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Abstract - **This paper reports a rotor position sensing technique for SynRM without using any discrete position sensors. An accuracy analysis of the new indirect SynRM position sensing scheme is presented. The analysis breaks down the position error to its fundamental causes in the position sensing scheme. The same basic approach can be taken to evaluate other SynRM rotor position sensing schemes.**

I. INTRODUCTION

Synchronous reluctance motors (SynRM) are suitable for many commercial drive applications because of their inherent simplicity and ruggedness. In particular it has been shown that a properly designed SynRM drive with field oriented control can perform as well as an induction motor drive when the field weakening range is not too wide [1,2,3]. However, the need for a rotor position sensor for a vector controlled SynRM drive tends to compromise the natural cost and simplicity advantage of these motors. Therefore, it is very desirable to eliminate the discrete shaft sensors, while producing the necessary position information for the drive controller. This paper presents the results of accuracy analysis of a new indirect rotor position sensing scheme for SynRM, which has been developed in the power Electronic Laboratory at Texas A&M University. The developed rotor position sensing scheme provides position information at the zero crossings of the phase current. Thus, for the six zero crossings within one electrical cycle of a three phase machine this technique provides six rotor position samples. However, these six samples per electrical cycle are not sufficient for any high performance applications. In order to increase the rotor position sampling rate the controller must employ extrapolation to derive rotor position information between two zero crossings of the phase current. Stator flux information can be used for this extrapolation purpose. However, an investigation of performance and fundamental accuracy of our six samples method is the topic in specific interest of this paper.

II. INDIRECT POSITION SENSING IN SYNCHRONOUS RELUCTANCE MOTOR

The SynRM possess unique features which make position sensing much simpler and reliable than either conventional squirrel cage induction machines or switched reluctance machines. In contrast to the induction machine the SynRM possess saliency which permits the rotor position to be sensed, since the inductance per phase is a function of rotor position. This allows sensing position at zero speed which is impossible for an induction machine. Secondly, in contrast to the switched reluctance motor, the stator windings of the SynRM are magnetically coupled. Hence voltages are induced in the stator windings which are open circuited. This allows sensing of the emf. These two features in combination can be used to obtain rotor position information for SynRM [4].

The rotor position information of a SynRM will be obtained by employing a special switching technique for the current regulated pulsewidth modulated (CRPWM) converter. In a regular CRPWM converter the phase switches are normally turned on and off in order to make the individual phase currents follow the desired reference within a desired band. However, in the modified switching technique, both of the switches of that phase are turned off when the current of a particular phase; e.g., phase A crosses zero. The remaining two phases (in this case phase B and C) are excited in series by turning on alternate pairs of switches (the lower switch of phase B and the upper switch of phase C (Fig. 1a) or the lower switch of phase C and the upper switch of phase B (Fig. 1b)). This modified switching pattern will extend the zero crossing of phase A for a short interval. Although current of phase A during this extended zero crossing period will zero, a voltage will be induced in phase A due to the currents in the other two phases. The induced voltage in phase A can be expressed as [4],

$$V_{ind} = \frac{d}{dt}(L_{ab}\, i_{bs} + L_{ac}\, i_{cs}) \quad (1)$$

where, L_{ab} and L_{ac} are the mutual inductances between phases A and B and between phases A and C respectively. Since phase B and phase C are now effectively in series, $i_{bs} = -i_{cs}$ and $d/dt(i_{bs}) = d/dt(i_{cs})$ and Eq. 1 becomes,

$$V_{ind} = (L_{ab} - L_{ac})\frac{d}{dt} i_{bs} + i_{bs}\frac{d}{dt}(L_{ab} - L_{ac}) \quad (2)$$

Since the mutual inductances depend on rotor positions [5], Eq. 2 can be written as,

$$V_{ind} = K_1\, sin(2\theta_r) + K_2\, cos(2\theta_r) \quad (3)$$

where,

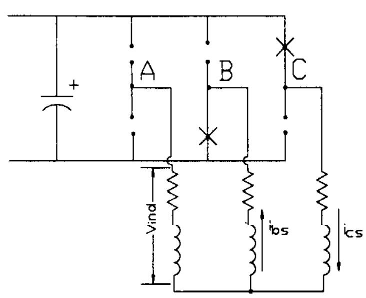
$$K_1 = -(2\, L_B\, sin\tfrac{2\pi}{3})\tfrac{d}{dt} i_{bs}$$

$$K_2 = -(4\, L_B\, \omega_r\, sin\tfrac{2\pi}{3})\, i_{bs}$$

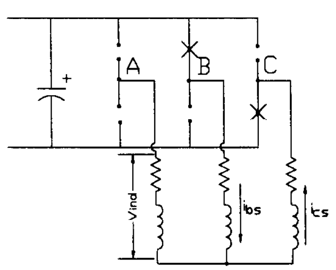
and L_B is a physical parameter of the motor.

