi - \frac{1}{3}\cos 3\phi\right] \\ N_b(\phi) &= \frac{4}{\pi}\frac{N}{2}\left[\cos\left(\phi - \frac{2\pi}{5}\right) - \frac{1}{3}\cos 3\left(\phi - \frac{2\pi}{5}\right)\right] \\ N_c(\phi) &= \frac{4}{\pi}\frac{N}{2}\left[\cos\left(\phi - \frac{4\pi}{5}\right) - \frac{1}{3}\cos 3\left(\phi - \frac{4\pi}{5}\right)\right] \\ N_d(\phi) &= \frac{4}{\pi}\frac{N}{2}\left[\cos\left(\phi + \frac{2\pi}{5}\right) - \frac{1}{3}\cos 3\left(\phi + \frac{2\pi}{5}\right)\right] \\ N_e(\phi) &= \frac{4}{\pi}\frac{N}{2}\left[\cos\left(\phi + \frac{4\pi}{5}\right) - \frac{1}{3}\cos 3\left(\phi + \frac{4\pi}{5}\right)\right] \end{aligned} \tag{8}$$

where N is the number of turns per pole per phase. In order to calculate the winding inductances, the flux linkages arising either from the winding itself or another winding must be computed.

Due to the saliency of the rotor, the airgap is not constant, as for the case of an induction motor, but it is a function of spatial angle ϕ. With the aid of flux plots, the gap length g can be measured as a function of ϕ. For this analysis, it is assumed that an inverse airgap function is defined as given by Fig. 3. In practical cases, the rotor is designed to have an even number of symmetrically shaped poles and an equal number of north and south poles. Therefore, the inverse gap function consists of a constant term plus even harmonics. Assuming τ_p is the rotor pole arc, the gap function is

$$g^{-1}(\phi-\theta) = a - \frac{2b}{k}\sin\frac{k\tau_p}{2}\cos k(\phi-\theta), \qquad k = 2, 4, 6, \cdots$$

where

$$a = \frac{1}{2}\left(\frac{1}{g_a} + \frac{1}{g_b}\right), \quad b = \frac{2}{\pi}\left(\frac{1}{g_a} - \frac{1}{g_b}\right).$$

In this analysis, components up to the third term are considered to include the effect of third harmonic of MMF. Therefore, the inverse gap function for $\tau_p = 90°$ is

$$g^{-1}(\phi - \theta) = a - b\cos 2(\phi - \theta) + \frac{b}{3}\cos 6(\phi - \theta) \tag{9}$$

and g_a and g_b are the minimum and maximum airgap length, respectively. Using the method presented in [15], the self inductances are readily calculated:

$$L_{aa} = L_{\ell a} + K\left\{\frac{10\pi a}{9} - \frac{\pi b}{6}\cos 2\theta + \frac{\pi b}{54}\cos 6\theta\right\}: \text{ etc} \tag{10}$$

where

$$K = \mu_0 r \ell \frac{4N^2}{\pi^2} \tag{11}$$

with r being the stator inner radius and ℓ the effective stator stack length. The quantity $L_{\ell a}$ is the leakage inductance of phase "a."

Similarly, the mutual inductance between phases "a" and "b" is given by

$$L_{ab} = K\left\{\pi a\left[\cos\frac{2\pi}{5} + \frac{1}{9}\cos\frac{4\pi}{5}\right] - \frac{\pi b}{6}\left[-2\cos\frac{4\pi}{5} + 3\right] \cdot \cos 2\left(\theta + \frac{4\pi}{5}\right) + \frac{\pi b}{54}\cos 6\left(\theta + \frac{4\pi}{5}\right)\right\}: \text{ etc.} \tag{12}$$

IV. Stator Voltage Equations

Since the inductance matrix L_{ss} varies with the position of the rotor, the second term of (1) can be written as

$$\frac{d\Lambda_s}{dt} = L_{ss}\frac{dI_s}{dt} + \frac{dL_{ss}}{dt}I_s. \tag{13}$$

The second term in the above equation can be written using the chain rule as

$$\frac{dL_{ss}}{dt}I_s = \frac{dL_{ss}}{d\theta_{rm}}\frac{d\theta_{rm}}{dt}I_s. \tag{14}$$

