

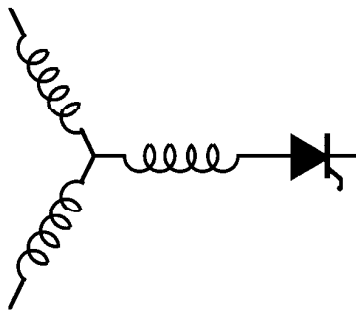
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**Analysis and Modeling of Five Phase Converters  
for Adjustable Speed Drive Applications**

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## ANALYSIS AND MODELING OF FIVE PHASE CONVERTERS FOR ADJUSTABLE SPEED DRIVE APPLICATIONS

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**Abstract.** Mathematical models based on complex voltage vectors for the theoretical studies of five phase voltage source inverters and five phase PWM inverters are presented. Complex vector representation is employed to help in visualizing the current regulation similar to the three phase systems. A dq model based on transformation theory is presented for five phase induction machines. Finally a detail implementation of an indirect type five phase field orientation control including the hysteresis type of PWM current regulator is described.

**Keywords.** Inverters, adjustable speed drives, five phase induction machines, field oriented controls.

### INTRODUCTION

Recent advances in the development of large power transistors have resulted in a new interest in variable speed drives, particularly those associated with the use of a direct voltage source. In these cases when the outputs are either a controlled direct or variable frequency alternating current, use of modern power transistors such as IGBT's could be operationally superior to the thyristors, because of the ease with which it can be turned off.

High phase number drives possess several advantages over conventional three phase drives. This has revived interest in the development of induction or synchronous reluctance machines and the corresponding inverters having more than three phases (1-15). Increasing number of phases has several advantages such as: reducing the amplitude and increasing the frequency of torque pulsations, higher reliability since loss of one of the phases does not prevent the motor from starting and running, reducing the rotor harmonic currents, reducing the current per phase without increasing the voltage per phase, and lowering the dc link current harmonics. By increasing the number of phases it is also possible to increase the torque per rms ampere for the same volume machine as presented by Dente [10] and Toliyat et al [12-14]. However, when a voltage source inverter is used to supply a motor, its stator harmonic current amplitudes are determined not only by the source amplitude and waveform, but also by the motor leakage inductances at harmonic frequencies. These inductances are a function of the number of phases and can be quite small so that certain harmonic currents have large amplitudes as mentioned by Ward and Harer [1], Abbas et al [5], and Klingshirn [6,7]. However, Pavithran et al [11] have shown that the five phase drive operates satisfactorily when fed from a pulse width modulation (PWM) inverter.

The high phase order drive is likely to remain limited to specialized applications where high reliability is demanded such as electric cars or high power applications where a combination of several solid state devices form one leg of the drive. Therefore, the requirement of  $n$  separate drive units in a multi phase system is not oppressive for large drives since many of the necessary components are present in the contemporary designs.

A major objective of the present work is to realize a mathematical model based on complex voltage vector for the theoretical studies of five phase voltage source inverter including both the ten step voltage source inverter (VSI) and the five phase PWM. Then a five phase hysteresis

current regulated PWM algorithm is developed. Also the necessary differential equations describing the performance of five phase induction machines are presented. Based on these findings a simple five phase induction machine field oriented control is given. Finally, the effect of losing two phases of the machine or inverter is illustrated.

### FIVE PHASE VOLTAGE SOURCE INVERTERS

In a five phase motor, the five stator phases are distributed with a spacing of 72 degree. Figure 1 illustrates the basic power circuit topology of five phase voltage source inverters. The figure is presented with insulated gate transistors (IGBT's) illustrated as the main power switches. The anti parallel diodes in the five phase VSI provide reverse current paths such that, when a particular IGBT is gated on, one output terminal and one input terminal will be connected. As a result, if 180 degree gating signals are used, a unique relation between input and output voltage exists. The input current  $I_i$  must be capable of reversal, therefore an input filter capacitor has been placed at the front end of the inverter. The basic operating principles of five phase VSI is developed assuming ideal commutation and zero device forward drop.

