

MAGNETIC CIRCUIT MODELING OF THE FIELD REGULATED RELUCTANCE MACHINE PART I: MODEL DEVELOPMENT

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Abstract - A transient magnetic circuit model of a field regulated reluctance machine is presented. The model is based on flux loop equations rather than node equations as have previously been employed. A key feature of this approach is that magnetic circuit models based on loop equations permit an entire machine to be represented by only one equivalent pole.

Keywords: Field Regulated Reluctance Machine, FRRM, Magnetic Circuit Model, MCM, flux loop equations, transient machine models

I. INTRODUCTION

The advent of solid state converters has opened the door to the possibility of new types of electric machines which exploit the ability of the converter to produce currents of arbitrary waveform. Such machines, for example variable reluctance machines, are being actively explored for specialized applications such as high speed generators, actuators, and even traction motors [1]. This paper presents a Magnetic Circuit Model (MCM) of one of these new machine types. This machine which utilizes concentrated windings was introduced by Weh [2] and subsequently investigated by Law et. al. [3], who termed it a Field Regulated Reluctance Machine (FRRM).

In the process of their investigations [3], the need for a new model was identified as a key need for satisfactory prediction of behavior of such machines which do not rely upon three phase, sinusoidal winding distributions for energy conversion. The most useful model would be one which could be used for both machine design and for simulation of motor transient and steady-state behavior when operating with a solid state power converter. However, since the FRRM is specifically designed to produce non-

sinusoidal air gap flux and air gap MMF, to best of the authors' knowledge no single model existed capable of predicting both design details (flux densities, MMFs, etc.) and transients with the effect of magnetic saturation included for the FRRM or any other similar type machine.

In this paper, a new general purpose MCM is developed which characterizes such a machine in terms of lumped magnetic circuit parameters, rather than through the use of finite elements or lumped electrical circuit parameters. While lumped circuit models have been used in the past for design, such a model expressed in terms of terminal parameters simply predicts terminal variables such as phase voltage and current and is not capable of correctly modeling the spatial saturation effects required for the design of an FRRM. On the other hand, finite elements could be used for design [4]. Transient simulations of electric machines using finite element based methods have been performed [5],[6]. This paper explores performing transient simulations of an FRRM using magnetic circuit modeling as an alternative general purpose approaches to machine characterization. Future work needs to be done to quantitatively compare the accuracy of results, computation, and time requirements of the two methods.

The new model developed in this paper is unique to magnetic circuit modeling in that it is based on flux loop equations rather than node equations as previously employed. Magnetic circuit models based on loop equations permit an entire machine to be represented by only one pole. To the best of the authors' knowledge, the literature does not contain methods of modeling only one pole of a machine when the magnetic circuit model is based on node equations.

II. LITERATURE REVIEW

Historically, the first major work devoted to magnetic circuit modeling is the classic book by Roters [7] who derived the permeances needed to represent the spatial properties of many typical types of electrical machines. However, these parameters were only used to refine the standard motor design procedure. The duality between electrical and magnetic circuit modeling, exploited in this paper, has been recognized for many years [8]. In the late 1960's Laithwaite [9] and Carpenter [10] further developed the duality of magnetic circuits. Of particular interest is the model proposed by Carpenter, based on magnetic currents, where magnetic current is defined as the time rate of change of the magnetic flux. The resulting equivalent

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circuit is intuitively pleasing in its similarity to electric circuits.

Beginning in the early 1980's, Ostovic [11] - [15] applied magnetic equivalent circuits to electric machine transient and steady-state analysis. In [15] the theory supporting his models was presented. The formulation of the magnetic circuit model has only been developed based on node equations, however, which does not inherently yield equations allowing the solution over one pole.

Carpenter and Macdonald [16] used equivalent magnetic circuits based on magnetic current to predict the commutation time in inverter-fed synchronous motor drives. The commutation behavior is explored as a function of the structure of solid salient pole machines. Their approach includes pole face eddy currents. However, their approach ignores non-linear saturation characteristics. Moreover, the ability to include multiple coils per phase belt and the technique for handling boundary conditions are not presented.