Defining rotor mechanical speed as

$$\omega_{rm} = \frac{d\theta_{rm}}{dt}$$

then

$$\frac{dL_{ss}}{dt}I_s = \omega_{rm}\frac{dL_{ss}}{d\theta_{rm}}I_s. \tag{15}$$

Therefore, (13) can typically be written in the form

$$\frac{d\Lambda_s}{dt} = L_{ss}\frac{dI_s}{dt} + \omega_{rm}\frac{dL_{ss}}{d\theta_{rm}}I_s. \tag{16}$$

Substituting (15) in (1) yields the voltage equation in the $abcde$ system:

$$V_s = R_s I_s + L_{ss}\frac{dI_s}{dt} + \omega_{rm}\frac{dL_{ss}}{d\theta_{rm}}I_s. \tag{17}$$

V. Transformation of the Machine Voltage Equations to the Synchronous Reference Frame

The set of equations previously developed are sufficiently detailed to describe both the transient and steady-state behavior of a five-phase synchronous reluctance machine. However, these equations are rather complex, due to the degree of coupling between windings. Also, the mutual inductances are a function of the rotor position. It is desirable to represent the machine with a simpler set of equations, without sacrificing any generality. When space harmonics are ignored, there are well-known transformations that can bring about simplification, especially for three-phase systems. However, when the space harmonics are included and the system has more than three phases, the task is rather formidable. Fortunately, for the five-phase synchronous reluctance machine under study, the effect of the third harmonic plays a significant role. Therefore, the necessary transformation required to simplify the system is as shown by (18) at the bottom of the page.

This transformation has the following pseudoorthogonal property:

$$T(\theta)^{-1} = \tfrac{5}{2}T(\theta) \tag{19}$$

which, explicitly, is given by (20) at the bottom of the next page.

Also, it can readily be shown that

$$T(\theta)\frac{dT(\theta)^{-1}}{dt} = \omega\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \omega x. \tag{21}$$

The use of the symbols "ωx" is used to call attention to the fact that this operation is equivalent to the vector cross product.

Similarly,

$$\frac{dT(\theta)}{dt}T(\theta)^{-1} = -\omega x. \tag{22}$$

$$T(\theta) = \frac{2}{5}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{5}\right) & \cos\left(\theta - \frac{4\pi}{5}\right) & \cos\left(\theta + \frac{4\pi}{5}\right) & \cos\left(\theta + \frac{2\pi}{5}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{5}\right) & \sin\left(\theta - \frac{4\pi}{5}\right) & \sin\left(\theta + \frac{4\pi}{5}\right) & \sin\left(\theta + \frac{2\pi}{5}\right) \\ \cos 3\theta & \cos 3\left(\theta - \frac{2\pi}{5}\right) & \cos 3\left(\theta - \frac{4\pi}{5}\right) & \cos 3\left(\theta + \frac{4\pi}{5}\right) & \cos 3\left(\theta + \frac{2\pi}{5}\right) \\ \sin 3\theta & \sin 3\left(\theta - \frac{2\pi}{5}\right) & \sin 3\left(\theta - \frac{4\pi}{5}\right) & \sin 3\left(\theta + \frac{4\pi}{5}\right) & \sin 3\left(\theta + \frac{2\pi}{5}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{18}$$

Multiplying (1) by the necessary transformation given by (18) in order to transform the voltage equations into the synchronous rotating reference frame yields

$$T(\theta)V_s = R_s T(\theta) I_s + T(\theta)\frac{d\Lambda_s}{dt}. \tag{23}$$

The second term of the right-hand side in the above equation can be written using the chain rule as

$$T(\theta)\frac{d\Lambda_s}{dt} = \frac{d}{dt}[T(\theta)\Lambda_s] - \frac{dT(\theta)}{dt}T(\theta)^{-1}T(\theta)\Lambda_s. \tag{24}$$

Upon multiplying the stator flux linkages by the transformation $T(\theta)$ yields the stator flux linkages in the d–q system, given by

$$T(\theta)\Lambda_s = T(\theta)L_{ss}T(\theta)^{-1}T(\theta)I_s = L_{dqs}I_{dqs} = \Lambda_{dqs} \tag{25}$$

where L_{dqs} is the stator inductance matrix in the d–q system and can be written as (26), at the bottom of the next page, where L_{ls} represents that portion of the leakage inductance which is not associated with flux in the airgap (slot and end winding leakage).

