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TRANSIENT BEHAVIOR COMPARISON OF SATURATED INDUCTION MACHINE MODELS

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Abstract-In this paper two different digital computer simulation models which take into account magnetic saturation of an induction machine are compared. In the first model the machine differential equations are expressed in terms of flux linkages as state variables, while in the second, the currents are used as the differential equations state variables. The d-q based machine equations are computed in a synchronously rotating reference frame. The two models are compared for various transient conditions which include line starting of the machine and a step change in either load torque or supply voltage. The simulation results obtained clearly show that the two machine models have identical behavior during both transient and steady state conditions, with the flux linkage state variable based model being superior in terms of compactness, computational time and representation of the real physical system.

1. INTRODUCTION

Throughout the past twelve years numerous papers have been written concerning the simulation of core saturated induction machines [e.g. 1-5]. In general, these papers use currents as their model state variables, usually without stating the reasoning behind their preference. In the process of analysis a so-called "cross-saturation" phenomenon bas been identified which is supposedly an important new discovery which had been "neglected" prior to this time. In actual fact, prior to 1981, flux linkages rather than currents were used to model saturation [6], primarily to avoid the difficulties "discovered" in these later references. The purpose of this paper is to investigate these two different simulation approaches by simulating a 5 HP induction machine in a synchronously rotating reference frame fixed to the supply voltage vector.

2. SATURATION CURVE

In round rotor machines (e.g. induction machine) saturation of the air gap flux is modeled in both axes (λ_{mdsat} & λ_{mqsat}). In this simulation approach [7] an approximate saturation curve defined by four straight lines with declining slopes is used. Having the curve in per unit as saturated (λ_{msat}) versus unsaturated air gap flux, (λ_{mu}) the slope of the first line (air gap line) is 45°. The point of 1.2 pu for unsaturated flux corresponds to 1 pu for saturated flux on this curve. Thus operation at this point represents the no-load magnetizing inductance for rated voltage.

3. SATURATION MODEL WITH FLUX LINKAGES AS STATE VARIABLES

The four differential equations used to simulate a saturation model of an induction machine in terms of flux linkages as state variables are :

$$\frac{d\lambda_{qs}}{dt} = V_{qs} - \frac{r_s}{L_{ls}}\left(\lambda_{qs} - \lambda_{mqsat}\right) - \omega\lambda_{ds} \qquad (1)$$

$$\frac{d\lambda_{ds}}{dt} = V_{ds} - \frac{r_s}{L_{ls}}\left(\lambda_{ds} - \lambda_{mdsat}\right) + \omega\lambda_{qs} \qquad (2)$$

$$\frac{d\lambda_{qr}}{dt} = V_{qr} - \frac{r_r}{L_{lr}}\left(\lambda_{qr} - \lambda_{mqsat}\right) - \left(\omega - \omega_r\right)\lambda_{dr} \qquad (3)$$

$$\frac{d\lambda_{dr}}{dt} = V_{dr} - \frac{r_r}{L_{lr}}\left(\lambda_{dr} - \lambda_{mdsat}\right) + \left(\omega - \omega_r\right)\lambda_{qr} \qquad (4)$$

Where ω and ω_r are the reference frame and rotor angular speed of rotation respectively. L_{ls} and L_{lr} are the stator and rotor leakage inductances. The subscripts qs,ds,qr,and dr represent stator and rotor total flux linkages in the direct and quadrature axes while the subscripts mqsat and mdsat denote saturated values of q and d axis magnetizing flux linkages.

Using the computer simulation language ACSL, a procedural statement whose block diagram is shown in Fig. 1 can be used to calculate the saturated air gap flux values (λ_{mqsat} & λ_{mdsat}) from the model state variables : λ_{qs}, λ_{qr}, λ_{ds} and λ_{dr}. First, the unsaturated flux components λ_{mdu} and λ_{mqu} are calculated. L_m^* is then defined as : $L_m^* = \frac{1}{\frac{1}{L_{mu}} + \frac{1}{L_{ls}} + \frac{1}{L_{lr}}}$ where L_{mu} is the unsaturated (air gap line) magnetizing inductance. As shown in the block diagram, a modified saturation curve is utilized in the procedure where the vertical axis is $f(\lambda_m) = \lambda_{mu} - \lambda_{msat}$. Since saturation does not result in a phase shift in the fundamental component of flux linkages and only decreases the amplitude; both the d and q components of saturated air gap flux are decreased by the same value.

