Research Report
94 - 35

Multiple Coupled Circuit Modeling of Synchronous Reluctance Machines

X. Luo*, A. El-Antably†, and T.A. Lipo*

*Electrical & Computer Engineering
University of Wisconsin-Madison
Madison WI 53706-1691

†Delco Remy Division
General Motors Corporation
Anderson IN 46018

Wisconsin
Electric
Machines &
Power
Electronics
Consortium

University of Wisconsin-Madison
College of Engineering
Electrical & Computer Engineering Department
2559D Engineering Hall
1415 Johnson Drive
Madison WI 53706-1691

MULTIPLE COUPLED CIRCUIT MODELING OF SYNCHRONOUS RELUCTANCE MACHINES

Xiaogang Luo‡, Ahmed El-Antably* and Thomas A. Lipo‡

‡ University of Wisconsin-Madison
1415 Johnson Drive
Madison, WI 53706-1691

* GM-Delco Remy
Indianapolis IN

Abstract—A new multiple coupled circuit model is presented for simulation of synchronous reluctance machines with both arbitrary winding layout and/or unbalanced operating conditions. The model is derived by means of magnetic circuit method and no symmetry is assumed. The parameters of the model are calculated directly from the geometry and winding layout of the machine. The behavior of a synchronous reluctance machine during starting is simulated using this model. The results are shown to be in good agreement with the solution obtained by a conventional d-q model for symmetric conditions. The new model is then extended to the solution of a wide variety of fault conditions such as open or short circuited stator coils.

Key Words: AC Machine Model, Simulation, Synchronous Reluctance Machine

I. INTRODUCTION

The well-known d-q model of AC machine is extensively used in AC machine analysis and simulation. Based on the assumption that the stator windings of AC machine are sinusoidally distributed, the d-q model is a powerful tool for static and dynamic analysis of AC machines, but is not suitable for study of a general machine with arbitrarily connected windings and internal winding faults such as coil open circuit and short circuit because the assumption of symmetrical and sinusoidal distribution of windings no longer hold. The multiple coupled circuit model of an AC machine, on the other hand, has been shown to be more desirable for the general purpose, time domain simulation of AC machines[1].

One feature of multiple coupled circuit model of AC machine is that in recognizing that inductances of an AC machine are time varying, it precalculates the inductance vs. rotor position curves. During real time simulation the parameters are evaluated (or looked up), while the secondary parameters such as resistances and leakage inductances are considered constants and precalculated in a conventional manner[2][3].

In multiple coupled circuit modeling of AC machine, the key to success is the evaluation of inductances. Although field theory and the finite element method are more accurate for inductance calculations and other analysis parameters, they are either too involved to obtain an analytical result or too costly in terms of setting up the model and computation time. In induction machine simulations, an approach termed the winding function is a convenient means to use for inductance calculation which assumes no symmetry in the placement of any motor coil in the slot[4]. However, the assumption that the stator and rotor laminations are isotropic makes it unsuitable for a reluctance machine with magnetic barriers in its rotor. The magnetic circuit approach, on the other hand, is more suitable for this case[5],[6]. This paper presents a new approach to coupled circuit modeling of synchronous reluctance machine using both winding functions and magnetic circuit modeling.

II. SYSTEM EQUATIONS AND SIMULATION MODELS

Consider initially, a synchronous reluctance machine having m stator circuits. Note that there are m stator circuits instead of m stator phases. The dynamic equations governing the behavior of the machine are then:

A. Voltage Equations:

The stator voltage equations are:

$$V_s = R_s I_s + \frac{d\Lambda_s}{dt} \quad (1)$$

where

$$\Lambda_s = L_{ss} I_s \quad (2)$$

and

$$I_s = \left[i_{s1} \; i_{s2} \cdots\cdots i_{sm} \right]^T \quad (3)$$

$$V_s = \left[V_{s1} \; V_{s2} \cdots\cdots V_{sm} \right]^T \quad (4)$$

The matrix R_s is a diagonal m by m matrix given by

$$R_s = \begin{bmatrix} r_{s1} & 0 & \cdots & 0 \\ 0 & r_{s2} & \cdots & 0 \\ . & . & \cdots & . \\ 0 & 0 & \cdots & r_{sm} \end{bmatrix} \quad (5)$$

where r_{si} is the resistance of circuit i.

The matrix L_{ss}, due to conservation of energy, is a symmetric m by m matrix of the following form:

$$L_{ss} = \begin{bmatrix} L_{s11} & L_{s12} & \cdots & L_{s1m} \\ L_{s21} & L_{s22} & \cdots & L_{s2m} \\ \cdot & \cdot & \cdots & \cdot \\ L_{sm1} & L_{sm2} & \cdots & L_{smm} \end{bmatrix} \qquad (6)$$

B. Torque Equation

$$T_e = \frac{1}{2} I_s^T \frac{\partial L_{ss}}{\partial \theta} I_s \qquad (7)$$

where θ is the mechanical angle.

