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A New Induction Machine Model for Analysis of Eccentric Rotor Magnetic Pull

Mohamed Osama and Thomas A. Lipo
Dept. of Electrical and Computer Engineering
University of Wisconsin-Madison
1415 Johnson Drive
Madison, WI 53706

WEMPEC

Department of Electrical and Computer Engineering
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Madison, Wisconsin 53706

Mohamed Osama and Thomas A. Lipo
Department of Electrical and Computer Engineering, University of Wisconsin-Madison
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SUMMARY

Because of the unbalanced magnetic field that exists in the air gap with even slight misalignment of the stator and rotor axes, large radial forces can appear (magnetic pull). To date, studies of the eccentric rotor magnetic pull have employed models which calculate the air gap flux density as a function of the spatial angle at a specific time instant in steady state. This paper introduces a new model to study this problem based on multiple coupled circuits. The approach uses the concept of winding functions to calculate all of the self and mutual inductances of the machine and hence obtain the electromagnetic force expressions. The induction machine model incorporates both the possibility of rotor eccentricity and currents in different coil groups being unequal. Transient and steady state waveforms are obtained and compared to previous experimental results. The dependence of UMP on supply voltage, eccentricity and load is also studied. Finally, the effects of a parallel/series stator winding connetcion on UMP is investigated.

INTRODUCTION

Careful alignment of the rotor within the stator is an important consideration for proper installation of any machine. For an induction machine with a uniform air gap, it is well known that the sum of the radial electromagnetic forces acting on the overall rotor periphery is zero. However, when the axis of rotation is not concentric with the center line of the stator, the rotor is said to be eccentric. A corresponding non-uniform air gap flux density occurs due to the eccentric rotor which causes a radial force in the direction of greatest flux density (smallest air gap). This radial force has been referred to in the literature as unbalanced magnetic pull (UMP)[2]. Investigations of the occurrence and the effects of eccentric rotors in induction machines have been carried out since 1900[1].

Several problems have been associated with rotor eccentricity including vibration accompanied by acoustic noise[3], and a rise of rotor temperature resulting in increase of losses[4]. Furthermore, positive feedback exists between eccentricity and UMP leading to increase of both, perhaps resulting in the rotor contacting the stator if there is not sufficient mechanical restraints. Accurate prediction of the magnitude of the unbalanced force is important as it necessitates an increase in shaft diameter and bearing size[5] and has the effect of reducing critical speed[6]. Reduction of UMP in induction machines with non-uniform air gap is usually achieved by using parallel connections of the stator coil groups in an attempt to eliminate (reduce) air gap flux density non-uniformity due to eccentricity [2,7].

Most methods of calculating UMP in use are based on similar theories and yield only steady state results[5,8,9]. The usual starting point is to consider the Maxwell stress distribution between two uniformly magnetized eccentric cylinders (the stator and rotor air gap surfaces) and then to evaluate the resulting net force along the plane of interest. Corrections are then introduced for the influence of saturation, rotor damping currents and parallel paths in stator winding. In this study, a more generally useful coupled circuit approach is used to model a four pole induction machine taking into account eccentricity and the resultant UMP. Digital computer simulations using this model are then carried out for various operating conditions.

DESCRIPTION OF MODEL

When the rotor of an induction machine is off-center relative to the stator bore (Fig. 1), the air gap becomes a function of the angular spatial position and can be expressed as:

$$g(\phi_m) = g_o\left(1 - \varepsilon_x \cdot \cos\phi_m - \varepsilon_y \cdot \sin\phi_m\right) \qquad (1)$$

where :

ϕ_m : angular position along the stator inner surface

g_o : radial air gap length in case of no eccentricity

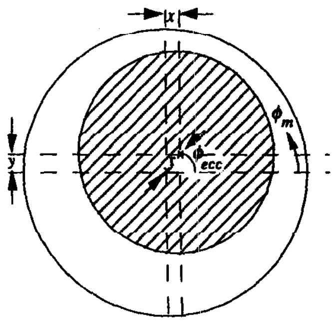

Fig. 1 Air gap in machine with eccentric rotor (exaggerated)

$\varepsilon_x = \frac{x}{g_o}$: the per unit eccentricity in the horizontal direction

$\varepsilon_y = \frac{y}{g_o}$: the per unit eccentricity in the vertical direction.