In Eq. (3), ω_r is the rotor speed in electrical rad/s, which is equal to the frequency applied to the stator. Thus by knowing the induced voltage V_{ind}, the slope and the instantaneous value of the current, i_{bs}, and using ω_r, it is theoretically possible to compute the instantaneous value of the rotor position θ_r.

Figure 2 shows a measured trace of phase A current with one extended zero crossing interval. During the extended zero crossing period of phase A current, phases B and C of the converter are switched in a special diagnostic manner. This diagnostic switching interval consists of following a constant reference current, in each phase, by means of hysteresis control. The level of constant reference currents in phases B and C are exactly the instantaneous value of these currents, respectively, at the instant of phase A current zero crossing. Figure 3 shows the constant PWM of phase B during that extended zero crossing of phase A. The sequence of coupled voltage pulses, shown in Fig. 4, have the instantaneous rotor angle encoded in their amplitudes, according to Eq.2. Each individual pulse amplitude shown in Fig. 4 can deliver one rotor angle sample by use of a look-up table. The table contains the inverse function of Eq. 3, solved for θ_r, in discrete form. For implementation, the voltage pulses are fed directly to a micro controller (INTEL 80C196KR) having a 16 MHz clock speed in order to get a highly accurate position information.



(a)



(b)

Fig.1 Circuit configurations during the extension of the zero crossing period of phase A current.

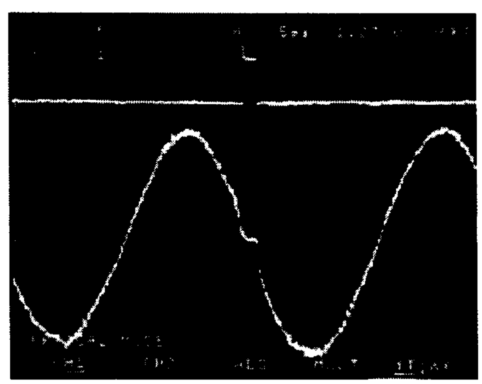


Fig.2 Lower trace: phase A current with extended zero crossing period. Upper trace: control pulse for disconnecting phase A .

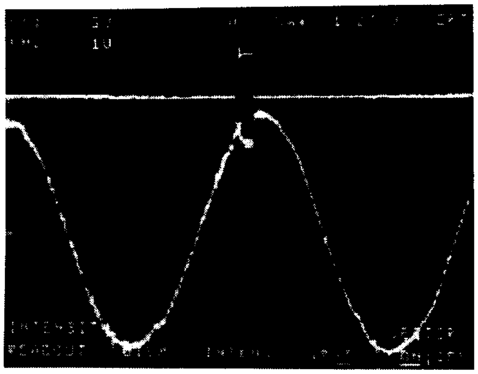


Fig. 3 Phase B current (lower trace) goes into constant PWM during the extended zero crossing of phase A.

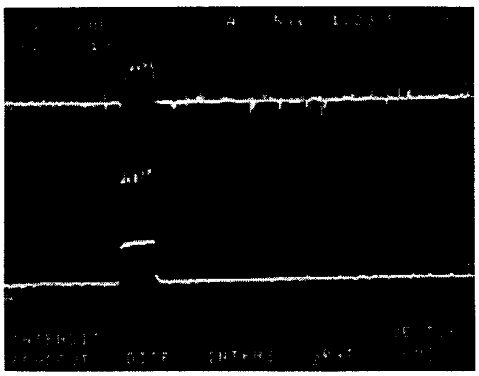


Fig. 4 Induced voltage measured in phase A. Upper trace: Induced voltage. Lower trace: control pulse.