C. Mechanical Equation

$$J \frac{d\omega}{dt} = T_e - T_L \qquad (8)$$

$$\frac{d\theta}{dt} = \omega \qquad (9)$$

where J is the inertia of the rotor, T_L is the load torque.

D. Ideal Current Source Model for Simulation

If an ideal sinusoidal current source is assumed as an excitation for the synchronous reluctance machine, then the state variables are ω and θ. ω is the angular speed of the rotor. Because I_s is known, the equations for simulation are straightforward:

$$T_e = \frac{1}{2} I_s^T \frac{\partial L_{ss}}{\partial \theta} I_s \qquad (10)$$

$$\frac{d\omega}{dt} = \frac{1}{J}(T_e - T_L) \qquad (11)$$

$$\frac{d\theta}{dt} = \omega \qquad (12)$$

E. Flux Model for Simulation

From the previous results the equations required for simulation can be readily written as:

$$I_s = L_{ss}^{-1} \Lambda_s \qquad (13)$$

$$\frac{d\Lambda_s}{dt} = V_s - R_s I_s \qquad (14)$$

$$T_e = \frac{1}{2} I_s^T \frac{\partial L_{ss}}{\partial \theta} I_s \qquad (15)$$

$$\frac{d\omega}{dt} = \frac{1}{J}(T_e - T_L) \qquad (16)$$

$$\frac{d\theta}{dt} = \omega \qquad (17)$$

III. CALCULATION OF INDUCTANCES

Obviously, the calculation of the machine inductances as defined by the inductance matrix in previous section is the key to the successful simulation of a synchronous reluctance machine. The machine inductances can be calculated by a variety of means including field theory, finite element method and various circuit approaches. However, magnetic circuit approach is the most suitable for our case due to the presence of the axially laminated rotor structure. The method assumes no symmetry in the placement of any coil in the stator slots.

It is clear that all necessary assumptions can not be removed if a reasonable simulation model is to be expected. The remaining basic assumptions made for the derivation of the stator winding inductances are:

1) No saturation (The effect of saturation is represented by a saturation coefficient of 1.8);

2) No eddy current and hysteresis losses, friction and windage losses are negligible;

3) No slotting and skewing. The slotting effect is taken into account by Carter's coefficient;

4) Uniform air gap;

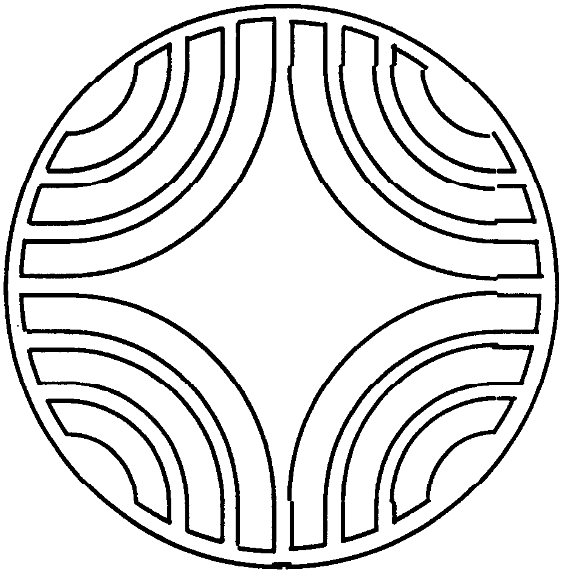


Fig. 1 Structure of Axially Laminated Rotor of Synchronous Reluctance Machine

Fig. 1 shows an axially laminated rotor structure of a synchronous reluctance machine. The extended view of the machine is shown in Fig. 2.