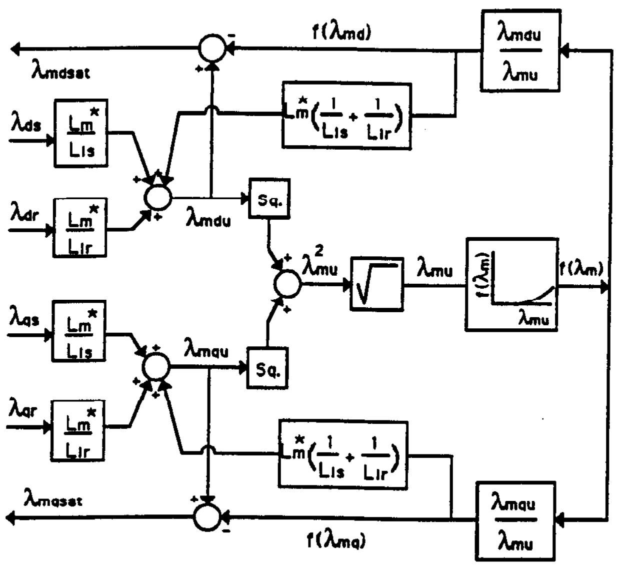

Fig. 1 Block diagram for procedural to calculate λ_{mdsat} & λ_{mqsat} in case of saturation model with flux linkages as state variables

Thus : $$f(\lambda_{mdu}) = \frac{\lambda_{mdu}}{\lambda_{mu}} f(\lambda_{mu})$$

& $$f(\lambda_{mqu}) = \frac{\lambda_{mqu}}{\lambda_{mu}} f(\lambda_{mu})$$

Finally the saturated values are obtained as :

$$\lambda_{mdsat} = \lambda_{mdu} - f(\lambda_{mdu})$$

& $$\lambda_{mqsat} = \lambda_{mqu} - f(\lambda_{mqu})$$

It is clear tat the introduction of main flux saturation in the model with flux linkages in the state equations did not introduce any new coupling between d and q variables terms with the only coupling being the "speed voltages" which already existed in the magnetically linear model.

3. SATURATION MODEL WITH CURRENTS AS STATE VARIABLES

Equations 1 through 4 can be rewritten as :

$$\frac{d\lambda_{qs}}{dt} = V_{qs} - r_s i_{qs} - \omega\lambda_{ds} \tag{5}$$

$$\frac{d\lambda_{ds}}{dt} = V_{ds} - r_s i_{ds} + \omega\lambda_{qs} \tag{6}$$

$$\frac{d\lambda_{qr}}{dt} = V_{qr} - r_r i_{qr} - (\omega - \omega_r)\lambda_{dr} \tag{7}$$

$$\frac{d\lambda_{dr}}{dt} = V_{dr} - r_r i_{dr} + (\omega - \omega_r)\lambda_{qr} \tag{8}$$

where :

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr} \tag{9}$$

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \tag{10}$$

$$\lambda_{dr} = L_r i_{dr} + L_m i_{ds} \tag{11}$$

$$\lambda_{qr} = L_r i_{qr} + L_m i_{qs} \tag{12}$$

where L_m, L_s and L_r are the magnetizing, stator self and rotor self inductances respectively.. Equations 5 through 8 can be written in matrix form as :

$$\left[\frac{d\lambda_{dq}}{dt}\right] = [V_{dq}] - [R_{dq}][I_{dq}] - [G_{dq}][I_{dq}] \tag{13}$$

which is the familiar relation stating that the rate of change of flux is equal to terminal voltage minus the iR drop and speed voltage terms. To express the machine model in terms of currents as state variables the $\frac{d\lambda}{dt}$ terms have to eliminated by defining a matrix [L] as :

$$\left[\frac{d\lambda_{dq}}{dt}\right] = [L]\left[\frac{dI_{dq}}{dt}\right] \tag{14}$$