It is important to note that the proposed technique can determine the rotor position even at zero speed. At zero speed all phase currents are zero. This provides one with the opportunity of sequentially inducing the diagnostic PWM signals on any pair of phases while the current in the other phase is zero. The additional advantage is that the zero crossing can be made to persists for as long as one wishes the diagnostic interval to be. Furthermore, speed term $i_{bs}\frac{d}{dt}(L_{ab} - L_{ac})$ in Eq. 2 is eliminated in the look-up table for θ_r, which can now be derived from the simple form given below,

$$V_{ind} = (L_{ab} - L_{ac})\tfrac{d}{dt} i_{bs} \qquad (4)$$

In order to calculate the starting rotor position of the experimental drive, three pairs of phases were diagnostically energized in sequence and the induced voltage was read across the third phase, as previously explained. Figure 5 shows the diagnostic constant zero current regulation flowing through the phases B and C of the experimental drive. Figure 6 shows the voltage induced across phase A during that diagnostic interval. A strong, easily measurable signal is clearly evident.

III. ERROR ANALYSIS OF THE NEW TECHNIQUE

The error analysis of the proposed technique is divided into two major sections. First, the error in individual rotor samples is calculated. This may be called the "sample error". Second, the error introduced due to extrapolation of the rotor samples between two zero crossings of phase currents is calculated. This may be called "inter-sample error". An expression for the drive acceleration is developed, so that, depending on the operating and load conditions, the maximum allowable drive acceleration can be estimated, without making

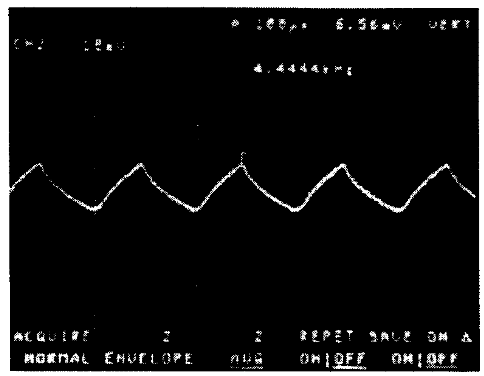


Fig. 5 Diagnostic current flowing through the phases B and C during the start-up operation.

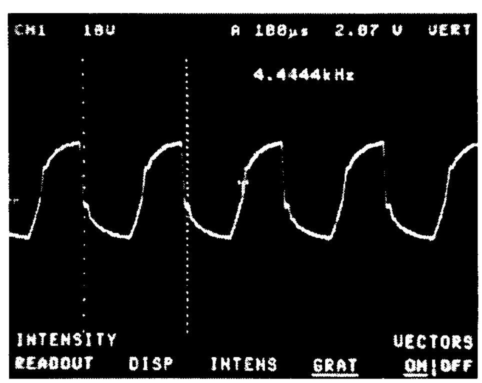


Fig. 6 Induced voltage in Phase A during the start up.

the inter-sample errors greater than the individual sample error.

IV. ERROR IN EACH INDIVIDUAL SAMPLE

The error in rotor samples comes from the quantization error in each of the rotor position samples. Thus, the worst case of the position estimation will occur at the worst case of the quantization error.

$$\Delta\theta_r = \Delta\theta_{sq} \tag{5}$$

where,
$\Delta\theta_{sq}$ = quantization sample error (mechanical degrees).

A. Quantization error, $\Delta\theta_{sq}$, calculation:

The variation of induced voltage is a nonlinear function of rotor positions and only the average sample error in the digitizing process can be obtained. The quantization error results from conversion of the induced voltage V_{ind}, the current slope, di_b/dt and the phase current, i_b (see Eq.(3)) to their integer digital representation and is always equal to 0.5 LSB. Using Eq.(3), the rotor position θ_r, can be written as a function of the measured variables: $\theta_r = \theta(V_{ind}, K_1, K_2)$. The quantization error in θ_r can be approximated by the first terms of its Taylor expansion in terms of quantization errors in the variables:

$$\Delta\theta_{sq} = \frac{\partial\theta_r}{\partial V_{ind}}\Delta V_{ind} + \frac{\partial\theta_r}{\partial K_1}\Delta K_1 + \frac{\partial\theta_r}{\partial K_2}\Delta K_2 \tag{6}$$

where, ΔV_{ind} and ΔK_2 are the individual variable quantization errors which depends on the A/D converter resolution and the full scale range of the individual variables. If V_{max} is the maximum measured induced voltage, then the quantization error for the induced voltage measurement, ΔV_{ind}, becomes,

$$\frac{V_{max}}{2^n}$$

where, n is the number of output bits of the A/D converter. Similarly, with the maximum phase current, I_{max}, the quantization error for current measurement, Δi, becomes,