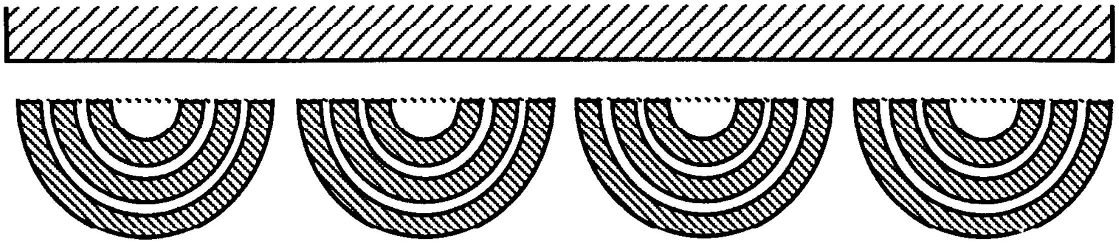


Fig. 2 Expanded View of Synchronous Reluctance Machine

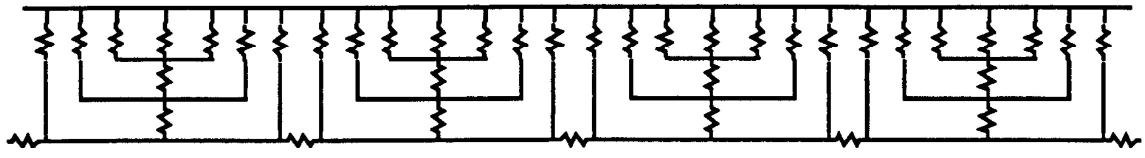


Fig. 3 Magnetic Circuit of SRM Without Excitation

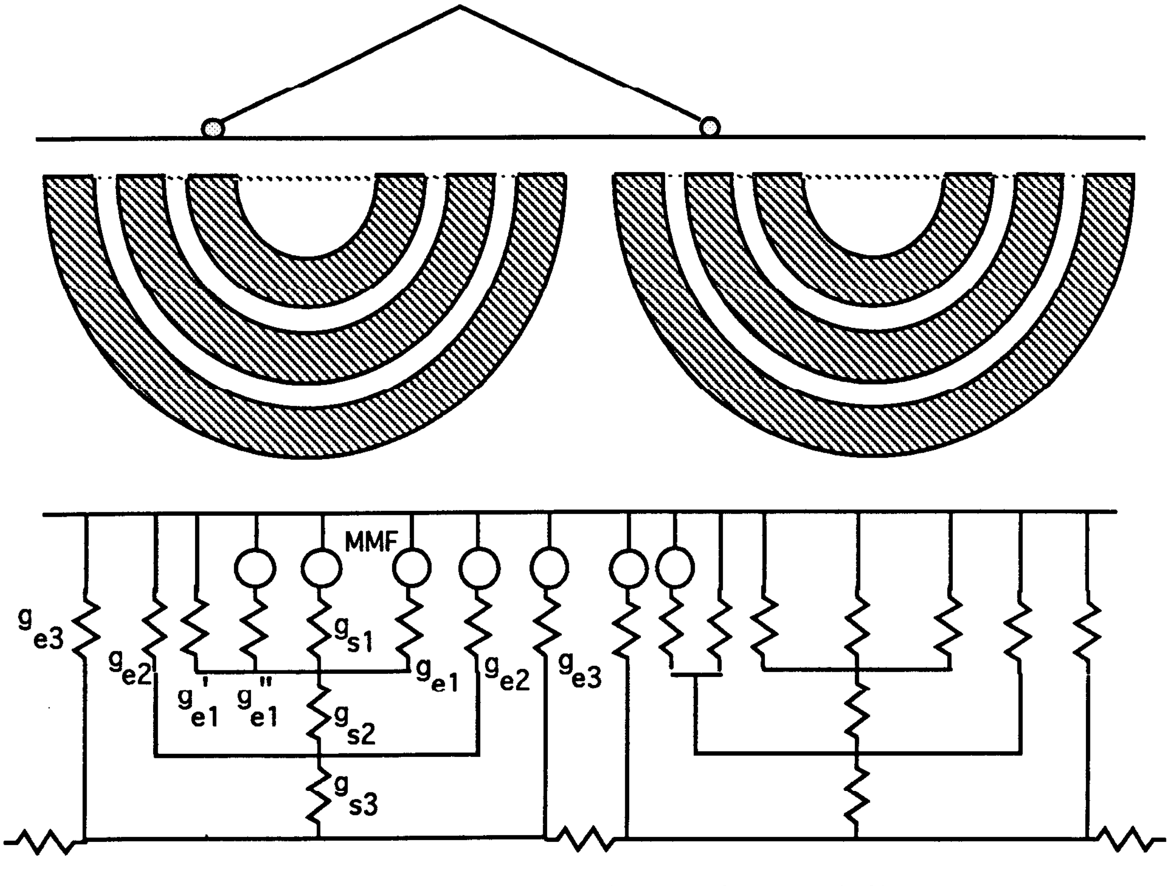


Fig. 4 A Coil Carrying Current and Corresponding Magnetic Circuit

A. Magnetic Circuit Model of the SRM

The basic assumption for an inductance calculation, is that all the iron/magnetic materials (that is, the stator and rotor laminations) have infinite permeability, so that they are treated as equal magnetic potential body in magnetic circuit method. Obviously, this is only an approximation when the magnetic paths are not saturated.

As the stator can be viewed as an integrated equal magnetic potential body, it is taken as the reference potential (ground). Each lamination in the rotor is represented in the circuit as a node. The reluctances between laminations and stator can be calculated according to the geometry of the rotor structure/air gap. In this manner the magnetic circuit is formed which is shown in Fig. 3. Note that there is no excitation (MMF) in the circuit. In the stator case, the teeth are neglected, that is to say, the stator is considered as a smooth cylinder. All the windings are assumed to be placed in the center of the air gap. Fig. 4 shows a case where a coil is carrying current while all the other coils are not. According to Fig.4, the node equation can be written as:

$$G_m \cdot P_m = J_m \tag{18}$$

where

$$G_m = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ g_{n1} & g_{n2} & \cdots & g_{nn} \end{bmatrix} \tag{19}$$

is the permeance matrix of the machine,

$$P_m = \left[\, p_1 \;\; p_2 \;\cdots\cdots\; p_n \right]^T \tag{20}$$

is the node magnetic potential vector, and

$$J_m = \left[\, j_1 \;\; j_2 \;\cdots\cdots\; j_n \,\right]^T \tag{21}$$

is the "flux source" vector, and n is the number of nodes in the circuit (excluding the ground) which is equal to the total number of laminations in the rotor of the SRM.