Slemon develops an equivalent (magnetic) circuit approach to analysis of synchronous machines with saliency and saturation in [17]. The author states that it is not clear as to where the stator current elements should be placed in the mesh. This ambiguity is due to the use of a nodal based approach. To get around this ambiguity the author uses a current sheet to represent the stator current. Xiao [18] uses the equivalent circuit of Slemon to analyze a synchronous machine. Xiao's paper demonstrates the validity and accuracy of the model developed by Slemon.

III. FRRM PRINCIPLES OF OPERATION

A cross-sectional view, Fig. 1, shows the basic elements of a typical field regulated reluctance machine. In such a machine rotor saliency is produced using poles constructed of axially oriented laminations [20].

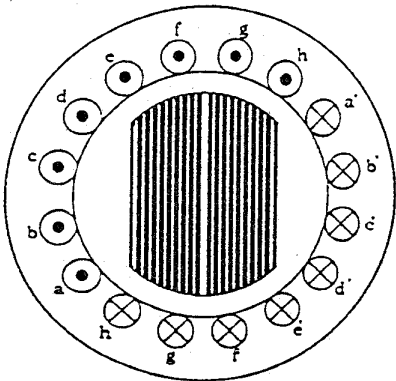


Figure 1. Simplified cross-sectional view of a field regulated reluctance machine.

An FRRM differs from conventional axially laminated reluctance machine in that it is constructed using full pitched concentrated windings embedded in a conventional slotted stator. The phase number is typically more than the value of three used in conventional machines.

Operation of the machine is such that all of the conductors are arranged to actively take part in torque production all of the time whereas in conventional three phase, square wave current source excitation, only two of the three phases are actively producing torque at any time.

Individual windings are operated in a time share mode. That is, for a portion of time each stator winding acts as the excitation (field) winding and for a portion of time as the armature winding. The mode of operation for a particular winding is dependent on the rotor position. During the period when the coils of a particular winding are located under the pole faces, the current flowing in these coils corresponds to and is controlled as the equivalent of an armature current of a dc motor. While the coils of a winding are located in the interpolar space, the current flowing in those coils corresponds to and is controlled as field current. This mode of exciting the stator windings allows for independent control of torque and regulation of the field (i.e. magnetizing) flux without need for a rotating field winding. Figure 2 shows the resulting idealized winding current and voltage waveforms versus time for motor operation. Note that the counter emf due to rotor rotation occurs only when the flux is changing.

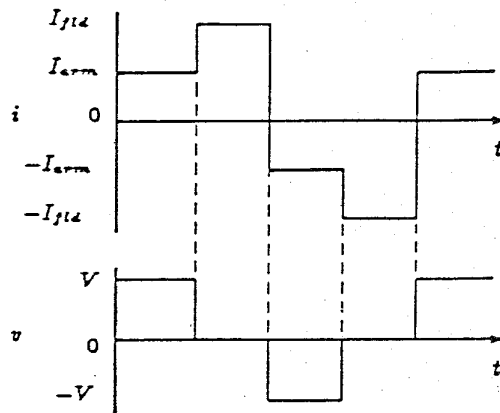


Figure 2. Idealized current and voltage waveforms of one phase of a field regulated reluctance motor.

IV. MAGNETIC CIRCUIT MODEL DEVELOPMENT

The approach used in the MCM is to characterize a machine in terms of lumped magnetic parameters, rather than magnetic fields or terminal parameters. The precise flux density profile given by finite element analysis is an excellent tool in the detailed geometric design of a machine. However, a precise flux density profile is not required for transient simulation of a well-designed conventional machine. The above statement is argued with the aid of Fig. 3.