$$\frac{I_{max}}{2^n}$$

The input variable K_1 depends on the current slope which makes the error ΔK_1 dependent on the current sampling rate. The slope of the phase current during the extended zero crossing period is calculated by measuring the phase currents at the switching instants of the phase pair and then dividing the difference of the measured currents by the time interval between the switching instants. If i_1 and i_2 are the currents at the two switching instants of the phase current, then with the quantization error of Δi, the worst case current difference becomes,

$$i_2 - i_1 + 2\Delta i$$

Consequently, the worst case error for current slope measurement (assuming linear current variation between i_1 and i_2) becomes,

$$\frac{2\Delta i}{1/(2f_s)} = 4\Delta i\, f_s$$

where, f_s is the PWM switching frequency during the extended zero crossing period of the phase current. This error in current slope measurement will contribute to the error ΔK_1, which can be written as,

$$\Delta K_1 = -(2 L_B \sin\tfrac{2\pi}{3})\,\Delta(\tfrac{d}{dt} i) = -(2 L_B \sin\tfrac{2\pi}{3})\cdot 4\Delta i\, f_s$$

The quantization error of the parameter K_2 depends on the current quantization error and can be written as,

$$\Delta K_2 = -(4 L_B \omega_r \sin\tfrac{2\pi}{3})\,\Delta i$$

Using Eq.(6), the quantization error for the individual rotor samples can be expressed as,

$$\Delta\theta_{sq} = \frac{\partial\theta_r}{\partial V_{ind}}\Delta V_{ind} + \frac{\partial\theta_r}{\partial K_1}(-8 L_B \sin\tfrac{2\pi}{3} f_s \Delta i) + \frac{\partial\theta_r}{\partial K_2}(-4 L_B \omega_r \sin\tfrac{2\pi}{3}\Delta i) \tag{7}$$

Equation 7 can now be expressed in terms of the maximum magnitude of the input variables and the resolution of the A/D converter as,

$$\Delta\theta_{sq} = \frac{\partial\theta_r}{\partial V_{ind}}\frac{V_{max}}{2^n} + \frac{\partial\theta_r}{\partial K_1}(-8 L_B \sin\tfrac{2\pi}{3} f_s \frac{I_{max}}{2^n}) + \frac{\partial\theta_r}{\partial K_2}(-4 L_B \omega_r \sin\tfrac{2\pi}{3}\frac{I_{max}}{2^n}) \tag{8}$$

Let us now calculate the quantization error of the individual samples according to the data obtained from one of our experiments. In our experiment, the maximum measured induced voltage was 80 volts and our A/D converter had a 8 bit resolution. This makes the quantization error of the induced voltage ΔV_{ind} equal to 0.31 volts. The rated phase current of the experimental motor is 10 amps making the quantization error for current measurement equal to 0.04 amps which consequently makes the error ΔK_2 equal to 0.18 at 1000 RPM. With 0.04 amps of current quantization error and 8 kHz of PWM frequency, the worst case error in current slope measurement is 1280 amps/sec, which makes the error ΔK_1 equal to 7.58. Using Eq.(8), the worst case error for rotor position estimation, $\Delta\theta_{sq}$, is found to be equal to or less than 0.80 mechanical degrees.

It should be mentioned that the estimated quantization error can be lowered by using 10 bit A/D conversion of the analog signals (Eq.(8)). However, the conversion time required for a 10 bit A/D will be longer.

The estimated quantization error will now be used in the next section to keep the inter-sample error within the individual sample error.

V. ERROR CALCULATION DUE TO EXTRAPOLATION

The proposed technique provides rotor samples at every zero crossings of the phase currents. Thus, extrapolation technique has to be applied in order to estimate rotor position between two zero crossings. A linear extrapolation will be sufficient to estimate rotor positions during the constant speed operation of the drive. But, during the acceleration period of the drive, linear extrapolation will introduce errors in rotor position estimation. Thus, acceleration of the drive has to be limited , so that the rotor estimation error between two zero crossings does not exceed a predefined error margin. In this analysis $\Delta\theta_{sample}$, which is equal to $\Delta\theta_{sq}$, is taken as the acceptable margin of error for extrapolation between two zero crossings.