From Fig. 4, the element of G_m, P_m and J_m can be calculated in the same fashion as we do in electric circuit equation. For example, g_{11} can be written as

$$g_{11} = g_{e1} + g'_{e1} + g''_{e1} + g_{s1} + g_{s2} \tag{22}$$

g_{12} can be written as

$$g_{12} = -g_{s2} \tag{23}$$

and j_1 can be expressed as

$$j_1 = g_{e1} \cdot MMF + g_{s1} \cdot MMF + g''_{e1} \cdot MMF \tag{24}$$

Obviously, this magnetic circuit model can be used to calculate inductances of the SRM. However, initially it is necessary to calculate all the permeances in the circuit.

1) Calculation of Permeances between Ends of Laminations and Stator

In the calculations to follow, the assumption made is that all flux lines go straight from one side of the gap to the other. The fringing effect is accounted for by enlarging the width of the end portion (refer to Fig. 5).

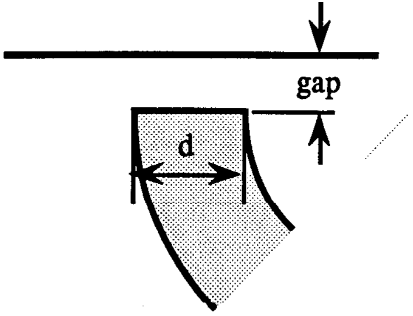


Fig. 5 Enlarged View of End of Lamination and Stator

Taking the fringing effect into account, the permeance between end of lamination and stator is written as

$$g_e = \mu_0 \cdot L \cdot \frac{d + gap}{gap} \tag{25}$$

where L is the length of the lamination and d is the width of the end.

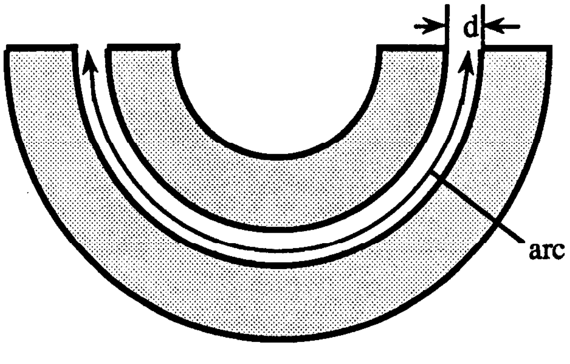


Fig. 6 Two Lamination Layers of Same Group

2) Calculation of Permeances between Sides of Laminations of the Same Group

Fig. 6 shows the geometry of two lamination layers of same group. It is reasonable to calculate the permeance between the two layers as

$$g_s = \mu_0 \cdot L \cdot \frac{arc}{d} \tag{26}$$

where arc is the average side length of the two sides of the lamination layers. Because the relative large length of arc compared to gap d, the fringe effect is neglected here.

3) Calculation of Permeances between Sides of Laminations of Different Groups

Although the above two cases have proved to be quite simple, the remaining two kinds of permeances are hard to estimate accurately due to the irregular geometry. Some degree of approximation is needed in the calculation.

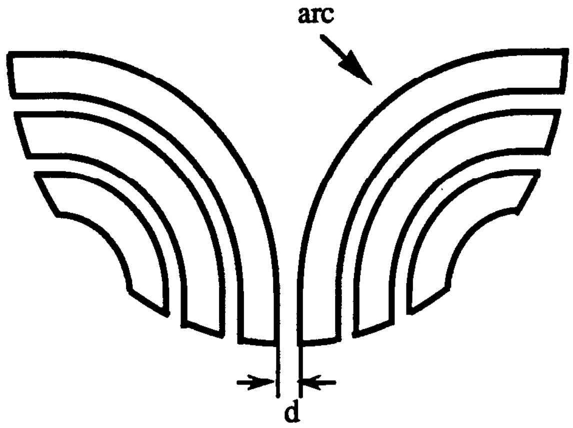


Fig. 7 Uneven Gap Length Between Two Group of Layers

If Eq. 26 is continued to be used to calculate the permeance, an approximation to the effective gap length and the width of the layer should be made. In this case, referring to Fig. 7, the gap length is taken as the minimum gap length, while the effective width of layer is taken to be 1/4 of the arc length of the outer side width of the layer which is a rough approximation. Hence, the permeance between sides of laminations of different groups is

$$g_s = \mu_o \cdot L \cdot \frac{arc_{eff}}{d} \qquad (27)$$

4) Calculation of Permeances between Sides of Laminations and Stator

As shown in Fig. 8, the gap length between the layer side and the stator is uneven. The approximations used to evaluate the effective gap length and gap width are