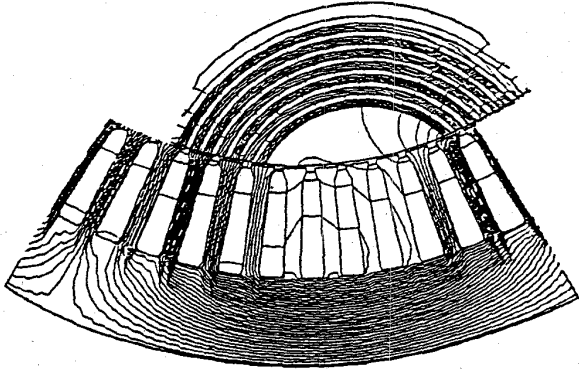


Figure 3: Flux plot of one pole of an FRRM obtained using Finite Element Analysis

Figure 3 shows a flux plot of an FRRM obtained by use of finite elements. It can be noted that the magnitude and direction of flux density, over easily identifiable sections of a machine, are relatively constant in space at a given time instant. For example, the magnetic flux density in a particular tooth is relatively constant in magnitude and direction. However, for a given set of field currents, the magnitude of flux density of different teeth varies, depending on their relative position with respect to the rotor and armature currents. The flux density in the stator back iron is relatively constant in space at a given time, in magnitude and direction over a single slot pitch. Flux density of an iron rotor lamination varies slowly over its length. The relatively long rotor arc length, and the thinness of the magnetic insulating material separating iron layers, allow armature reaction to force flux between iron layers. The resulting q-axis flux in the rotor iron laminations can be accounted for with lumped magnetic paths between, and perpendicular to, the iron laminations.

By explicitly taking into account this *a priori* knowledge of the flux behavior, an electrical machine can be described with reasonable accuracy by a much less detailed lumped parameter magnetic circuit. Figure 4 shows a cross-sectional view of the significant features of one pole of an FRRM.

It can be noted that all the turns in one slot carry the same magnitude of current in an FRRM. Therefore, Ampere's law can be written as

$$\oint_{12341} \mathbf{H} \cdot d\mathbf{l} = Ni_c \quad (1)$$

where N is the number of turns, and i_c is the magnitude of the winding currents.

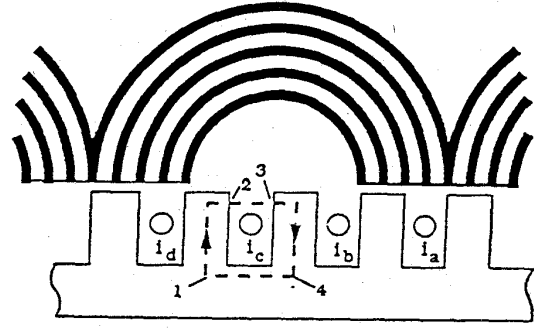


Figure 4: A cross-sectional view of the significant features of one pole of an FRRM

Assuming that the field intensity, H , is constant along a particular section of, and everywhere co-linear with, the integration path, Eq. 1 can be rewritten as

$$H_{12}l_{12} + H_{23}l_{23} + H_{34}l_{34} + H_{41}l_{41} = Ni_c \quad (2)$$

Approximating the cross-sectional area, A , and the magnetic flux, ϕ , as constant along each sub-length of the original integration path, the field intensities are expressed in terms of ϕ 's and A 's in Eq. 3.

$$\left[\frac{l_{12}\phi}{\mu_{12}A_{12}} \right] + \left[\frac{l_{23}\phi}{\mu_{23}A_{23}} \right] + \left[\frac{l_{34}\phi}{\mu_{34}A_{34}} \right] + \left[\frac{l_{41}\phi}{\mu_{41}A_{41}} \right] = Ni_c \quad (3)$$

Recognizing the bracketed terms as reluctances multiplied by flux allows Eq. 3 to be rewritten as

$$R_{12}\phi + R_{23}\phi + R_{34}\phi + R_{41}\phi = Ni_c \quad (4)$$

Correction factors can be readily applied to lengths and areas to account for differences between the physical machine and the approximations just invoked. In addition, it is clear that certain of the machine sections just described may have to be subdivided, or new sections added, for special machines or machines operating with unusual excitation so that a designer's experience is important in developing a suitable model.