### Ten step voltage source inverters

In the ten step VSI the gating signals and commutation circuits result in 180 degree conduction for each IGBT's with five IGBT's "on" at all times. The pole voltages are illustrated in Figure 2. The full cycle can be divided into ten separate modes as illustrated in Figure 3. In each mode all line voltages are uniquely related to the inverter input voltages also there is always three output line to line short circuit and possible short circuit currents in the resulting closed loops. Since the inverter input variables are isolated from these loops, the short circuit current is determined entirely by the load on the inverter. By varying the initiation and length of the individual conduction modes, the frequency and phase of the output waves are easily controlled. Amplitude control is accomplished by varying the amplitude of the input dc voltage using a control front end converter. Another method is to employ pulse width modulation of the inverter power switches. In [2] a dq model of a five phase concentrated winding synchronous reluctance motor including the effect of third harmonic of space harmonics was presented. The dq model for a five phase VSI system is realized by applying the transformation given in the Appendix to the inverter output voltages on a mode by mode basis noting that ten modes exist.

Figure 3 illustrates the transformed voltages to stationary dq frame. The complex vector form of these dq equations for the five phase VSI take on very simple form and are useful in visualizing the converter operation and explaining various converter properties. It is useful to define the complex dq equations for the voltages in the stationary frame as

$$V_{dq1}^s = V_{ds1}^s - j V_{qs1}^s \quad (1)$$

The values of these vectors for the ten modes of operation are as follows  
Mode 1 through 10

$$V_{dq1}^s = V_1 \frac{4}{5} \cos \frac{\pi}{5} e^{j \frac{k\pi}{5}} \quad k=0, 1, 2, \dots, 9 \quad (2)$$

Clearly these ten modes show that in each mode the complex vector for voltage has constant amplitude of  $(V_1 \frac{4}{5} \cos \frac{\pi}{5})$ . Moreover a constant phase angle equal to  $(k \frac{\pi}{5})$ .

#### Five phase pulse width modulation inverters

The induction motor is a non linear multi-variable system which may interact with the source impedances when is supplied by the converter. The analytical study of such system is difficult and therefore digital computer simulations become favorable. PWM operation makes possible the implementation of very fast current control which is essential to high performance drives. PWM operation of a five phase VSI has several additional modes including modes which all five phases are connected together through positive or negative dc bus to yield a five phase line to line short circuit on the load plus twenty more other modes of switching. Therefore, the five phase PWM is actually a thirty two state device. The existing modes including the modes exist for the ten step VSI are:

$V_1(A^+B^+C^-D^-E^-)$	$V_2(A^+B^+C^-D^-E^-)$	$V_3(A^+B^+C^+D^-E^-)$
$V_4(A^-B^+C^+D^-E^-)$	$V_5(A^-B^+C^+D^+E^-)$	$V_6(A^-B^-C^+D^+E^-)$
$V_7(A^-B^-C^+D^+E^+)$	$V_8(A^-B^-C^-D^+E^+)$	$V_9(A^+B^-C^-D^+E^+)$
$V_{10}(A^+B^-C^-D^-E^+)$	$V_{11}(A^+B^-C^-D^-E^-)$	$V_{12}(A^+B^+C^+D^-E^+)$
$V_{13}(A^-B^+C^-D^-E^-)$	$V_{14}(A^+B^+C^+D^+E^-)$	$V_{15}(A^-B^-C^+D^-E^-)$
$V_{16}(A^-B^+C^+D^+E^+)$	$V_{17}(A^-B^-C^-D^+E^+)$	$V_{18}(A^+B^-C^+D^+E^+)$
$V_{19}(A^-B^-C^-D^-E^+)$	$V_{20}(A^+B^+C^-D^+E^+)$	$V_{21}(A^-B^+C^-D^-E^-)$
$V_{22}(A^+B^+C^-D^+E^-)$	$V_{23}(A^+B^-C^-D^-E^-)$	$V_{24}(A^-B^+C^+D^-E^-)$
$V_{25}(A^-B^+C^-D^+E^-)$	$V_{26}(A^+B^-C^+D^+E^-)$	$V_{27}(A^-B^-C^+D^-E^-)$
$V_{28}(A^-B^+C^-D^+E^+)$	$V_{29}(A^+B^-C^-D^+E^+)$	$V_{30}(A^+B^-C^+D^-E^+)$
$V_{31}(A^+B^+C^+D^+E^+)$	$V_{32}(A^-B^-C^-D^-E^-)$	

