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Yuefeng Liao
Emerson Motor Technology Ctr.
Emerson Electric Co.

Thomas A. Lipo
Dept. of Electrical and Computer Engineering
University of Wisconsin-Madison
1415 Johnson Drive
Madison, WI 53706

WEMPEC

Department of Electrical and Computer Engineering
1415 Johnson Drive
Madison, Wisconsin 53706
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SIZING AND OPTIMAL DESIGN OF DOUBLY SALIENT PERMANENT MAGNET MOTORS

Yuefeng LIAO* and Thomas A. LIPO

Dept. of Electrical and Computer Engineering, University of Wisconsin-Madison, USA
*Emerson Motor Technology Center, Emerson Electric Co. USA

INTRODUCTION

Recent development of the doubly salient permanent magnet (DSPM) motor has shown that the DSPM motor is of high efficiency, high power density with an extremely simple structure [1-3]. This new type of PM motor has evolved from the doubly salient Flux-Switch generator [4]. However, like the Variable Reluctance Motor (VRM), since the airgap field of this type of motor is of pulsating instead of rotating nature, it is less intuitive as to how to size the motor, and to compare the torque density of the motor with that of other well-established types of motors.

The objective of this paper is to develop a methodology for sizing and optimal design of DSPM motors to meet various customer specified performance requirements. The approach is modeled on the one proposed by Honsinger for induction machines [5]. This is a synthesizing process, linking the main dimensions of the motor to the specified performance. The basic principles of the DSPM motor are first reviewed. Next, the sizing relationships of the DSPM motor are explored, which serves as a means for design optimization of the motor. Design of a prototype DSPM motor is then given and, finally, a nonlinear program algorithm , the sequential nonlinear quadratic program (SNQP) is used to double check the methodology proposed.

BASIC PRINCIPLES OF OPERATION AND CONTROL

Configuration of the DSPM Machine

Figure 1 shows the cross-section of a 3-phase, 6/4-pole DSPM motor with stationary magnets. This 3-phase, 6/4-pole configuration is the most simple pattern for motoring operation requiring satisfactory starting performance. However, two phase or even single phase schemes can be conceived for use as a generator. For low speed and high torque applications, the machine can be constructed in repetitive fashion, for example a 12/8 pole configuration.

It can be noted that the rotor of the DSPM machine is identical to that of the three phase variable reluctance machine. The stator structure is also similar to that of the SRM except that two pieces of PM are buried in the core and therefore introduced into the main flux path of the stator windings. In order to achieve a high flux concentration in such machines, use is made of the corners of the stator lamination which are normally discarded. This changes the physical appearance of the motor to either a square or "football" shaped cross-section and adds slightly to the weight and space occupied by the machine. High performance PM material with a linear demagnetizing characteristic is used to sustain the magnetization and demagnetization of the armature reaction so as to keep a nearly constant flux level within the air gap. The stator pole arc is set to be $\pi/6$ mechanical radians and the rotor pole arc selected to be slightly greater than the stator pole arc to allow for current reversal. As configured, the airgap reluctance seen by the PM excitation is invariant of rotor position if fringing is negligible. Therefore, there is essentially no cogging torque produced at no-load. A linear variation of the PM induced flux linkage and thus a trapezoidal back EMF is induced in each of the stator windings at no-load as shown in Fig. 1.

When the machine is loaded, armature reaction flux is produced in the windings in addition to the PM induced flux. It is important to note, however, that the existence of the PM constitutes a very high reluctance path for the armature reaction flux and thus forces the bulk of the armature reaction flux to circulate through another overlapped pole pair. As a result, the active stator phase winding will possess small inductance at <u>both</u> aligned and unaligned positions, and the maximum inductance appears when the poles are, in fact, essentially half overlapped, as illustrated in Fig. 2. In contrast to the VRM this small aligned inductance makes it possible to reverse the current rapidly at the aligned position. Therefore, torque can be produced both by applying positive current to the winding when its PM-induced flux is increasing and by applying negative current while the flux is decreasing, as shown in Fig. 1.