In order to simplify the calculations, the actual iron is replaced with fictitious iron of infinite permeability (no mmf drop) and the actual gap (g) is replaced by a slightly larger gap (g_{eff}) so that the mmf drop across the iron part of the flux path is incorporated into the mmf drop across the gap. Hence, the effective air gap function for an eccentric rotor can be defined as :

$$g_{eff}(\phi_m) = g_{eff} - (x \cdot \cos\phi_m + y \cdot \sin\phi_m) \\ = g_{eff} - g_o(\varepsilon_x \cdot \cos\phi_m + \varepsilon_y \cdot \sin\phi_m) \quad (2)$$

Thus the effective inverse air gap function can be approximated as:

$$g_{eff}^{-1}(\phi_m) = \frac{1}{g_{eff}} + \frac{1}{g_o}\left[\varepsilon_x \cdot \cos\phi_m + \varepsilon_y \cdot \sin\phi_m \right. \\ \left. + \varepsilon_x^2 \cdot \cos^2\phi_m + \varepsilon_y^2 \cdot \sin^2\phi_m \right] \quad (3)$$

Conventionally, induction machines are modeled using the d-q model. When the rotor and stator are not concentric, the air gap varies along the periphery of the machine resulting in the inductances being a function of the horizontal and vertical position of the rotor in addition to the angular position. Furthermore, the different currents of one phase in case of parallel connections of stator poles are not necessarily equal for an eccentric rotor. To account for those two complications the induction machine must be simulated in the actual physical rather than the transformed variables using the multiple coupled circuit approach. In this model, the coupled circuits will become one or more connected coil groups or the entire phase winding either for stator or for rotor quantities referred to the stator.

Defining the winding function $N(\phi_m)$ as the mmf distribution for a unit winding current, the mutual inductance between any two windings i & j can be expressed as :

$$L_{ij} = \frac{\lambda_{ij}}{i_j} = \mu_o r l \int_0^{2\pi} \frac{N_i(\phi_m) \cdot N_j(\phi_m)}{g_{eff}(\phi_m)} d\phi_m \quad (4)$$

where :

l: is the axial length of the machine
r: is the outer radius of rotor or inner radius of stator.

Using Eq. (3), the inductance expression can be expanded as :

$$L_{ij} = Lo_{ij} + \varepsilon_x \cdot Kx_{ij} + \varepsilon_y \cdot Ky_{ij} + \varepsilon_x^2 \cdot Kxx_{ij} \\ + \varepsilon_y^2 \cdot Kyy_{ij} + 2 \cdot \varepsilon_x \cdot \varepsilon_y \cdot Kxy_{ij} \quad (5)$$

where :

$$Lo_{ij} = \frac{\mu_o r l}{g_{eff}} \int_0^{2\pi} N_i(\phi_m) \cdot N_j(\phi_m) d\phi_m$$

$$Kx_{ij} = \frac{\mu_o r l}{g_o} \int_0^{2\pi} N_i(\phi_m) \cdot N_j(\phi_m) \cdot \cos(\phi_m) d\phi_m$$

$$Ky_{ij} = \frac{\mu_o r l}{g_o} \int_0^{2\pi} N_i(\phi_m) \cdot N_j(\phi_m) \cdot \sin(\phi_m) d\phi_m$$

$$Kxx_{ij} = \frac{\mu_o r l}{g_o} \int_0^{2\pi} N_i(\phi_m) \cdot N_j(\phi_m) \cdot \cos^2(\phi_m) d\phi_m$$

$$Kyy_{ij} = \frac{\mu_o r l}{g_o} \int_0^{2\pi} N_i(\phi_m) \cdot N_j(\phi_m) \cdot \sin^2(\phi_m) d\phi_m$$

$$Kxy_{ij} = \frac{\mu_o r l}{g_o} \int_0^{2\pi} N_i(\phi_m) \cdot N_j(\phi_m) \cdot \cos(\phi_m) \cdot \sin(\phi_m) d\phi_m$$

which can be re-written in matrix form as:

$$[L] = [Lo] + \varepsilon_x \cdot [Kx] + \varepsilon_y \cdot [Ky] + \varepsilon_x^2 \cdot [Kxx] \\ + \varepsilon_y^2 \cdot [Kyy] + 2 \cdot \varepsilon_x \cdot \varepsilon_y \cdot [Kxy] \quad (6)$$