The corresponding voltage vectors are:

Mode 11 through 20

$$V_{dq1}^s = V_1 \frac{2}{5} e^{j \frac{k\pi}{5}} \quad k=0, 1, \dots, 9 \quad (3)$$

Mode 21 through 30

$$V_{dq1}^s = V_1 \frac{4}{5} \cos \frac{2\pi}{5} e^{j \frac{k\pi}{5}} \quad k=0, 1, \dots, 9 \quad (4)$$

Figure 4 illustrates the complex vector voltages given by equations (2-4).

#### HYSTERESIS CONTROLLER

Generally there are two types of hysteresis controllers [16]. One variation uses five independent controllers, one for each phase. When the line current becomes greater (less) than the current reference by the hysteresis band, the inverter leg is switched in the negative (positive) direction. However, one current control may be removed because the current in one phase will be determined by the other two phases in a system with no neutral connection. Therefore, the other type utilizes only four independent hysteresis controllers with removing the switches in the fifth leg for lower inverter switching frequencies and using the divided dc bus. Notice that when an inverter leg switches state, the resulting voltage vector is dependent on the state of the other four inverter legs. For example, if phase a switches from high to low, the following inverter voltage vectors can result.

$V_1(A^+B^+C^-D^-E^-)$	→	$V_{21}(A^-B^+C^-D^-E^-)$
$V_2(A^+B^+C^-D^-E^-)$	→	$V_{13}(A^-B^+C^-D^-E^-)$
$V_3(A^+B^+C^+D^-E^-)$	→	$V_4(A^-B^+C^+D^-E^-)$
$V_4(A^-B^+C^+D^-E^-)$	→	$V_{14}(A^+B^+C^+D^+E^-)$
$V_5(A^-B^-C^+D^+E^-)$	→	$V_{26}(A^+B^-C^+D^+E^-)$
$V_6(A^-B^-C^+D^+E^-)$	→	$V_{18}(A^+B^-C^+D^+E^+)$
$V_7(A^-B^-C^-D^+E^+)$	→	$V_9(A^+B^-C^-D^+E^+)$
$V_8(A^-B^-C^-D^+E^+)$	→	$V_{19}(A^-B^-C^-D^-E^+)$
$V_{10}(A^+B^-C^-D^-E^+)$	→	$V_{32}(A^-B^-C^-D^-E^-)$
$V_{11}(A^+B^-C^-D^-E^-)$	→	$V_{24}(A^-B^+C^+D^-E^-)$
$V_{12}(A^+B^+C^+D^-E^+)$	→	$V_{31}(A^+B^+C^+D^-E^-)$
$V_{15}(A^-B^-C^+D^-E^-)$	→	$V_{23}(A^+B^+C^+D^+E^-)$
$V_{16}(A^-B^+C^-D^-E^-)$	→	$V_{25}(A^+B^+C^+D^+E^+)$
$V_{17}(A^-B^-C^-D^+E^+)$	→	$V_{29}(A^+B^-C^-D^+E^+)$
$V_{20}(A^+B^+C^-D^+E^+)$	→	$V_{28}(A^-B^+C^-D^+E^+)$
$V_{22}(A^+B^+C^-D^+E^-)$	→	$V_{25}(A^-B^+C^-D^+E^-)$
$V_{27}(A^-B^-C^+D^-E^-)$	→	$V_{30}(A^+B^-C^+D^-E^+)$

It is important to mention that if the load has no neutral connection, then the individual line current will depend not only on the state of the corresponding inverter leg, but also on the state of the other four inverter legs. As a result the hysteresis current controller might experience interaction between the phases.