Substituting (14) in (13) results in :

$$\left[\frac{dI_{dq}}{dt}\right] = [L]^{-1}\{[V_{dq}] - [R_{dq} + G_{dq}][I_{dq}]\} \tag{15}$$

Matrix equation (15) represents the four electrical differential equations for modeling the induction machine in terms of currents as state variables. In a magnetically linear model L_m (and consequently L_s and L_r) is constant independent of the change of flux with time, thus [L] can be expressed as a decoupled matrix with constant terms:

$$[L] = \begin{bmatrix} 0 & L_s & 0 & L_m \\ L_s & 0 & L_m & 0 \\ 0 & L_m & 0 & L_r \\ L_m & 0 & L_r & 0 \end{bmatrix}$$

For a magnetically non-linear model two values of L_m exist both depending on flux and current levels, namely, instantaneous magnetizing inductance (L_{inst}) and Incremental magnetizing inductance (L_{inc}) as defined by Melkebeek [4]. L_{inst} is defined by the slope of the chord from origin to operating point on saturation curve. :$\lambda_{msat} = L_{inst}\, i_m$

L_{inc} is defined by slope of the tangent at the operating point on saturation curve: $\frac{d\lambda_{msat}}{dt} = L_{inc}\frac{di_m}{dt}$

where i_m is the net magnetizing current.

Matrix [L] becomes : $$\begin{bmatrix} L_1 + L_{ls} & L_3 & L_1 & L_3 \\ L_3 & L_2 + L_{ls} & L_3 & L_2 \\ L_1 & L_3 & L_1 + L_{lr} & L_3 \\ L_3 & L_2 & L_3 & L_2 + L_{lr} \end{bmatrix}$$

where : $$L_1 = L_{inst}\left[\frac{i_{md}}{i_m}\right]^2 + L_{inc}\left[\frac{i_{mq}}{i_m}\right]^2$$

$$L_2 = L_{inst}\left[\frac{i_{mq}}{i_m}\right]^2 + L_{inc}\left[\frac{i_{md}}{i_m}\right]^2$$

$$L_3 = (L_{inc} - L_{inst})\frac{i_{mq} i_{md}}{i_m^2}$$

The matrix is no longer decoupled and its elements are a function of the magnetizing currents I_{md}, I_{mq} and I_m.

Figure 2 shows a block diagram for the procedural used in the simulation program to calculate $[L]^{-1}$ knowing the synchronous frame d-q currents and the normalized saturation curve. First $\lambda mu = Lmu\ i_m$ is calculated. The given saturation curve is utilized to obtain $L_{inst} = \frac{\lambda_{msat}}{i_m}$ and $L_{inc} = \frac{\partial \lambda_{msat}}{\partial \lambda_{mu}} L_{mu}$.

Later matrix [L] is formed then a matrix inversion subroutine is used to obtain $[L]^{-1}$.

4. SIMULATION RESULTS

In order to compare the two saturation modelling approaches, the two saturated machine models were assumed to be fed from a balanced sinusoidal three phase voltage supply. The supply frequency is assumed constant at 60 Hz. Figures 3 and 4 show the simulation results for flux linkages and currents as state variables in plots a and b respectively. In Fig. 3 the machine is started at no load then after 0.8 seconds a step torque is applied for 0.4 seconds. In Fig. 4 a step reduction in supply voltage occurs for 0.4 sec. Thus three different transient conditions are compared : line starting at synchronous speed, application and removal of load at rated voltage, step change in voltage at a loaded condition. The electromagnetic torque, motor speed and stator current $I_s = \sqrt{I_{qs}^2 + I_{ds}^2}$ are plotted. Clearly, the transient and steady state responses are identical in both models. The main difference between the two models results is the computational time, which is 278 seconds for flux linkages based model compared with 651 seconds for the currents based model for a 1.4 seconds simulation, the main reason being the matrix inversion which has to be done each time step.