Assuming magnetic linearity, the magnetic energy stored in the induction machine can be written as

$$W_f = \frac{1}{2}[I]^T [L][I] \quad (7)$$

The net electromagnetic force in the horizontal direction is

$$F_x = \frac{1}{2}[I]^T \frac{\partial [L]}{\partial x}[I] \\ = \frac{1}{2g_o}[I]^T \frac{\partial [L]}{\partial \varepsilon_x}[I] \\ = \frac{1}{2g_o}[I]^T \{[Kx] + 2 \cdot \varepsilon_x \cdot [Kxx] + 2 \cdot \varepsilon_y \cdot [Kxy]\}[I] \quad (8)$$

Similarly, the net electromagnetic force in the vertical direction is

$$F_y = \frac{1}{2}[I]^T \frac{\partial [L]}{\partial y}[I] \\ = \frac{1}{2g_o}[I]^T \frac{\partial [L]}{\partial \varepsilon_y}[I] \\ = \frac{1}{2g_o}[I]^T \{[Ky] + 2 \cdot \varepsilon_y \cdot [Kyy] + 2 \cdot \varepsilon_x \cdot [Kxy]\}[I] \quad (9)$$

The magnitude of the resulting unbalanced magnetic pull is, finally :

$$UMP = \sqrt{F_x^2 + F_y^2} \quad (10)$$

Equations (8) and (9) show that a net electromagnetic force can exist due to eccentricity alone or due to current unbalance within the same phase or due to both.

For an induction machine with no eccentricity, the inductances and hence the impedances of all coil groups are equal. This leads to equal currents in all coil groups and equal voltages across them whether they are connected in series, series/parallel or all in parallel. So no matter what the coil group connection is, there will be one winding function (hence one coupled circuit) corresponding to each phase winding which is sinusoidal (if space harmonics are neglected). In case of eccentricity, the inductances (and hence impedances) associated with different coil groups are not necessarily equal. Thus currents in parallel circuits can be different and are thus simulated as separate coupled circuits. The winding function corresponding to each coupled circuit would not be sinusoidal. Closed form solutions for the integrals to calculate all terms of Eq. (5) are not feasible -except in special cases-, hence numerical integrations inside the simulation program are required.

Under balanced operation all the coupled circuits equivalent impedances are symmetrical hence the phase (line-to-neutral) voltages can be obtained directly from the line-to-line voltages. Under unbalanced operation such as stator or rotor faults or in this paper, rotor eccentricity, the impedances are no longer symmetrical hence the machine model is adjusted to employ line-to-line voltages as inputs[10]. On the stator side equivalent line-to-line flux linkages and inductances (which do not necessarily have a physical meaning) are used.

It should be noted that this model has two limitations that effect its accuracy for obtaining the various components of UMP due to eccentricity. Slotting on both the stator and rotor has been neglected. However, previous experimental results have shown that a component of UMP exists at rotor slot frequency. The model also assumes magnetic linearity. In case of eccentricity, the saturation level instead varies along the machine periphery according to the flux density value. Thus higher saturation occurs in both the core and teeth at the smaller air gap side than at the larger air gap side.

SIMULATION RESULTS

A. TRANSIENT WITH ALL POLES SERIES CONNECTED

The 4 pole 100 HP machine -whose parameters are given in the appendix- was simulated with all coil groups connected in series at no-load. A static rotor eccentricity in the horizontal direction is assumed such that the per unit eccentricities are $\varepsilon_x=0.1$ and $\varepsilon_y=0$. Figure 2 shows the motor speed and the two electromagnetic force components in horizontal and vertical directions for the line starting transient. Both force components start with high oscillation values which die out as the machine reaches steady state speed. F_x has a steady state average of 3262 newtons while F_y is practically zero in steady state. The three phase currents remain

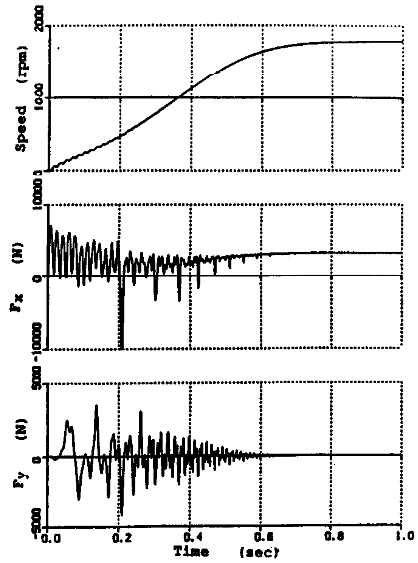

Fig. 2 UMP starting transient with eccentric rotor

symmetrical in case of eccentricity as each of them is flowing in a winding that spans the whole motor periphery so the total winding impedances are equal.