Torque Production in the DSPM Machine

As a first approximation, the variation of the winding inductance and the PM induced flux linkage of an active stator phase winding are assumed to be piece-wise linear and spatially dependent only, as shown in Fig. 2. The terminal voltage equation for an active stator phase winding is then

$$v = Ri + e \approx e = \frac{d\lambda}{dt} \quad (1)$$

The flux linkage λ is composed of the PM induced flux linkage λ_m and the armature reaction flux linkage (Li):

$$\lambda = Li + \lambda_m \quad (2)$$

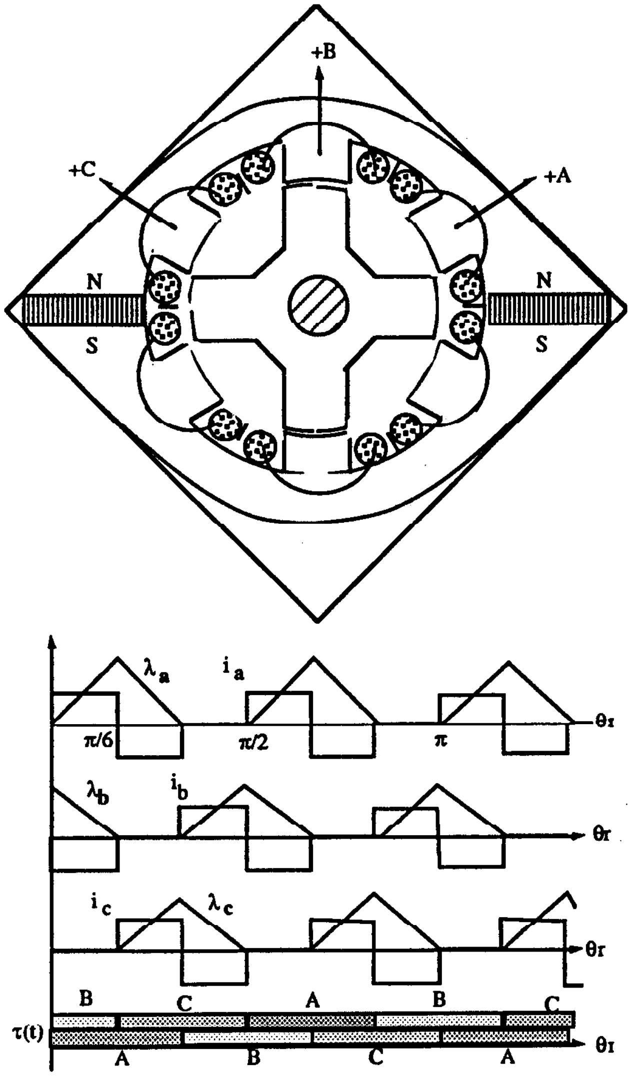

Fig. 1 Illustration of Operating Principles of the DSPM Motor

Therefore,

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dt} + \frac{d\lambda_m}{dt}$$

$$= L\frac{di}{dt} + e_r + e_m \qquad (3)$$

The electrical power entering any of the windings is, neglecting ohmic and iron losses,

$$P_e = ei = iL\frac{di}{dt} + i^2\frac{dL}{dt} + i\frac{d\lambda_m}{dt}$$

$$= \frac{d}{dt}\left(\frac{1}{2}Li^2\right) + \left(\frac{1}{2}i^2\frac{\partial L}{\partial\theta_r} + i\frac{\partial\lambda_m}{\partial\theta_r}\right)\omega_r \qquad (4)$$

Power balance gives

$$P_e = \frac{d}{dt}W_f + T\omega_r \qquad (5)$$

Hence, the torque can be written as the sum of two components, reluctance torque and reaction torque,

$$T = \frac{1}{2}i^2\frac{\partial L}{\partial\theta_r} + i\frac{\partial\lambda_m}{\partial\theta_r} = T_r + T_m \qquad (6)$$

Up to rated speed, if current regulation is maintained, the reluctance torque will be of zero average because of the triangle-shaped variation of the winding inductance. Hence the rated averaged torque will be

$$T = T_m = 2i\frac{\partial\lambda_m}{\partial\theta_r} = 2k_d\,\lambda_m I_{pk} \qquad (7)$$

where the factor of 2 allows for the fact that two phases are conducting at a time, and λ_m is the PM flux linkage per phase while k_d represents flux leakage and is approximately equal to 0.9.