w_{eff} = arc of stator over the region

d_{eff} = gap + 0.5 d

so that the permeance is

$$g_{ss} = \mu_o \cdot L \cdot \frac{arc_{eff}}{d_{eff}} \qquad (28)$$

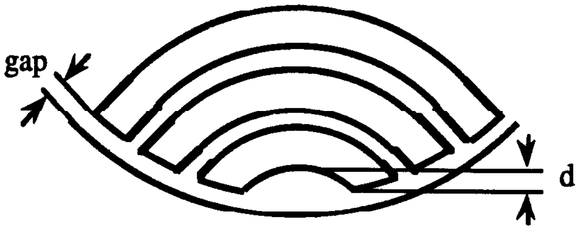


Fig. 8 Uneven Gap between Layer Side and Stator

5) Calculation of g' and g"

As can be noted in Equation (24) and Fig. 4, when a coil side falls between two edges of a layer end (or a side) as shown in Fig. 9, it is necessary to split the permeance of the end (or the side) into two parts in order to calculate the "flux source" of the circuit equation. Fortunately due to our assumption this can be done without difficulty.

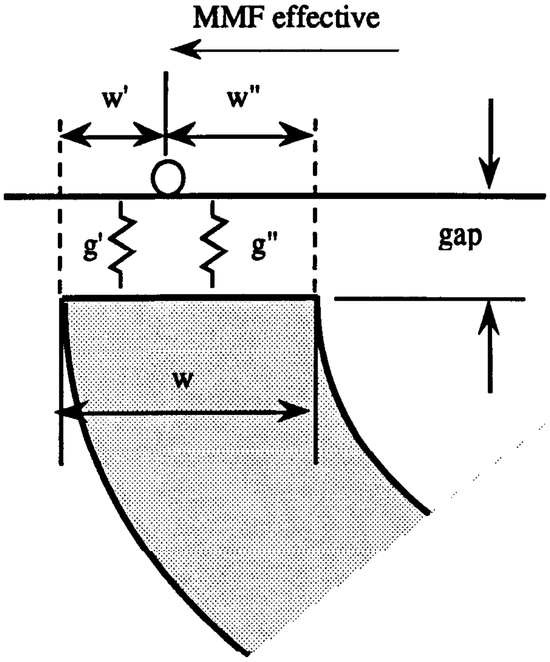


Fig. 9 Illustration of Calculation of g' and g"

Referring to Fig. 9, the permeance of the region outside the MMF effective region is

$$g' = \frac{w'}{w} g \qquad (29)$$

while the permeance of the region inside the MMF is

$$g'' = \frac{w''}{w} g \tag{30}$$

It is obvious that

$$g = g' + g'' \tag{31}$$

B. Calculation of Inductances between all Coils

Having derived the magnetic circuit model, one is now ready to calculate the actual inductances of the SRM. The first step is to calculate inductances between all the coils.

Step 1: One must calculate all the permeances of the SRM from the geometry of the SRM from design data. Also calculate J_m vector of equation (18) by letting only one coil (coil 1) carry unit current. In calculating J_m, g' and g" will be used if the coil side falls between two edges of one layer end (or a side) as shown in Fig. 9. In this manner equation (18) is formed.

Step 2: Solve equation (18) by the Gauss method for the magnetic potential.

Step 3: Find all the flux in the circuit path from stator to rotor following equation (32):

$$\text{flux} = g \cdot (\text{MMF} - p) \tag{32}$$

where g is the permeance of that path and p is the magnetic potential of the node opposite to the stator which is obtained from step 2. Note that MMF is produced by the unit current in coil 1 picked in step 1. If g' and g" are used in step 1, they are also used here. The flux is assumed to be perpendicular to the stator inner surface and evenly distributed within the region specified by the path when the permeance is calculated as illustrated in Fig. 10.

Step 4: Select coil 2. Calculate the flux linking the coil by summing the flux calculated in step 3 over the coil span. The inductance between coil 1 and coil 2 is the product of the number of turns of coil 2 and the flux linking the coil since unity current is assumed to be applied to coil 1.

All the inductances between coils are calculated in the same fashion.

C. Calculation of Inductances between all Circuit

As it is already known that this simulation is based on circuits, it is important to calculate inductances between all the circuits.

The equation for calculating the nductance between circuit m and n is given as[2]:

$$L_{smn} = \sum_{i=1}^{k} \sum_{j=1}^{l} \pm L_{minj} \tag{33}$$

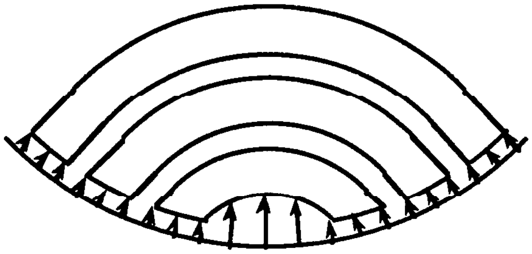


Fig. 10 Flux in the Air Gap

where k is the number of coils of circuit m, l is the number of coils of circuit n, L_{minj} is the mutual inductance between coils mi and nj. The sign before L_{minj} is determined according to the manner of connection of these two phase-belt windings. When m=n Eq. (36) gives the self inductance of circuit m.