B. INFLUENCE OF OPERATING CONDITIONS ON UMP

In this case the 100 HP motor was simulated under a variety of conditions by varying rotor eccentricity, supply voltage or load torque. For each simulation run, the average steady state UMP was calculated.

B.1 Variation of UMP with Eccentricity

The per unit eccentricity was varied from 0 to 0.25 for both no load and full load torque. Figure 3 shows the resulting average UMP. It is clear that the UMP is linearly proportional to per unit eccentricity for both cases. Correlating this result with Eq. (8) shows that the

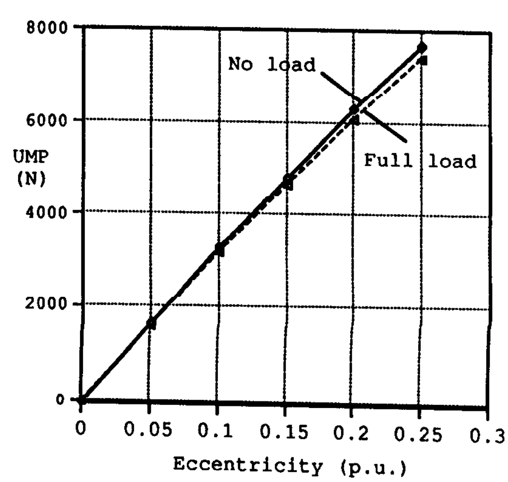

Fig. 3 Variation of UMP with eccentricity

term containing $[Kx]$ is zero and that is due to the currents being symmetrical. Previously published experimental results confirm the linear relationship between per unit eccentricity and UMP but they differ in establishing the value of eccentricity at which this relationship departs from linearity. Reference [8] claims that linearity is valid only until 0.1 per unit eccentricity while reference [5] has linear results up to 0.45 per unit eccentricity.

B.2 Variation of UMP with Voltage

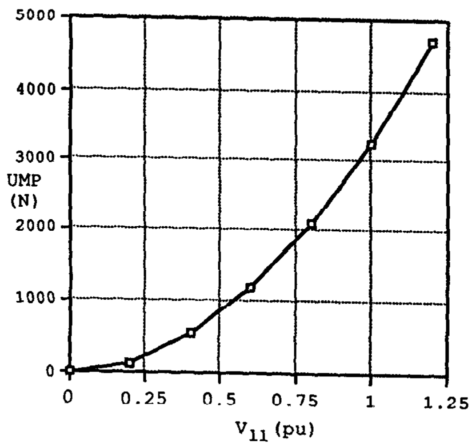

Fig. 4 Variation of UMP with voltage

In this case the line-to-line voltage was varied from 92 volts (0.2 per unit) to 552 volts (1.2 per unit) in steps of 0.2 per unit, at an eccentricity of 0.1 and no-load conditions. Figure 4 shows the resulting UMP as a function of per unit supply voltage. The UMP is directly proportional to the square of the line voltage. This confirms with previous predictions and experimental results[5] and is explained by the fact that UMP is proportional to the square of the air gap density. At higher voltage levels the air gap flux density is no longer proportional to line voltage due to main flux saturation so the rate of increase of UMP would be limited. This model does not incorporate magnetic saturation and so is not valid for voltage level above the simulated range.

B.3 Variation of UMP with load

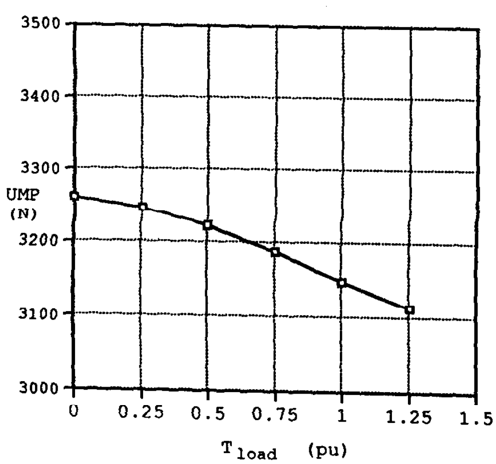

Fig. 5 Variation of UMP with load

The load torque was then varied from 0 to 523.75 Nm (1.25 per unit) at a per unit eccentricity of 0.1 and rated supply voltage. The resulting average steady state UMP is shown in Fig. 5 There is a slight drop of UMP with load which appears to contradict earlier experimental results[5,8] which obtain an increase in the UMP with load.