It is clear that the DSPM motor is, in principle, similar to the PM brushless DC (PM-BLDC) motor with a 120^o quasi-square current waveform. The only difference is that the two 120^o conducting current blocks are drawn together in the case of the DSPM motor. At high speed, the current cannot be maintained constant due to the excessive PM induced back EMF. In this case the current peaks in the first half stroke where the inductance is increasing and drops rapidly in the second half stroke where the inductance is decreasing. This uneven distribution of the phase current, however, gives rise to a considerable amount of reluctance torque which ultimately contributes to extending the constant power capability of the DSPM motor.

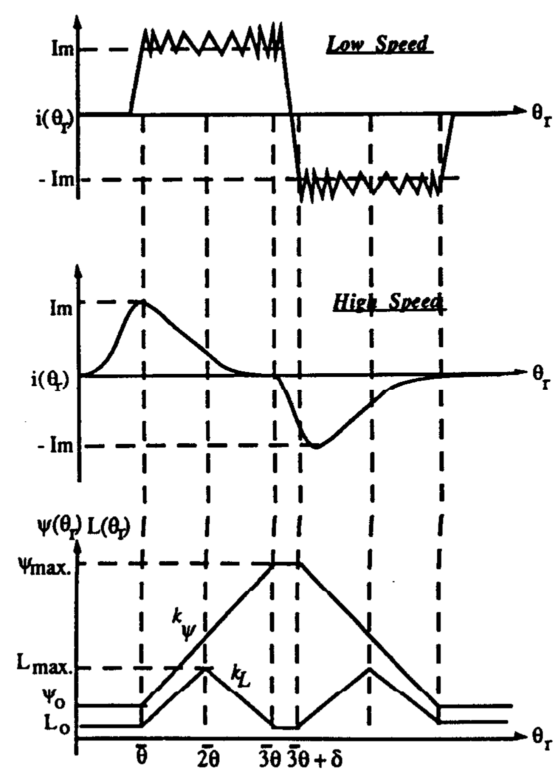

Fig. 2 Current Waveforms of the DSPM Motor

SIZING RELATIONSHIPS OF THE DSPM MOTOR

Power Density of the DSPM Motor

The sizing equation can now be derived from the torque equation (7) in a manner very similar to that used for classical motors, thereby bringing the designer's experience to bear on this new machine. The rated output power is related to the averaged torque and rotor speed as

$$P_n = T\omega_r = T_m\left[\frac{2\pi}{60}n_e\right] \tag{8}$$

where P_n := output power in horsepower
n_e := speed of the motor in rpm

Substituting the expression for T_m into Eqn. (8) and further assuming a 6/4 pole combination and one parallel circuit for the stator windings, we have

$$P_n = 2k_d\left\{\left[\frac{N_s}{\frac{\pi}{6}}\right]\left[\frac{\pi}{6}\left(\frac{D_r}{2}\right)L_{eff}B_g\right]\left[\frac{2\pi}{60}n_e\right]\right\}I_{pk}$$

$$=\sqrt{2}k_d\left[\frac{\pi^2}{120\sqrt{3}}\right]\left[\frac{3\left(\sqrt{\frac{2}{3}}I_{pk}\right)(2N_s)}{\pi D_r}\right]B_g[D_r^2L_{eff}]\,n_e$$

$$=\sqrt{2}k_d\left[\frac{\pi^2}{120\sqrt{3}}\right]B_g A_s[D_r^2L_{eff}]\,n_e \tag{9}$$

where N_s is the number of series turns of the stator windings and the rest notations have their usual meanings.