Since this component of leakage inductance is identical for all five phases, it passes unchanged under the d–q transformation in much the same manner as the stator resistance. The mutual inductances are given by

$$\begin{aligned} L_{mq1} &= \tfrac{5}{2}K\pi[a + \tfrac{1}{2}b], \quad L_{md1} = \tfrac{5}{2}K\pi[a - \tfrac{1}{2}b] \\ L_{mq3} &= \tfrac{5}{18}K\pi[a - \tfrac{1}{6}b], \quad L_{md3} = \tfrac{5}{18}K\pi[a + \tfrac{1}{6}b] \\ L_{m13} &= \tfrac{5}{2}K\pi[\tfrac{1}{6}b]. \end{aligned} \tag{27}$$

It is important to notice that the transformation presented by (18) resulted successfully in a matrix inductance in the synchronous reference frame which is no longer a function of the rotor position in much the same manner as for three-phase machines.

Using (24) and (25) in (23) yields

$$V_{dqs} = R_s I_{dqs} + \frac{d\Lambda_{dqs}}{dt} + \omega x \Lambda_{dqs}. \tag{28}$$

In summary, the voltage equations in terms of flux linkages can, therefore, be written as

$$v_{qs1} = r_s i_{qs1} - \omega\lambda_{ds1} + \frac{d\lambda_{qs1}}{dt} \tag{29}$$

$$v_{ds1} = r_s i_{ds1} + \omega\lambda_{qs1} + \frac{d\lambda_{ds1}}{dt} \tag{30}$$

$$v_{qs3} = r_s i_{qs3} - 3\omega\lambda_{ds3} + \frac{d\lambda_{qs3}}{dt} \tag{31}$$

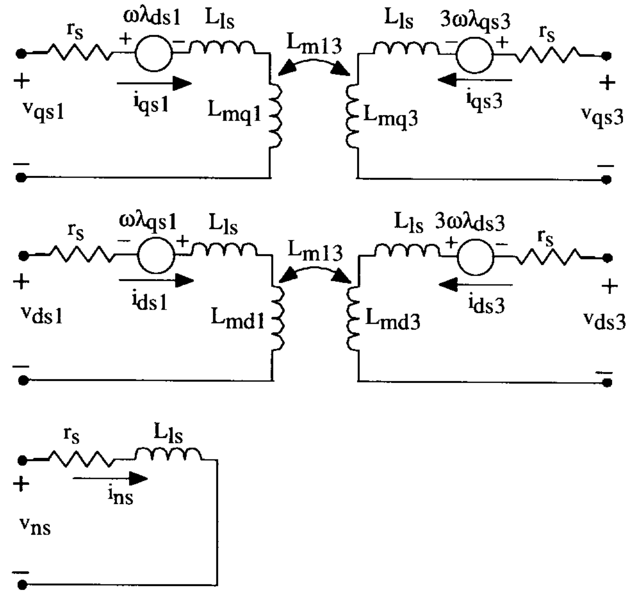

Fig. 4. The equivalent circuit of a five-phase synchronous reluctance machine in the synchronous reference frame.

$$v_{ds3} = r_s i_{ds3} + 3\omega\lambda_{qs3} + \frac{d\lambda_{ds3}}{dt} \tag{32}$$

$$v_{ns} = r_s i_{ns} + \frac{d\lambda_{ns}}{dt}. \tag{33}$$

In a five-phase balanced system without a neutral connection, the zero-sequence current (n-axis current) does not exist, resulting only in four equations. Also, it is important to mention that, if no third harmonic of current is applied, λ_{ds3} and λ_{qs3} still do exist, which is due to the choice of a 90° pole arc for the rotor.

Using (29)–(33), the equivalent circuit of the five-phase synchronous reluctance machine in the synchronous d–q frame is given in Fig. 4.