A magnetic circuit for a simplified FRRM, Fig. 5, results from applying the techniques employed above to the remainder of the machine. Crosses in the magnetic circuit indicate the location of currents. Loop equations give a simple relationship between the total flux linking a phase and the flux loop encircling a slot current, and a formulation which takes advantage of the half-wave symmetry over a pole-pair. The reluctance topology in Fig. 5, developed systematically from Fig. 4, is an equivalent magnetic circuit for a single pole of a simplified FRRM, presented for purposes of development. The actual model for a real FRRM is developed further in the Part II of this paper.

Next, loop equations can be developed for a simplified two pole FRRM with a two layer, full pitch, concentrated winding with all of the coils connected in series. Initially,

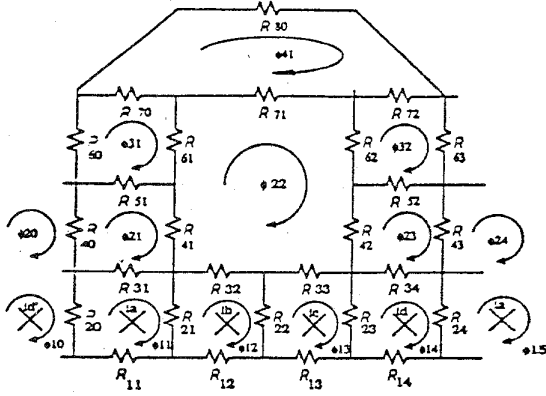


Figure 5: Magnetic circuit model of a field regulated reluctance machine

the machine is restricted to one coil per phase belt. Note that half-wave symmetry conditions give:

$$\begin{aligned}\phi_{10} &= -\phi_{14}, & \phi_{15} &= -\phi_{11}, \\ \phi_{20} &= -\phi_{23}, & \phi_{24} &= -\phi_{21}.\end{aligned}$$

Given the previous loop flux relationships, it is easily shown that R_{20} is equal to R_{24} , and R_{40} is equal to R_{43} . Taking advantage of these symmetry conditions allows description of the whole machine by modeling just one pole.

By inspection of Fig. 5, it is apparent that the flux within the bore linking a particular phase current, ϕ_{ph_b} , is simply

$$\Phi_{kph_b} = 2\phi_{1k} \quad \text{for } k = 1 \text{ to } 4 \quad (5)$$

Defining

$$\Phi_{ph_b} \equiv 2 [\phi_{11} \ \phi_{12} \ \phi_{13} \ \phi_{14}]^T$$

$$\Phi_{loop} \equiv 2 [\phi_{21} \ \phi_{22} \ \phi_{23} \ \phi_{31} \ \phi_{32} \ \phi_{41}]^T$$

$$\mathbf{R}^{(1)} \equiv [R_{11} + R_{24} + R_{21} + R_{31} \quad -R_{21} \quad 0 \quad R_{24}]$$

$$\mathbf{R}^{(2)} \equiv [-R_{21} \quad R_{12} + R_{21} + R_{22} + R_{32} \quad -R_{22} \quad 0]$$

$$\mathbf{R}^{(3)} \equiv [0 \quad -R_{22} \quad R_{13} + R_{22} + R_{23} + R_{33} \quad -R_{23}]$$

$$\mathbf{R}^{(4)} \equiv [R_{24} \quad 0 \quad -R_{23} \quad R_{14} + R_{23} + R_{24} + R_{34}]$$

$$\mathbf{R}_{11} \equiv [\mathbf{R}^{(1)} \ \mathbf{R}^{(2)} \ \mathbf{R}^{(3)} \ \mathbf{R}^{(4)}]^T$$

$$\mathbf{R}_{12} \equiv \mathbf{R}_{21}^T \equiv \begin{bmatrix} -R_{31} & 0 & 0 & 0 & 0 & 0 \\ 0 & -R_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & -R_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & -R_{34} & 0 & 0 \end{bmatrix} \quad (6)$$

and \mathbf{R}_{22} in a similar manner to \mathbf{R}_{11} allows loop equations describing the magnetic circuit to be expressed in matrix form as

$$\begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \end{bmatrix} \begin{bmatrix} \Phi_{ph_b} \\ \Phi_{loop} \end{bmatrix} = N \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \quad (7)$$

where \mathbf{I} is a vector comprised of the phase currents, $\mathbf{0}$ is a 6×1 vector of zeros, and N is the total number of turns per phase. The vector Φ_{loop} contains the loop flux linkages that do not encircle any slot currents. The vector Φ_{ph_b} contains the loop flux linkages that encircle the slot currents.