Equation (9) can be written as

$$\frac{P_n}{n_e} = \xi D_r^2L_{eff} \tag{10}$$

with

$$\xi = \sqrt{2}k_d\left[\frac{\pi^2}{120\sqrt{3}}\right]B_g A_s = C_m B_g A_s$$

Equation (10) is of the same form as the conventional $D_r^2L_{eff}$ equation which has been used for decades to size electrical machinery. As an observation, comparing Eqn. (9) with the well-known sizing equation for the induction machine

$$P_n = \left[\frac{\sqrt{2}\pi^2}{120}\right]k_w B_g A_s[D_r^2L_{eff}]\,n_e \tag{11}$$

noting that B_g for the DSPM motor is about the same as the flux density in the stator poles, thus about twice that for the induction machine, we can see immediately that an increase of $\frac{2}{\sqrt{3}}$ in torque production results even if A_s is kept the same.

A case has already been made for comparing the performance of the DSPM motors and other types of AC motors [1], showing that the DSPM motor is even substantially superoir over the PM-BLDC motors in terms of torque density and efficiency. For brevity, the result is not repeated here and the reader is referred to Ref. [1] for more details.

It is important to note that Eqn. (10), while relating output power P_n and rated speed n_e to the rotor volume through an output coefficient ξ containing the air gap quantities, i.e. flux density B_g and surface current density A_s, does not consider several key factors. For example, machine size is affected by the complete stator geometry which includes the relative proportions of the stator inner and outer diameters, slot and tooth dimensions, flux densities in the iron parts and the actual current densities in the conductors. There are no relationships in Eqn. (10), however, connecting these air gap quantities with the flux and current densities existing in the machine's interior. The complete stator geometry is determined passively after this computation, which often results in a design that is anything but optimal.

The $D_o^3L_{eff}$ Sizing Equation for DSPM Motors

In 1987 Honsinger proposed a new sizing equation which eliminates the deficiencies just mentioned [5]. The key element in Honsinger's approach is the establishment of the relationship of the amount of steel (or overall volume of the motor) and copper used as a function of the ratio of the outer diameter to the inner diameter of the motor λ (=D_i/D_o), while parameterizing broad design main dimensions. It has the form $\frac{P_n}{n_e} = \xi_o D_o^3 L_{eff}$. The equation is based primarily on stator geometry. The output coefficient ξ_o includes the iron flux densities and conductor current densities. It also contains a maximizing function $f_o(\lambda)$ which is extremely important to the maximization of the torque density of the motor. The reader is referred to Ref. [5] and for more details.

Honsinger's work is extended here to the DSPM motors. Instead of repeating the procedure to the case of DSPM motors, we can rearrange the output equation to have the same form as that of the induction motor to preserve all the relationships available.

Re-arranging the output equation we have

$$P_n = 2k_d\left\{\left[\left(\frac{N_s}{\frac{\pi}{6}}\right)\Phi_p\right]\omega_r\right\}I_{pk}$$

$$=\left[2k_d\frac{6}{\sqrt{3}\pi}\right]\left\{3\frac{1}{\sqrt{2}}N_s\Phi_p\,\omega_r\left[\sqrt{\frac{2}{3}}I_{pk}\right]\right\}$$

$$=\chi'\left[3\frac{1}{\sqrt{2}}N_s\Phi_p\,\omega_r I_{rms}\right] \tag{12}$$

Equation (12) has the same form as that of the 3-phase induction motor, with the only difference being in the definition of the performance constant χ'.

Further, by defining a fictitious flux density

$$B_g' = \frac{\pi}{6} B_g$$

it can be shown that that all the expressions for B_t, B_c and Φ_p will have the same form as that of the 3-phase induction motor. Therefore, the output equation can be expressed in the form as

$$\frac{P_n}{n_e} = \xi_o D_o^3 L_{eff} \tag{13}$$

where

$$\xi_o = 0.0390\, B_g' J_s f_o(\lambda)\, \chi' k_{cu} * 10^{-11}$$

with B_g' := airgap flux density

J_s := volume current density

χ' := $2 k_d * 6/\sqrt{3}\pi$

k_{cu} := the slot fill factor

and $f_o(\lambda) = \lambda f_{cu}(\lambda)$ (14)

with $f_{cu}(\lambda) = \alpha\lambda^2 - 2\beta\lambda + 1$

$$\alpha = (G_t + G_c)^2 - (1 - G_t)^2$$

$$\beta = G_t + G_c$$

$$G_t = k_s B_g / B_t$$

$$G_c = \left(\frac{2}{P}\right) k_s B_g / B_c$$

B_t := stator tooth flux density

B_c := stator yoke flux density

k_s := stacking factor

It is clear that the function $f_o(\lambda)$ incorporates a number of important sizing relations. Moreover, it can be shown that $f_o(\lambda)$ is essentially the ratio of the slot area to the entire cross-section of the motor. Therefore, the volume of the copper used (core only) can be expressed as

$$V_{cu} = k_{cu} f_o(\lambda) D_o^2 L_{eff} \tag{15}$$

and the slot area can also be found from $V_{cu} = A_{cu} L_{eff}$.