VI. Electromagnetic Torque

The mechanical equation of motion depends upon the characteristics of the load, which may differ widely from one application to the next. We will assume here, for simplicity, that the torque which opposes that produced by the machine consists only of an inertial torque and an external load torque, which are known explicitly. In this case, the mechanical

$$T(\theta)^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & \cos 3\theta & \sin 3\theta & \frac{1}{\sqrt{2}} \\ \cos\left(\theta - \frac{2\pi}{5}\right) & \sin\left(\theta - \frac{2\pi}{5}\right) & \cos 3\left(\theta - \frac{2\pi}{5}\right) & \sin 3\left(\theta - \frac{2\pi}{5}\right) & \frac{1}{\sqrt{2}} \\ \cos\left(\theta - \frac{4\pi}{5}\right) & \sin\left(\theta - \frac{4\pi}{5}\right) & \cos 3\left(\theta - \frac{4\pi}{5}\right) & \sin 3\left(\theta - \frac{4\pi}{5}\right) & \frac{1}{\sqrt{2}} \\ \cos\left(\theta + \frac{4\pi}{5}\right) & \sin\left(\theta + \frac{4\pi}{5}\right) & \cos 3\left(\theta + \frac{4\pi}{5}\right) & \sin 3\left(\theta + \frac{4\pi}{5}\right) & \frac{1}{\sqrt{2}} \\ \cos\left(\theta + \frac{2\pi}{5}\right) & \sin\left(\theta + \frac{2\pi}{5}\right) & \cos 3\left(\theta + \frac{2\pi}{5}\right) & \sin 3\left(\theta + \frac{2\pi}{5}\right) & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{20}$$

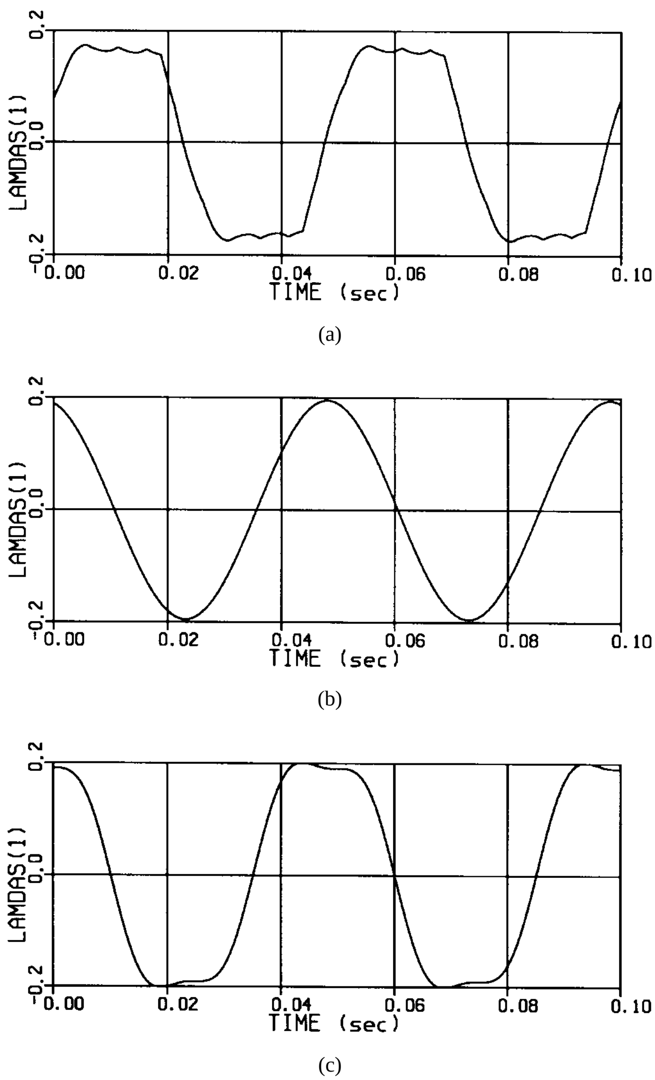

Fig. 5. Stator flux linkages of phase a at full load under only fundamental current excitation. (a) Case I. (b) Case II. (c) Case III.