In the simulation program an input matrix is needed to describe the connection of stator circuits in terms of phase-belt windings and in turn an input matrix describing the forming of phase belt winding is also needed for calculating the circuit inductances.

D. Calculation of Curves of Inductances Between all Circuits

In order to implement a simulation, it is necessary to determine the inductances and their derivatives at each simulated rotor angle. The curves of $L_{ss}(\theta)$ and $\frac{dL_{ss}(\theta)}{d\theta}$ must thus be calculated in advance so that in simulation iterations the program only need a look up table to interpolate to find the inductances and their derivatives at the present rotor angular position. The steps for calculating the curves are:

Step 1: Calculate all the permeances of the SRM. Form G_m of equation (18). Set initial rotor angular position at zero.

Step 2: For each rotor angular incremental position from 0 to 2π/number of poles, do following:

- calculate coil inductances,
- calculate circuit inductances, add to circuit self inductances with the leakage inductances calculated in conventional way,
- calculate inductance derivatives.

IV. SIMULATION PROGRAM

Using the model derived, a simulation study has been conducted examining the performance of a SRM and the results are presented herein.

For purposes of comparison the model is used to simulate the acceleration transient of a symmetrical three phase machine from rest under current regulated pulse width modulation (CRPWM) excitation and the results are shown in Fig. 11. Fig. 12 shows the calculated circuit self inductance vs. rotor electrical angular position curve. In Fig. 13 the same machine is simulated using the conventional *d-q* model under the same conditions for comparison. Comparison of the two simulation traces clearly shows very good correlation.

The model which has been developed can also be used with simulation techniques to solve nearly any asymmetrical conditions including stator winding circuit open circuits, and circuit terminal to terminal short circuit faults. The open circuit faults are assumed to occur when the relevant current reaches zero because of the existence of inductances in all current paths.

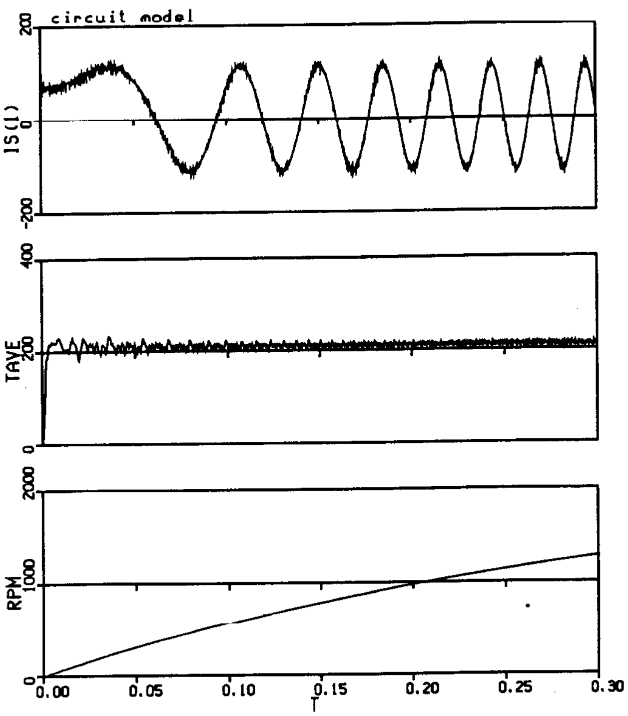


Fig. 11 Acceleration Transient Using New Model with Excitation from a CRPWM

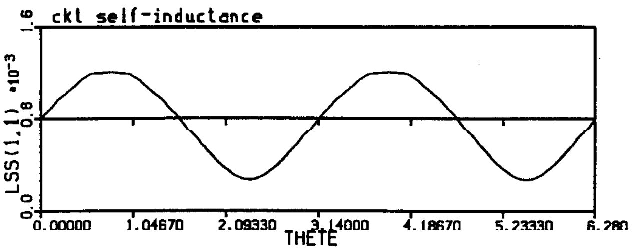


Fig. 12 Calculated Circuit Self Inductance vs. Rotor Electrical Angular Position Curve