What is most important in this approach is that there exists a special value of λ where the function $f_o(\lambda)$ achieves its maximum value. Thus the overall volume of the machine is minimum with this value of λ. This point is given by

$$\frac{d f_o(\lambda)}{d\lambda} = 0$$

hence

$$\lambda_{(max)} = \frac{2\beta - \sqrt{4\beta^2 - 3\alpha}}{3\alpha} \tag{16}$$

It is very clear that the choice of λ can affect the performance significantly in both cases. Obviously, λ is one of the major optimizing variables.

Furthermore, multiplying the sizing equation for $D_o^3 L_{eff}$, i.e. Eqn. (13), and the traditional sizing equation for $D_r^2 L_{eff}$, i.e. Eqn. (10), and taking the square root yields another equation.

$$\frac{P_n}{n_e} = \sqrt{\xi_o \xi} \sqrt{D_0^3 D_r^2 L_{eff}} \tag{17}$$

Eqn. (17) can be expressed in terms of D_o after replacing D_r with $D_i = \lambda D_o$.

$$\frac{P_n}{n_e} = \sqrt{\xi_o \xi_r \lambda^2}\, D_o^{2.5} L_{eff} = \xi'_o D_o^{2.5} L_{eff} \tag{18}$$

The factor ξ'_o in Eqn. (18) is

$$\xi'_o = 0.0779 B_g \chi' \sqrt{J_s A_s} \sqrt{k_{cu} \lambda^2 f_o(\lambda)} * 10^{-11} \tag{19}$$

What is special to Eqn. (19) is the term $\sqrt{J_s A_s}$. It can be shown that the product is proportional to the stator I^2R loss (core only) per unit of stator core area

$$\rho_s J_s A_s = \frac{m_1 I_{rms}^2 R}{\pi D_i L_{eff}}$$

where ρ_s is the resistivity of the stator conductors. Therefore, it is closely related to the temperature rise of the machine, especially in the case of totally enclosed, naturally cooling. Equation (19) is thus very suitable for sizing of machines under this cooling condition. Also, the function $\lambda^2 f_o(\lambda)$ in the output coefficient of Eqn. (19), like $f_o(\lambda)$, displays a maximum given by

$$\lambda_{(max)} = 0.8\, \frac{\beta - \sqrt{\beta^2 - 0.9375\alpha}}{\alpha} \tag{20}$$

where the overall volume of the machine is a minimum.

While absent in Honsinger's paper, the effect of the aspect ratio on the power density can be obtained easily from Eqn. (13), noting that the volume of the steel is $D_o^2 L_{eff}$,

$$\tau = \frac{\frac{P_n}{n_e}}{D_o^2 L_{eff}} = \frac{\xi_o D_o^3 L_{eff}}{D_o^2 L_{eff}} = \xi_o D_o \tag{21}$$

A similar result can be obtained from the $D_o^{2.5} L_{eff}$ sizing equation.

It is clear that the torque density increases with the outer diameter, thus a smaller aspect ratio is desirable to achieve high torque density. However, this argument excludes the end effect of the motor. It is obvious that an exceptionally small aspect ratio would cause excessive increase in the end-turn windings and the end region space required, which adversely affects the overall size and weight of the motor.