#### DIGITAL COMPUTER SIMULATION OF FIVE PHASE VECTOR CONTROLLED DRIVE

Generally two types of vector controlled known as direct and indirect method exists. In the first scheme which is probably the most accurate induction motor control method direct sensing of the air gap flux vector is required. This technique is essentially insensitive to variation in motor parameters however, suffers from high cost and unreliability of the flux measurement. In the second method the rotor flux is estimated using the stator vector, voltage vector and/or rotor speed. In this paper the indirect vector control approach is used.

Suppose that the rotor flux linkages in the synchronously rotating reference frame is entirely in the d axis, resulting

$$\lambda_{qr1}^s = 0 \quad (5)$$

As a result the electromagnetic torque equation (A-37) has the same form as the torque equation of a DC machine. Therefore, like a shunt DC machine the electromagnetic torque and the rotor flux are controlled

independently by regulating the d and q components of stator current together with slip frequency resulting from the constraint equation (5) in the equations given in appendix A for five phase induction machines. The final necessary equations are

$$S \omega_e^* = \frac{4}{5p} r_r \frac{T_e^*}{(\lambda_{dr1}^*)^2} \quad (6)$$

$$i_{qs1}^{e*} = \frac{4}{5p} \frac{L_r T_e^*}{L_m \lambda_{dr1}^*} \quad (7)$$

$$i_{ds1}^{e*} = \frac{1}{L_m} \lambda_{dr1}^{e*} + \frac{L_r}{r_r L_m} \frac{d}{dt} \lambda_{dr1}^{e*} \quad (8)$$

where  $T_e^*$  and  $\lambda_{dr1}^{e*}$  are the commanded values of torque and rotor flux.

The stator frequency command can be obtained from the following equation

$$\omega_e^* = \omega_r + S \omega_e^* \quad (9)$$

A block diagram configuration for indirect vector control of five phase induction motor is shown in Figure 5.

### SIMULATION RESULTS

Figure 6 shows the waveforms of a 2 kW [1], five phase induction machine driven by the current control type voltage source inverter with slip frequency control type field oriented controller. The induction motor is accelerated from zero speed to full speed under the rated load and constant flux command. Later, the load torque has been suddenly changed. A rather instantaneous response of field oriented type controller is evident.

### CONCLUSIONS

Five phase current regulated induction motor is considered. A mathematical model based on complex voltage vectors for five phase VSI and PWM inverters is developed. It is shown that five phase current regulated PWM inverter assume thirty two states of switching. The differential equations describing the performance of five phase induction motors is derived. This paper has presented the details of a digital computer implementation of five phase indirect type of vector controlled drive. Among several benefits in using five phase induction machine is its capability to start and run even with one, two, or even three phases open circuited. For example if phase d and phase e are open circuited due to device failures or machine windings a rotating forward field is still possible to be achieved if the currents in the remaining phases are regulated as

$$i_{as} = 5 I_{\max} \frac{\cos \frac{\pi}{5}}{(\sin \frac{2\pi}{5})} \cos(\omega t + \frac{2\pi}{5})$$

$$i_{bs} = 5 I_{\max} \frac{2(\cos \frac{\pi}{5})^2}{(\sin \frac{2\pi}{5})} \cos(\omega t - \frac{2\pi}{5})$$

$$i_{cs} = 5 I_{\max} \frac{\cos \frac{\pi}{5}}{(\sin \frac{2\pi}{5})} \cos(\omega t + \frac{4\pi}{5})$$

where initially they were

$$i_{as} = I_{\max} \cos \omega t$$

$$i_{bs} = I_{\max} \cos(\omega t - \frac{2\pi}{5})$$

$$i_{cs} = I_{\max} \cos(\omega t - \frac{4\pi}{5})$$

In a future paper the performance of five phase induction motor drives under loss of one or several inverter legs which recently has got attention in special applications such as electric cars will be addressed.