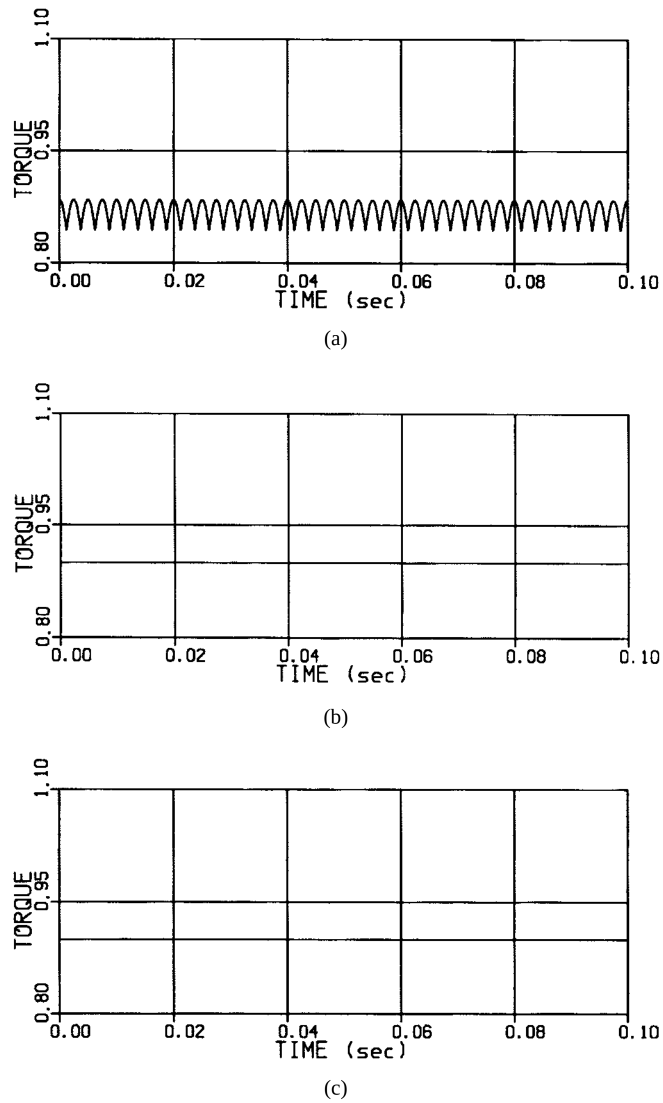

Fig. 6. Electromagnetic torque developed under conditions of Fig. 5 in Newton meters. (a) Case I. (b) Case II. (c) Case III.

equation of motion is simply the usual expression

$$J\frac{d^2\theta_{rm}}{dt^2} + T_L = T_e \tag{34}$$

where θ_{rm} is the angular displacement of the rotor, T_L is the load torque, and T_e is the electromagnetic torque produced by the machine.

The electrical torque can be found from the magnetic coenergy W_{co}

$$T_e = \left[\frac{\partial W_{co}}{\partial \theta_{rm}}\right]_{(I_s \text{ constant})}. \tag{35}$$

In a linear magnetic system, the coenergy is equal to the stored magnetic energy

$$W_{co} = \tfrac{1}{2} I_s^t L_{ss} I_s. \tag{36}$$

Therefore, the electromagnetic torque will be

$$T_e = \frac{1}{2} I_s^t \frac{\partial L_{ss}}{\partial \theta_{rm}} I_s. \tag{37}$$

Thus far, we have assumed, for simplicity, that the machine has only two poles. In general, let P denote the number of motor poles. It is clear that any inductance which is a function of angular displacement undergoes $P/2$ complete cycles as

$$L_{dqs} = \begin{bmatrix} L_{\ell s} + L_{mq1} & 0 & L_{m13} & 0 & 0 \\ 0 & L_{\ell s} + L_{md1} & 0 & L_{m13} & 0 \\ L_{m13} & 0 & L_{\ell s} + L_{mq3} & 0 & 0 \\ 0 & L_{m13} & 0 & L_{\ell s} + L_{md3} & 0 \\ 0 & 0 & 0 & 0 & L_{\ell s} \end{bmatrix} \tag{26}$$