OPTIMAL DESIGN OF THE DSPM MOTOR

The sizing approach is, unfortunately, very elementary, with many secondary effects still excluded from the process. Moreover, it cannot handle conflicting performance requirements, such as efficiency, torque-to-inertia ratio, etc. Design optimization based on nonlinear programming algorithms, on the other hand, has made significant progress recently. The sequential quadratic nonlinear program-ming (SQNP) technique, which has been recognized as an ideal algorithm for medium size problems and has been successfully applied to electric machine design optimization [6], has been chosen for optimal design of the DSPM motor. However, it should be noted that although the SQNP has very good convergence properties, it cannot guarantee global optimum because of the complicated shape of the objective function. Therefore, the sizing approach, empirical as it is, does provide a supplement to the SQNP in providing a good starting point.

A prototype DSPM motor is optimized using this optimization approach to achieve a minimum size (D_o^2L) while keeping the output power and other performance almost unchanged. The following main dimensions of the DSPM motor are selected as principal variables for design optimization:

- stator outer diameter D_o;
- stator inner diameter D_i;
- stacking length L_{eff};
- stator pole height d_{ss};
- rotor pole height d_{rr};
- number of series turns N_s;
- magnet height l_m.

The constraints are chosen as follows:

- efficiency at rated operating point η;
- total weight of the motor wt;
- total material cost of the motor Cst;
- RMS stator winding current density J_s;
- temperature rise of the windings - ΔT;
- current reversal period $\Delta\theta$;
- slot fill factor f_{slot}.

Comparison of the original design and the optimal design is shown in Table I. The size of the motor has been reduced by 20% as a result of this optimization. Note that the optimal design takes on a much smaller aspect ratio, which is consistent with the sizing relationships as revealed in last section. Also note that *the total weight is reduced only by 10%, which clearly* shows the effect of the end turn winding.

CONCLUSION

The sizing relationships of the newly developed DSPM motor have been explored, showing that the DSPM motor is comparable to the PM-BLDC motor in terms of power density and efficiency, and that the selection of the ratio of stator OD to ID has a dominant effect on motor performance, and thus is a critical factor.

Table I COMPARISON OF DSPM MOTOR DESIGNS

Parameters	Initial Design	Optimal Design
Stator OD (Do)	77.8 mm	94 mm
StatorID (Di)	40 mm	50 mm
Stack (L_{eff})	50 mm	28. mm
Airgap (g)	0.45 mm	(same)
βs	30 Deg.(mech.)	(same)
βr	36 Deg.(mech.)	(same)
Rotor ID (Dri)	12 mm	(same)
k_{fe}	0.95	(same)
Turns Number	130	170
M.L.T.	152 mm	121 mm
A.W.G.	20	(same)
Slot Fill (k_s)	36%	(same)
Copper Weight	0.61lbs	0.63 lbs
Magnet length	2.0 mm	2.0 mm
Magnet width	28 mm	37.6 mm
Magnet weight	0.10 lbs	0.08 lbs
Volume (D_o^2L)	302.6 cm^3	247.4 cm^3
(normalized)	1.0	0.82
Total Weight	3.23 lbs	2.97 lbs
(normalized)	1.0	0.91

Together with the SQNP nonlinear programming technique, this optimal design methodology proves to be an invaluable tool for design of the DSPM motor for certain applications.

ACKNOWLEDGMENT

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REFERENCES

[1] Y. Liao and T.A. Lipo:" A new doubly salient permanent magnet motor for adjustable speed drives" SPEEDAM, Positano, Italy, 1992

[2] Y. Liao, F. Liang and T.A. Lipo:"A novel permanent magnet motor with doubly salient structure", *IEEE IAS Annual Meeting 1992*, Houston, Texas, 1992

[3] Y. Liao: *"Design and Performance Evaluation of a New Class of Permanent Magnet Motors with Doubly Salient Structure"*. Ph.D. Thesis, Univ. of Wisconsin-Madison, 1992

[4] S.E. Rauch and L.J. Johnson:"Design principles of flux-switch alternators", *AIEE Trans.* Dec. 1955, pp.1261-68

[5] V.B. Honsinger: 'Sizing Equations for Electric Machinery" *IEEE Trans. on Energy Conversion*, vol. EC-2,no.1, March 1987, pp.116-121

[6] H. Huang, E.F. Fuchs:"Successive nonlinear quadratic programming for high power density, high efficiency PM motors", *ICEM-90*, Cambridge, Massachusetts.