Suppose the rotor speed ω^* is constant and let the rotor position be θ_1 at time t_1 (see Fig.7), then using the linear

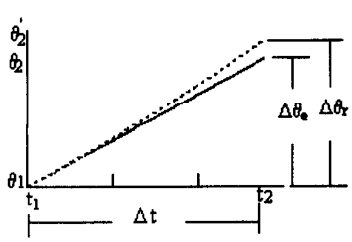


Fig.7 Rotor position variation with and without drive acceleration.

extrapolation, the new rotor position θ_2 at time t_2 can be written as,

$$\theta_2 = \int_{t_1}^{t_2} \omega^* dt \; + \theta_1$$

$$\theta_2 = \omega^* \Delta t + \theta_1 \qquad (9)$$

where, Δt is the time interval between two zero crossings of the phase current. Thus the change in rotor angle , $\Delta\theta_e$ in the interval Δt is,

$$\Delta\theta_e = \omega^* \Delta t \qquad (10)$$

Now, let us consider a constant acceleration, K, for the motor. Then, the new rotor speed, ω, after time t, can be written as,

$$\omega = \omega^* + Kt \qquad (11)$$

With this new acceleration, K, the new rotor position θ'_2, at the end of the time interval can be written as (see Fig.7),

$$\theta'_2 = \tfrac{1}{2}K(\Delta t)^2 + \omega^* \Delta t + \theta_1 \qquad (12)$$

Thus, the rotor angle variation with this acceleration is,

$$\Delta\theta_r = \tfrac{1}{2}K(\Delta t)^2 + \omega^* \Delta t \qquad (13)$$

To keep this angle variation within an acceptable margin, we can limit the acceleration of the drive by defining the boundary condition as,

$$\Delta\theta_r - \Delta\theta_e \le \Delta\theta_{sample}$$

$$\tfrac{1}{2}K(\Delta t)^2 + \omega^* \Delta t - \omega^* \Delta t \le \Delta\theta_{sample}$$

$$K \le 2 \cdot \frac{\Delta\theta_{sample}}{(\Delta t)^2} \qquad (14)$$

$$K \le 72 \cdot \Delta\theta_{sample} \cdot f_e^2 \qquad (15)$$

where, $\Delta t = 1/(6 \cdot f_e)$ and f_e is the electrical frequency of the drive.

The following observations can be made from the drive acceleration given by Eqs. (14) and (15):

- The allowable acceleration is proportional to the error margin.

- Starting acceleration has to be low due to the long time interval between two consecutive zero crossings (long Δt).

- High acceleration is possible at high speed operation (short Δt).

In the previous section the $\Delta\theta_r$ was calculated to be 0.80 degrees at the operating frequency of 33 Hz. The operating speed was 1000 RPM. For this particular case the acceptable acceleration becomes,

$$K = 1090 \ rad / sec^2$$

The following equation, along with Eq.(14) or (15), can be used to identify the drives for which this new technique is suitable for rotor sensing with a desired accuracy,

$$T = \frac{WR^2}{308} \frac{(rpm_2 - rpm_1)}{\Delta t} \qquad lb - ft \qquad (16)$$

where, T is the torque required for the acceleration between two speeds rpm_1 and rpm_2. WR^2 is the inertia of the system in $lb\text{-}ft^2$.

Equations (15) and (16) show that the applicability of the technique depends on the system inertia and the motor torque. A suitable system for this technique will have a high enough system inertia to keep the acceleration low during operation. However, the system acceleration can increase at higher speeds as shown by Eq.(14). Pump, fan and compressor applications satisfy this low starting torque and high inertia requirements of the new technique. For example, a typical 50 hp compressor drive ($WR^2 = 135\ lb\text{-}ft^2$) with a starting torque of $230\ lb\text{-}ft$ has a starting acceleration of $50\ rad/sec^2$, which is low enough for the new technique to provide accurate rotor position samples. The acceleration can increase as the drive gradually picks up speed. Thus the extrapolation technique, with out any modifications, is suitable for fan, pump and compressor applications using large machines (> 50 hp).

VI. CONCLUSIONS

The error in rotor position estimation resulting from the application of the proposed technique is calculated. The individual sample error at the phase current zero crossings comes from the quantization error of the induced voltage and the quantization and the sampling rate errors of the phase current. It is shown that the rotor position sensor, as we have at the present is adequate for $\leq 1^o$ accuracy in rotor position sensing during the extended zero crossing periods of the phase currents.

In order to keep the inter-sample error within this error margin ($\leq 1^o$), an expression for the drive acceleration is developed, which shows the relationship between the inter-sample error and the other system components. It is found that the proposed technique is suitable for slowly changing speeds; e.g., in pump and fan applications. However, the proposed technique can easily be applied to fast speed changing applications, when some other means is adopted (for example flux estimation), to estimate the rotor position between the zero crossings of the phase currents.

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