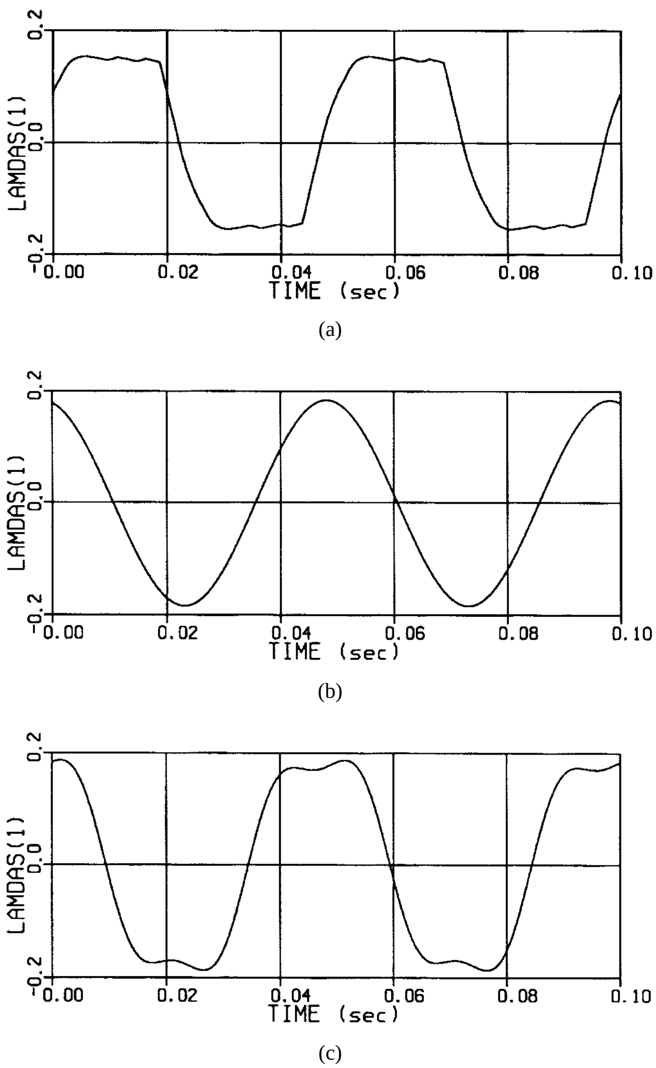

Fig. 7. Stator flux linkages of phase a at full load under combined fundamental and third harmonic of current excitation. (a) Case I. (b) Case II. (c) Case III.