### APPENDIX A

#### Five phase induction machines equations

The voltage equations for the stator phases in the natural coordinate of systems are

$$V_s = R_s I_s + \frac{d\Lambda_s}{dt} \quad (A-1)$$

where stator flux linkages are given by

$$\Lambda_s = L_{ss} I_s + L_{sr} I_r \quad (A-2)$$

and

$$I_s = \begin{bmatrix} i_{as} & i_{bs} & i_{cs} & i_{ds} & i_{es} \end{bmatrix}^t \quad (A-3)$$

$$I_r = \begin{bmatrix} i_{ar} & i_{br} & i_{cr} & i_{dr} & i_{er} \end{bmatrix}^t \quad (A-4)$$

$$V_s = \begin{bmatrix} v_{as} & v_{bs} & v_{cs} & v_{ds} & v_{es} \end{bmatrix}^t \quad (A-5)$$

The matrix  $R_s$  is a diagonal 5 by 5 matrix given by,

$$R_s = r_s I$$

where,  $I$  is a 5 by 5 identity matrix and  $r_s$  is the resistance of each coil assuming all coils are similar. Due to conservation of energy, the matrix

$L_{ss}$  is a symmetric 5 by 5 matrix of the form,

$$L_{ss} = \begin{bmatrix} L_{sas} & L_{sab} & L_{sac} & L_{sad} & L_{sae} \\ L_{abs} & L_{abs} & L_{abs} & L_{abs} & L_{abs} \\ L_{acs} & L_{acs} & L_{acs} & L_{acs} & L_{acs} \\ L_{ads} & L_{ads} & L_{ads} & L_{ads} & L_{ads} \\ L_{aes} & L_{aes} & L_{aes} & L_{aes} & L_{aes} \end{bmatrix} \quad (A-6)$$

The mutual inductance matrix  $L_{sr}$  is a 5 by 5 matrix comprised of the mutual inductances between the stator phases and the rotor phases.

$$L_{rr} = \begin{pmatrix} L_{aas} & L_{abtr} & L_{aor} & L_{adsr} & L_{aor} \\ L_{bas} & L_{bbr} & L_{bor} & L_{bdsr} & L_{bor} \\ L_{cas} & L_{cbr} & L_{cor} & L_{cdsr} & L_{cor} \\ L_{das} & L_{dbr} & L_{dcr} & L_{dds} & L_{dcr} \\ L_{eas} & L_{ebr} & L_{eor} & L_{edsr} & L_{eor} \end{pmatrix} \quad (A-7)$$

Rotor voltage equation is given by

$$V_r = R_r I_r + \frac{d\Lambda_r}{dt} \quad (A-8)$$

where,

$$V_r = \begin{bmatrix} v_{ar} & v_{br} & v_{cr} & v_{dr} & v_{er} \end{bmatrix}^t$$

In case of a cage rotor  $v_r = 0$ . The matrix  $R_r$  is a diagonal 5 by 5 matrix

given by,

$$R_r = r_r I$$

The rotor flux linkages  $\Lambda_r$  can be written as,

$$\Lambda_r = L_{rr}^t I_r + L_{rs} I_s \quad (A-9)$$

where the matrix  $L_{rr}^t$  is the transpose of the matrix  $L_{rr}$  and the rotor

inductance matrix  $L_{rr}$  is given by equation (A-6), however, simply the

subscript  $s$  is replaced by  $r$ .

The magnetizing inductance of each phase is given by

$$L_{ms} = \frac{\mu_0 r l \pi}{4g} N_s^2 \quad (A-10)$$

where  $N_s$  is the effective number of stator turns per phase,  $g$  is the air gap,  $r$  is the average value of the stator inner and rotor outer radius, and  $l$  is the length of the coil. The elements of matrix  $L_{ms}$  are as follows

$$\begin{aligned} L_{aas} &= L_{bbs} = L_{ccs} = L_{dds} = L_{ees} = L_{fs} + L_{ms} \\ L_{abs} &= L_{bcs} = L_{cds} = L_{des} = L_{aes} = \cos\left(\frac{2\pi}{5}\right) L_{ms} \\ L_{acs} &= L_{bds} = L_{ces} = L_{ads} = L_{hes} = \cos\left(\frac{4\pi}{5}\right) L_{ms} \end{aligned} \quad (A-11)$$