C. REDUCTION OF UMP

In order to decrease the UMP in case of an eccentric rotor, the eccentricity-induced air gap flux density distortion should be reduced or eliminated. Three different methods have been previously proposed to achieve this[8] :

- Using equalizing windings on the stator[2].
- Using short circuited damper windings on the stator.
- Reconnection of stator coil groups to obtain parallel paths for winding currents[3]. The most economical of the above methods is to use parallel connection of stator coil groups.

Simulations were then performed for the same conditions as case (A) but with the stator windings connected to have two parallel paths with the adjacent coil groups (poles) in series. The six no-load stator currents are shown in Fig. 6. It is obvious that for each phase the currents in the parallel circuits are distorted and unequal. Figure 7 shows the steady state UMP in both the horizontal and vertical directions. As expected, the average magnetic pull in the direction of minimum air gap is greatly attenuated compared to previous results with a single circuit. It should be noted that computation of the effect of attenuation can be accurately obtained by the coupled circuit model of this paper but is only roughly estimated by the previous steady state approaches.

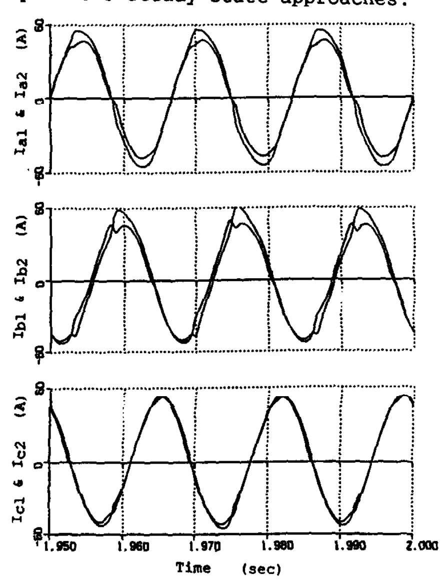

Fig. 6 Currents in parallel paths for adjacent poles in series and opposite poles in parallel connection

The mechanism by which the average UMP is reduced in case of adjacent poles connected in series and opposite poles connected in ırallel is explained in reference [7]. ıe inductance is lower in those circuits ocated where the air gap is larger than ominal, relative to the inductance of ircuits located where the air gap is maller than nominal. The currents in the ircuits facing the larger air gap are lightly greater than those facing the maller air gap resulting in reduction of he eccentricity-induced distortion of the ir gap flux density. Correlating this esult with Eq. (8) shows that due to urrents being unequal in the parallel aths, the term containing $[Kx]$ is egative, resulting in a reduction of the otal value of F_x given by that expression. imilar results are expected in the case here all coil groups are connected in arallel

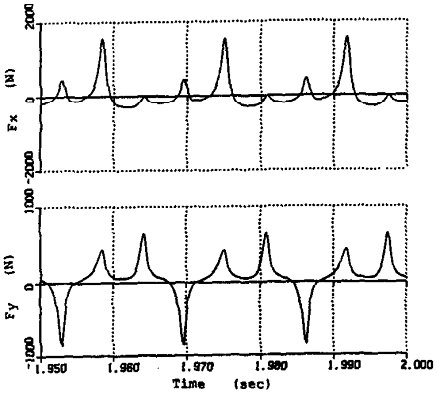

Fig. 7 Steady State UMP for adjacent poles in series and opposite poles in parallel connection

CONCLUSIONS

A new induction machine model proposed to study unbalanced radial forces due to static rotor eccentricity is developed in this paper. This model which calculates forces based on inductance variation has two main advantages over the conventional methods which calculate the resulting force vector based on flux density distribution. Clearly, transient as well as steady state simulations can be carried out with this model. In addition, a high flexibility exists in the formulation to simulate a variety of different stator winding connections. Digital computer simulations have been shown to yield satisfactory results which are in close agreement with experimental results of previous studies. The possibility of reducing eccentric UMP using parallel paths in the stator winding has also been demonstrated via the use of computer simulations.

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APPENDIX

Parameters of the machine used in the simulation study:

460 volt, 60 Hz, 100 HP, 4 pole induction machine with parameters:

L_{ls} = 0.4 mh	L_{lr} = 0.4 mh
R_s = 0.031 Ω	R_r = 0.134 Ω
L_m = 18.9 mh	J = 4.449 kg m^2
g_o = 0.1143 cm	