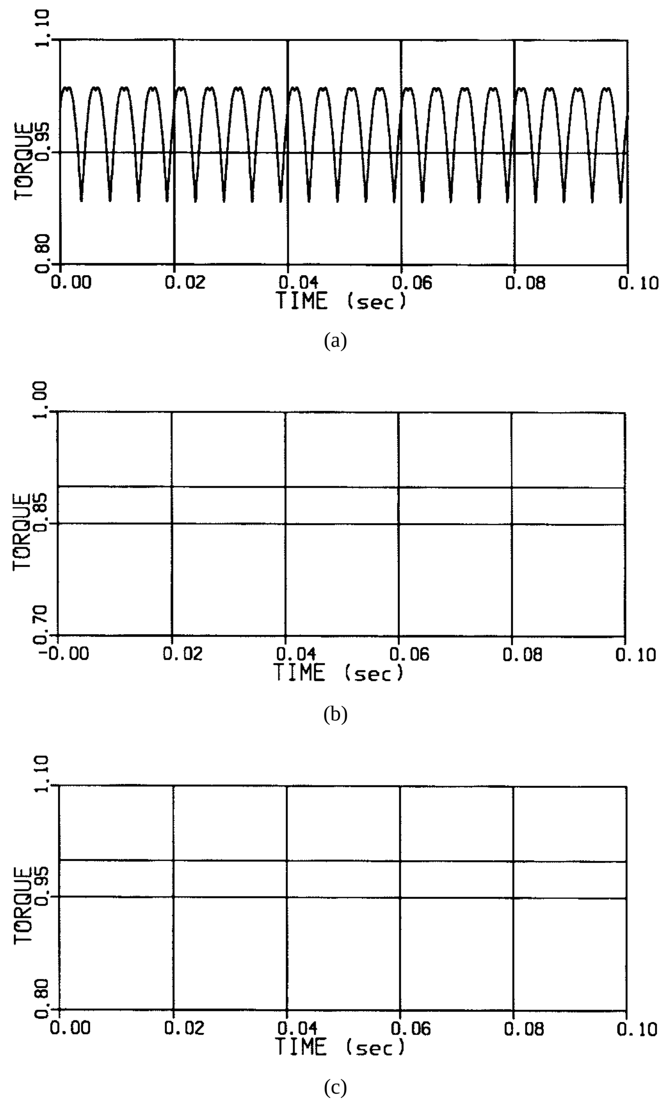

Fig. 8. Electromagnetic torque developed under conditions of Fig. 7 in Newton meters. (a) Case I. (b) Case II. (c) Case III.

θ_{rm} varies from 0 to 2π. That is,

$$\theta_r = \frac{P}{2}\theta_{rm}. \tag{38}$$

The angle θ_r is called the rotor displacement in electrical radians. Using the transformation in the following equation results in

$$T_e = \frac{P}{4}[T(\theta)I_s]^t \frac{5}{2} T(\theta)\frac{\partial L_{ss}}{\partial \theta_r} T(\theta)^{-1}[T(\theta)I_s]. \tag{39}$$

Simplifying the above equation, electromagnetic torque for a five-phase current-regulated synchronous reluctance motor is given by the following algebraic equation:

$$\begin{aligned} T_e = \frac{P}{2}\frac{5}{2}\{&[L_{mq1} - L_{md1}]i_{qs1}i_{ds1} \\ &- 2L_{m13}[i_{qs1}i_{ds3} - i_{ds1}i_{qs3}] \\ &+ 3[L_{mq3} - L_{md3}]i_{qs3}i_{ds3}\}. \end{aligned} \tag{40}$$

This equation clearly represents the dependence of the electromagnetic torque on the fundamental and third harmonic of the d- and q-axes currents $(i_{qs1}, i_{ds1}, i_{qs3}, i_{ds3})$. For our choice of rotor structure with no flux barrier, the effect of third harmonic magnetizing inductances $(L_{mq3}, L_{md3}, L_{m13})$ is shown with constant terms in (40). In cases where the machine is subjected to only fundamental current, then the resultant electromagnetic torque equation will be the familiar equation given for the synchronous reluctance machine operated from a sinusoidal supply modified for five phases. Another result of (40) is the fact that, if the third harmonic of current does not exist, there will not be any associated third harmonic component of torque. However, a third harmonic of inductance will still have an influence on the terminal voltage and stator current.

VII. Digital Computer Simulation Results

In order to demonstrate the importance of this new model, a four-pole synchronous reluctance machine with five phases was simulated. In this study, the machine was assumed to