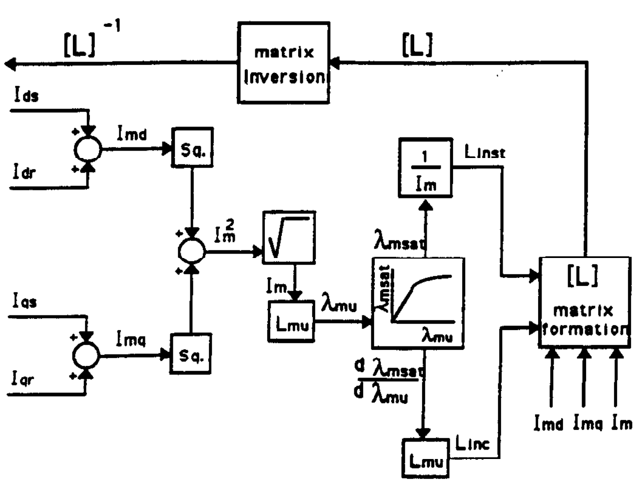

Fig. 2 Block diagram for procedural to calculate $[L]^{-1}$ matrix in case of saturation model with currents as state variables

5. CONCLUSION

Though the simulation models led to the same results, the flux linkages based model requires less computations leading to a significantly less simulation time. Another reason for the flux based model being favorable is that the d-q variables remained decoupled as they are in the linear model while in the currents based model the introduction of core saturation resulted in the disappearance of the decoupling between d and q variables and the emergence of "cross-coupling" terms. Another advantage of using the flux linkages based model is that it provides a better understanding of the physical operation of the machine as it is based on the "real" air gap flux compared with the "hypothetical" instantaneous and incremental magnetizing inductances used in the current based model.

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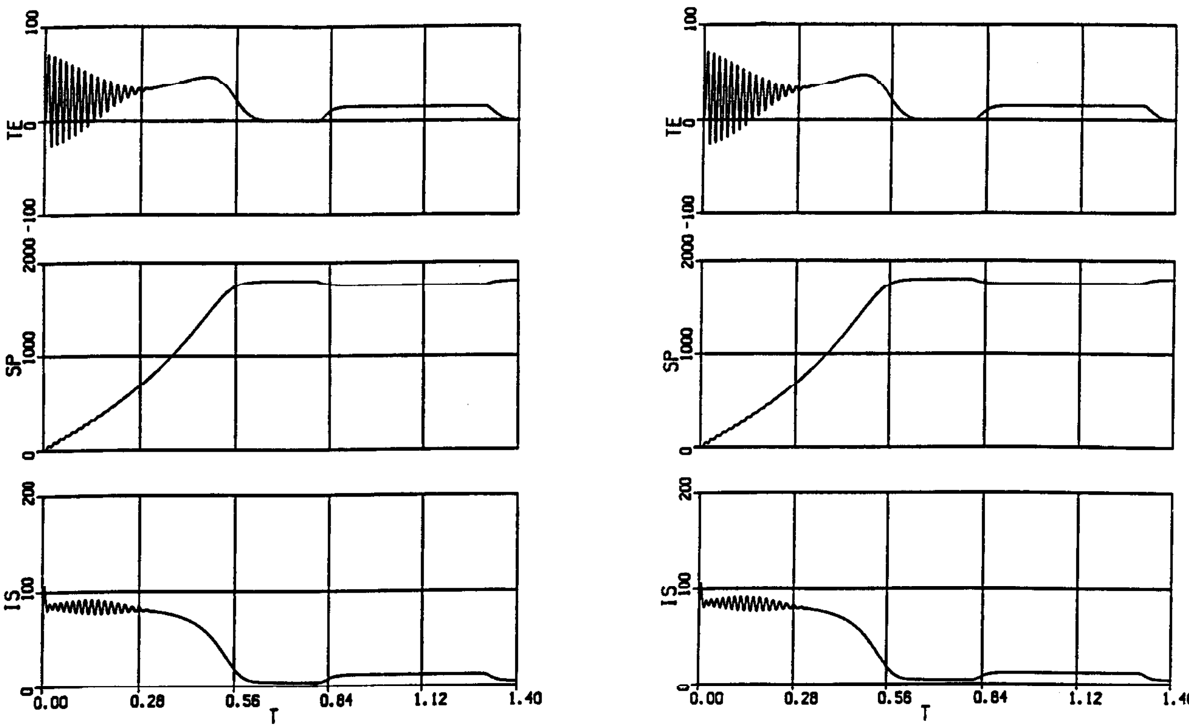
APPENDIX