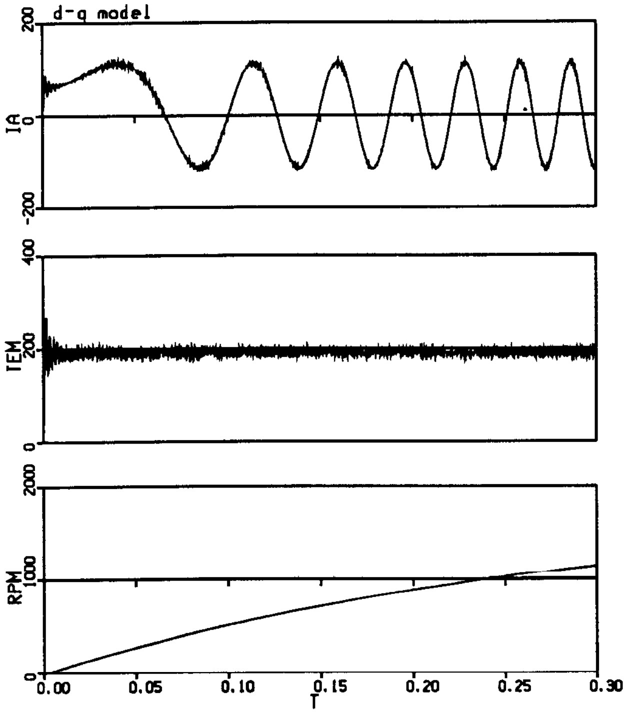
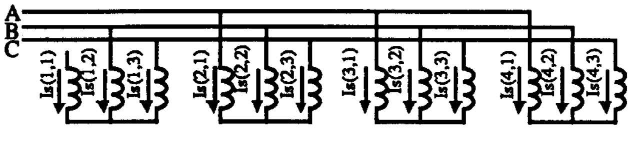
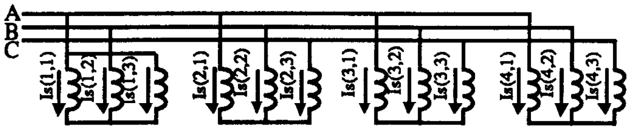


Fig. 13 Acceleration Transient Using Conventional D-Q Model under CRPWM



(a) one circuit open circuit



(b) circuit line to line short circuit

Fig. 14 Stator Winding Connection and Fault Conditions

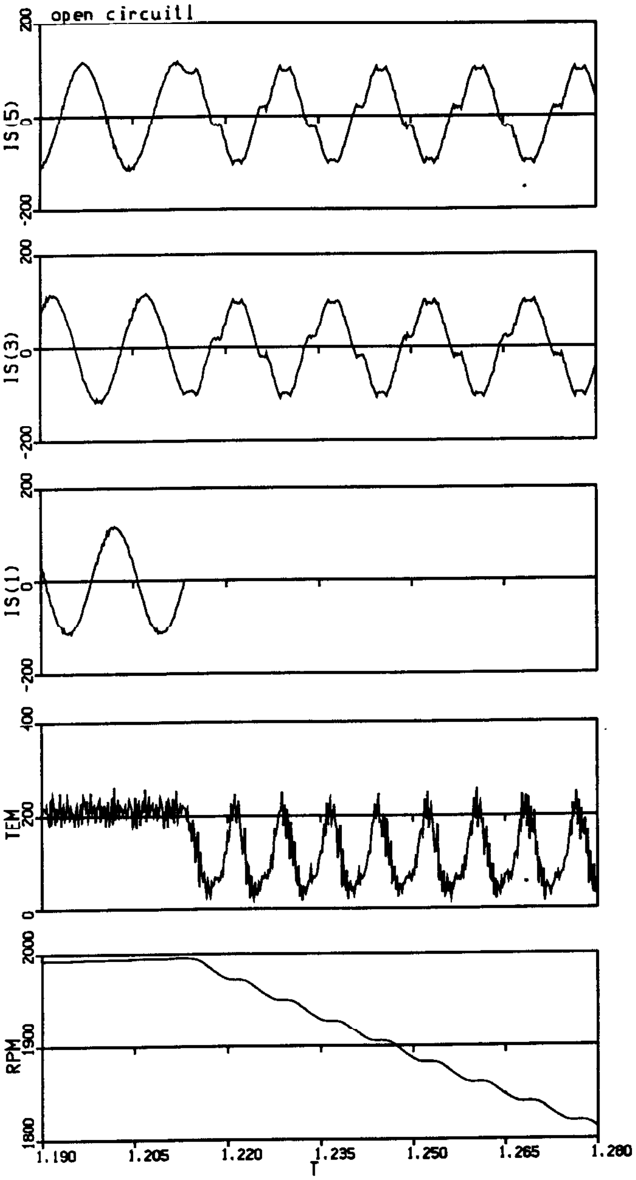


Fig. 15 A Sudden Open Circuit of One of Four Parallel Circuits Making up One of the Three Stator Phases.