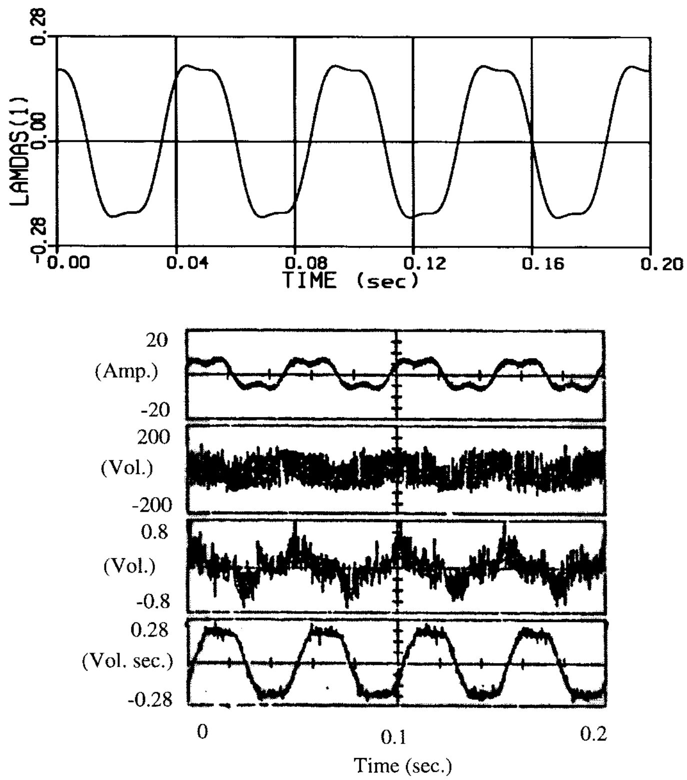

Fig. 9. Comparison of calculated and measured stator flux linkages at no load. Top trace: digital computer result in volt seconds. Bottom trace: test results; from top to bottom are stator current, phase voltage, search coil voltage, stator phase flux linkages.

have 24 stator slots. The rotor was considered to have four poles with a 90° pole arc. It was assumed that the motor shaft is rotating at constant speed. Three different cases have been studied. Case I is based on the method presented in [3]. Case II assumes a sinusoidal variation of the inductances with respect to rotor position. Case III uses the d–q method including the third harmonic of the winding functions already presented in this paper. Fig. 5 illustrates the total flux linking phase $\boldsymbol{a}$, assuming that the machine is excited with fundamental current only at full load. The result is the same as that predicted from the usual three-phase d–q model, except that the torque is appropriately increased due to the use of five rather than three phases. Fig. 6 shows the corresponding electromagnetic torque showing the same average torque for all cases.

Shown in Fig. 7 are the flux linkages when the machine is excited with a combined fundamental and 33% third harmonic of current, whereby the rms current is maintained at the same value as in Fig. 5. Since the rms current is the same as in Figs. 5 and 7, the total copper losses are, therefore, equal. Fig. 8 illustrates the electromagnetic torque for this situation. Notice the error in the output torque which exists when only sinusoidal variation of the inductances are considered. However, since the machine is excited with the combined fundamental and third harmonic of the current, the fundamental torque is reduced because of the addition of third harmonic in the current. On the contrary, if the effect of third harmonic in the inductances is considered, as shown in Fig. 8, there is calculated at least a 10% improvement in the torque production, which has already been confirmed by finite-element calculations [3].

VIII. Experimental Results

In order to verify the new model that has been developed, a five-phase synchronous reluctance motor was designed and fabricated in a standard induction motor frame size rated at 5.6 kW, four poles, 60 Hz, 460 V, three phase. However, the stator of the five-phase machine has 40 slots, rather than 36, as was the case for the original stator for the induction machine. The machine is wound with full-pitch single-layer coils and has four poles, which results in two slots per pole per phase. The salient pole rotor was milled from the corresponding three-phase squirrel-cage induction motor. Half of the pole pitch area for each pole was milled away to resemble the 90 electrical degrees pole arc achieved in the previous analysis.

The machine was excited with only half of the windings energized, meaning one slot per pole per phase. In this case, the MMF produced by exciting a coil at one instant of time resembles a rectangular waveform. Therefore, the assumption of rectangular waveforms for the winding functions are more realistic. However, energization of the entire machine results in a stepwise MMF for each coil, which requires a certain amount of modification in the winding functions models. The bottom trace in Fig. 9 gives an experimental result showing the flux linking one stator coil after integration of the voltage induced in a search coil parallel to that phase. The high-frequency noise is due to the switching. The machine is running at no load under a combined excitation of fundamental and 33% third harmonic. For the sake of comparison, the digital computer result using d–q modeling is given in the top trace in Fig. 9. The similarity between the experimental result and the simulation using the d–q model is obvious.

IX. Conclusion

In this paper, a detailed analysis of the d–q equations for a five-phase synchronous reluctance machine including the third harmonic of the airgap MMF was presented. An equivalent circuit based on these equations was developed. A digital computer simulation was used to confirm the model and as a means of comparison between the d–q model and the equations in the natural system (abc variable). The computer results were compared to the experimental traces obtained from a specially constructed machine. Agreement between measured and calculated waveforms was demonstrated.

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