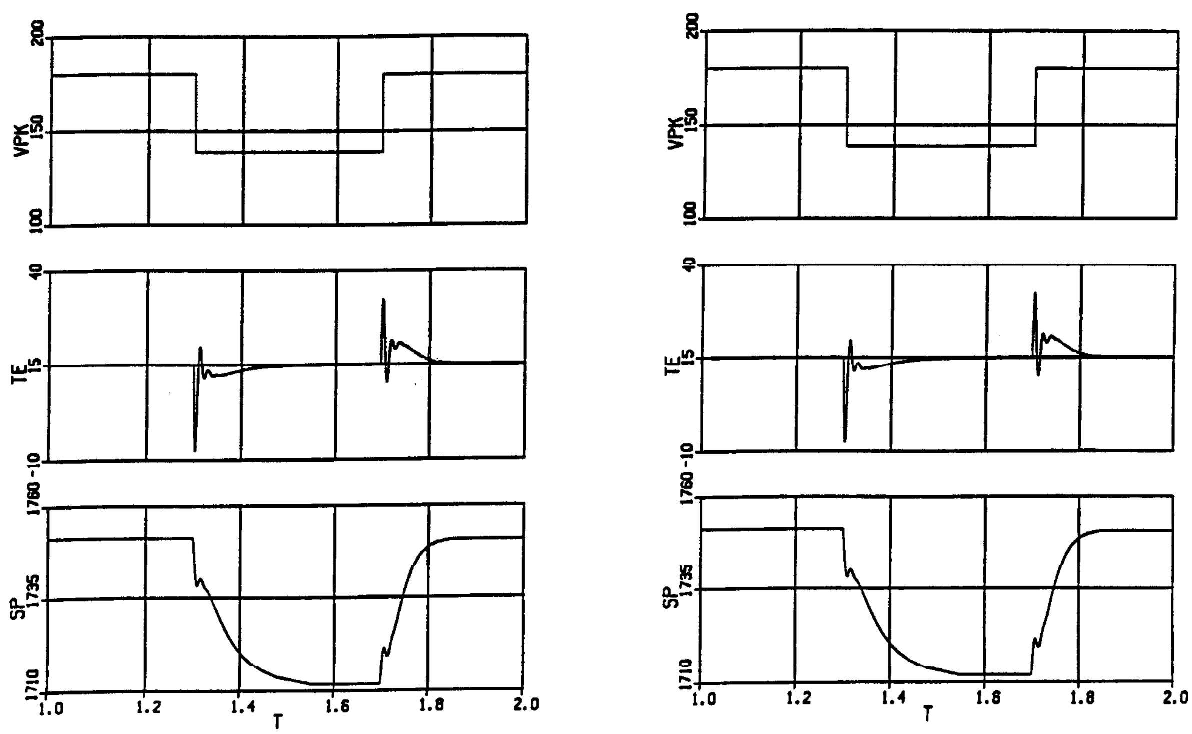
Parameters of machine used in simulation:
220 volt, 60 Hz, 5 HP 4 pole Induction machine

L_{ls}=2.52 mh	L_{lr}=2.52 mh
r_s=0.531 Ω	r_r=0.408 Ω
L_{mu}=101.64 mh	J=0.1 Nm sec^2/rad



a) Fluxes as state variables b) Currents as state variables

Fig. 3 Simulation results for motor start-up and sudden change of load



a) Fluxes as state variables b) Currents as state variables

Fig. 4 Simulation results for sudden change of terminal voltage.