Similarly the stator to rotor mutual inductances are

$$\begin{aligned} L_{aar} &= L_{bar} = L_{cor} = L_{dsr} = L_{esr} = \frac{N_r}{N_s} L_{ms} \cos \theta_r \\ L_{abr} &= L_{bar} = L_{cdr} = L_{dsr} = L_{esr} = \frac{N_r}{N_s} L_{ms} \cos\left(\theta_r + \frac{2\pi}{5}\right) \\ L_{acr} &= L_{bdr} = L_{cor} = L_{dsr} = L_{esr} = \frac{N_r}{N_s} L_{ms} \cos\left(\theta_r + \frac{4\pi}{5}\right) \end{aligned}$$

$$L_{adr} = L_{bar} = L_{cor} = L_{dsr} = L_{esr} = \frac{N_r}{N_s} L_{ms} \cos\left(\theta_r + \frac{4\pi}{5}\right)$$

$$L_{aer} = L_{bar} = L_{cdr} = L_{dsr} = L_{esr} = \frac{N_r}{N_s} L_{ms} \cos\left(\theta_r + \frac{2\pi}{5}\right) \quad (A-12)$$

The rotor self and magnetizing inductances are

$$L_{rar} = L_{br} = L_{cr} = L_{dr} = L_{er} = L_r + \left(\frac{N_r}{N_s}\right)^2 L_{ms}$$

$$L_{abr} = L_{bcr} = L_{cdr} = L_{dsr} = L_{esr} = \cos\left(\frac{2\pi}{5}\right) \left(\frac{N_r}{N_s}\right)^2 L_{ms}$$

$$L_{acr} = L_{bdr} = L_{cor} = L_{dsr} = L_{esr} = \cos\left(\frac{4\pi}{5}\right) \left(\frac{N_r}{N_s}\right)^2 L_{ms} \quad (A-13)$$

Using the following transformation of variables in the synchronously rotating reference frame for the stator variables

$$\begin{bmatrix} f_{qs1}^e & f_{ds1}^e & f_{qr2}^e & f_{ds2}^e & f_{os}^e \end{bmatrix}^t = T(\theta_e) \begin{bmatrix} f_{as} & f_{bs} & f_{cs} & f_{ds} & f_{es} \end{bmatrix}^t \quad (A-14)$$

where

$$\begin{aligned} \cos \theta_e & \quad \cos\left(\theta_e - \frac{2\pi}{5}\right) & \quad \cos\left(\theta_e - \frac{4\pi}{5}\right) & \quad \cos\left(\theta_e + \frac{4\pi}{5}\right) & \quad \cos\left(\theta_e + \frac{2\pi}{5}\right) \\ \sin \theta_e & \quad \sin\left(\theta_e - \frac{2\pi}{5}\right) & \quad \sin\left(\theta_e - \frac{4\pi}{5}\right) & \quad \sin\left(\theta_e + \frac{4\pi}{5}\right) & \quad \sin\left(\theta_e + \frac{2\pi}{5}\right) \\ T(\theta_e) = \frac{2}{5} & \quad \cos \theta_e & \quad \cos\left(\theta_e + \frac{4\pi}{5}\right) & \quad \cos\left(\theta_e - \frac{2\pi}{5}\right) & \quad \cos\left(\theta_e - \frac{4\pi}{5}\right) \\ & \quad \sin \theta_e & \quad \sin\left(\theta_e + \frac{4\pi}{5}\right) & \quad \sin\left(\theta_e - \frac{2\pi}{5}\right) & \quad \sin\left(\theta_e - \frac{4\pi}{5}\right) \\ & \quad \frac{1}{2} & \quad \frac{1}{2} & \quad \frac{1}{2} & \quad \frac{1}{2} \end{aligned} \quad (A-15)$$

and the corresponding transformation of variables for the rotor variables

$$\begin{bmatrix} f_{qr1}^e & f_{dr1}^e & f_{qr2}^e & f_{dr2}^e & f_{or}^e \end{bmatrix}^t = T(\theta_o - \theta_r) \begin{bmatrix} f_{ar} & f_{br} & f_{cr} & f_{dr} & f_{er} \end{bmatrix}^t \quad (A-16)$$

results in the following five phase induction machine equations in the synchronously rotating reference frame.