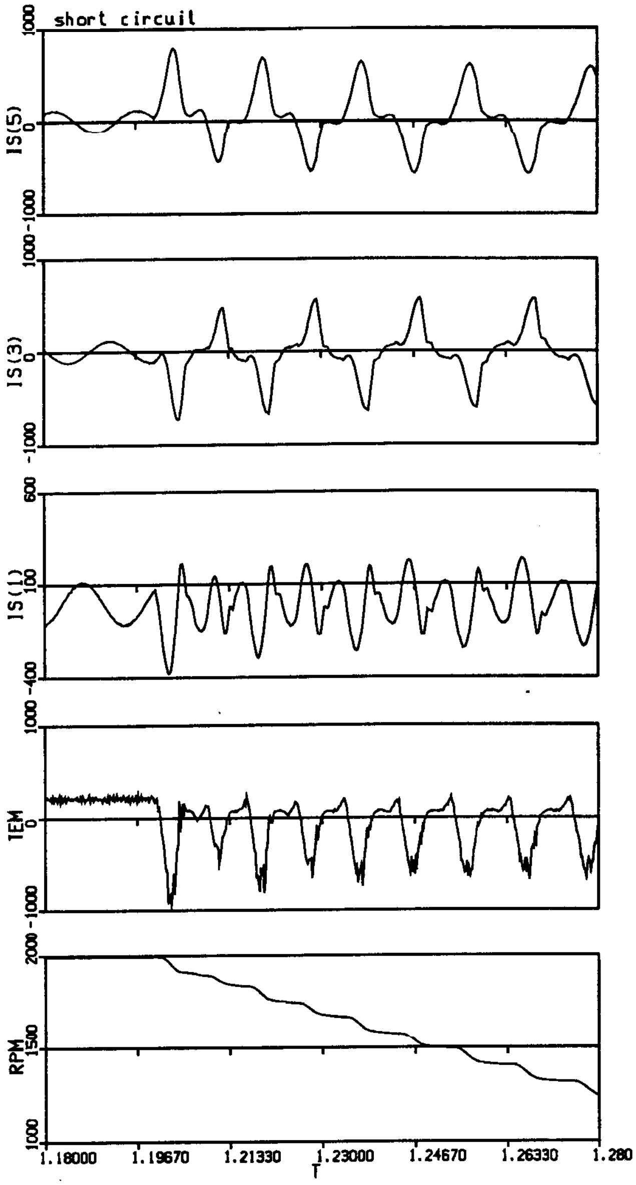


Fig. 16 A Sudden Line to Line Short Circuit of Two Circuits.

Figure 14 shows two fault conditions imposed on the stator windings. In Fig. 14(a) it is shown how one of the 4 parallel circuits of one of the phase windings is open circuited and the simulation results are shown in Fig. 15. It can be seen that when one circuit is open circuited a substantial unbalanced operation occurs but is not so significant as the case when the entire phase is open circuited.

Finally, a line to line short circuit fault which is depicted in Fig. 14(b) was simulated and the results are shown in Fig. 16. In this case, one of the four circuits of one phase of the machine is shorted corresponding to the circuit of another phase. A large pulsation in torque is observed followed by a continual double frequency torque pulsation. These selected fault conditions clearly illustrate the great flexiblity available to the analyst to investigate motor asymmetrical conditions using this analysis approach.

V. CONCLUSION

A new approach to synchronous reluctance machine modeling has been introduced in this paper. The model is based directly on the geometry of the synchronous reluctance machine and the physical layout of the stator windings. Calculation of inductances is further carried out on a coil-by-coil basis. Because the coupled circuit model takes into account arbitrary winding distributions, it should prove very useful for fault analysis wherein the fault occurs within the winding itself. A general purpose simulation program has been developed employing this model and several fault operations of the synchronous reluctance machine have been simulated to demonstrate its versatility. It has also been shown that the results are consistent with the results from the conventional d-q method for balanced, symmetrical operation.

VI. ACKNOWLEDGMENT

The support and encouragement of Mr. Ron Martin of the GM/Delco-Remy during the course of this project is gratefully acknowledged.

VII. REFERENCES

[1] Hamid A. Toliyat, "Analysis of Concentrated Winding Induction and Reluctance Machines for Adjustable Speed Drive Applications", Ph.D. Thesis, Department of Electrical and Computer Engineering, University of Wisconsin-Madison, 1991.

[2] Xiaogang Luo, Yuefeng Liao› Hamid Toliyat, Ahmed El-Antably and Thomas A. Lipo,"Multiple Coupled Circuit Modeling of Induction Machines", IAS '93 Annual Meeting.

[3] T.A. Lipo,"AC Machine Design", Course Notes for ECE 713, Department of Electrical and Computer Engineering, University of Wisconsin-Madison, 1991.

[4] T.A. Lipo, "Analysis And Control of Synchronous Machines", Course Notes for ECE 511, Department of Electrical and Computer Engineering, University of Wisconsin-Madison, 1986.

[5] A. Vagati, "Design of Synchronous Reluctance Motors", in "Synchronous Reluctance Motors and Drives - A New Alternative?", Tutorial, IAS Annual Mtg., Oct. 5, 1992.

[6] J.C. Moreira, T. A. Lipo, "Simulation of a Four Phase Switched Reluctance Motor Including the Effects of Mutual Coupling", Beijing International Conference on Electrical Machines (BICEM), Aug. 1987.