$$v_{qs1}^e = r_s i_{qs1}^e + \frac{d\lambda_{qs1}^e}{dt} + \omega_e \lambda_{ds1}^e \quad (A-17)$$

$$v_{ds1}^e = r_s i_{ds1}^e + \frac{d\lambda_{ds1}^e}{dt} - \omega_e \lambda_{qs1}^e \quad (A-18)$$

$$v_{qr2}^e = r_s i_{qr2}^e + \frac{d\lambda_{qr2}^e}{dt} \quad (A-19)$$

$$v_{dr2}^e = r_s i_{dr2}^e + \frac{d\lambda_{dr2}^e}{dt} \quad (A-20)$$

$$v_{os}^e = r_s i_{os}^e + \frac{d\lambda_{os}^e}{dt} \quad (A-21)$$

$$v_{qr1}^e = r_r i_{qr1}^e + \frac{d\lambda_{qr1}^e}{dt} + (\omega_e - \omega_r) \lambda_{dr1}^e \quad (A-22)$$

$$v_{dr1}^e = r_r i_{dr1}^e + \frac{d\lambda_{dr1}^e}{dt} - (\omega_e - \omega_r) \lambda_{qr1}^e \quad (A-23)$$

$$v_{qr2}^e = r_r i_{qr2}^e + \frac{d\lambda_{qr2}^e}{dt} \quad (A-24)$$

$$v_{dr2}^e = r_r i_{dr2}^e + \frac{d\lambda_{dr2}^e}{dt} \quad (A-25)$$

$$v_{or}^e = r_r i_{or}^e + \frac{d\lambda_{or}^e}{dt} \quad (A-26)$$

where the flux linkages are

$$\lambda_{qs1}^e = L_{ls} i_{qs1}^e + L_m (i_{qs1}^e + i_{qr1}^e) \quad (A-27)$$

$$\lambda_{ds1}^e = L_{ls} i_{ds1}^e + L_m (i_{ds1}^e + i_{dr1}^e) \quad (A-28)$$

$$\lambda_{qs2}^e = L_{ls} i_{qs2}^e \quad (A-29)$$

$$\lambda_{ds2}^e = L_{ls} i_{ds2}^e \quad (A-30)$$

$$\lambda_{os}^e = L_{ls} i_{os}^e \quad (A-31)$$

$$\lambda_{qr1}^e = L_{lr} i_{qr1}^e + L_m (i_{qs1}^e + i_{qr1}^e) \quad (A-32)$$

$$\lambda_{dr1}^e = L_{lr} i_{dr1}^e + L_m (i_{ds1}^e + i_{dr1}^e) \quad (A-33)$$

$$\lambda_{qr2}^e = L_{lr} i_{qr2}^e \quad (A-34)$$

$$\lambda_{dr2}^e = L_{lr} i_{dr2}^e \quad (A-35)$$

$$\lambda_{or}^e = L_{lr} i_{or}^e \quad (A-36)$$

The electromagnetic torque is given by

$$T_e = \frac{3P}{2} \frac{L_m}{(L_r + L_m)} (\lambda_{dr1}^e i_{qs1}^e - \lambda_{qr1}^e i_{ds1}^e) \quad (A-37)$$

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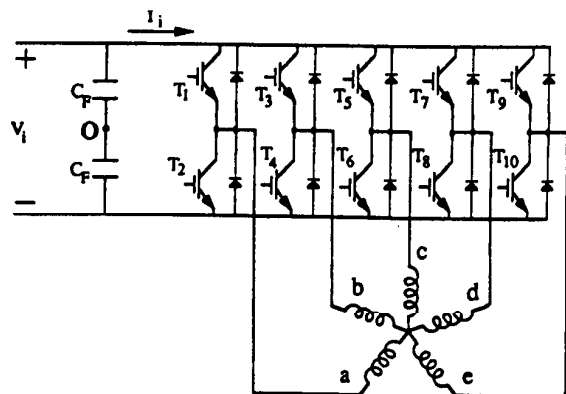


Figure 1 Power circuit configuration of five phase VSI.