V. MULTIPLE COILS PER PHASE BELT

Most machines are wound with multiple coils per phase belt. The coils of a phase belt are connected in series. Series connection of the coils of a particular phase constrains the current in those coils to be the same. Given series connected coils that link different flux, a method must be found to divide the bore flux linking a phase belt between the coils of that phase belt.

Defining m equal to the number of phases, n equal to one less than the number of coils per phase belt, and l equal to the dimension of the vector Φ_{loop} simplifies the following development.

Equation 7 is restated here with a change of variables.

$$\begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \end{bmatrix} \begin{bmatrix} \Phi_{coil} \\ \Phi_{loop} \end{bmatrix} = N \begin{bmatrix} \mathbf{I}_{coil} \\ \mathbf{0} \end{bmatrix} \quad (8)$$

where \mathbf{I}_{coil} is a vector of the coil currents, Φ_{coil} is a vector of flux linking the coils, and $\mathbf{0}$ is a vector of zeros.

The scalar equations represented by the matrix Eq. 8 are ordered such that the first m elements of \mathbf{I}_{coil} are each from a different phase. The current vector \mathbf{I}_{coil} is partitioned into two vectors, \mathbf{I}_1 and \mathbf{I}_2 . The vector \mathbf{I}_1 contains the first m elements of the vector \mathbf{I}_{coil} . The vector \mathbf{I}_2 contains the elements $m+1$ through $m+n$ of the vector \mathbf{I}_{coil} . The flux linkage vector, Φ_{coil} , is partitioned into two vectors, Φ_1 and Φ_2 in a similar manner. Partitioning the reluctance matrices of Eq. 8 such that the new reluctance matrices correspond to the partitioning of Φ_{coil} and \mathbf{I}_{coil} yields

$$\begin{bmatrix} \mathbf{R}'_{11} & \mathbf{R}'_{12} & \mathbf{R}'_{13} \\ \mathbf{R}'_{21} & \mathbf{R}'_{22} & \mathbf{R}'_{23} \\ \mathbf{R}'_{31} & \mathbf{R}'_{32} & \mathbf{R}'_{33} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_{loop} \end{bmatrix} = N \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{0} \end{bmatrix} \quad (9)$$

The bore flux linkage vector is given by

$$\Phi_{ph_b} = \Phi_1 + \mathbf{W}\Phi_2 \quad (10)$$

where \mathbf{W} is the $m \times n$ winding matrix. The winding matrix is defined by two rules.

1. Row k of \mathbf{W} contains a 1 in column j if the j^{th} element of Φ_2 links current belonging to the same phase as the current being linked by the k^{th} element of Φ_1 .

2. Row k of W contains a 0 in column j if the j^{th} element of Φ_2 does not link current belonging to the same phase as the current being linked by the k^{th} element of Φ_1 .

Utilizing the constraint that the coils of the same phase have the same current yields the following equation.

$$0 = I_2 - W^T I_1 \quad (11)$$

Therefore, pre-multiplying Eq. 9 by

$$\begin{bmatrix} U_{m \times m} & 0_{m \times n} & 0_{m \times l} \\ -W_{n \times m}^T & U_{n \times n} & 0_{n \times l} \\ 0_{l \times m} & 0_{l \times n} & U_{l \times l} \end{bmatrix} \quad (12)$$

where $U_{r \times s}$ is an $r \times s$ identity matrix and $0_{r \times s}$ is an $r \times s$ matrix full of zeroes yields

$$\begin{bmatrix} R'_{11} & R'_{12} & R'_{13} \\ R'_{21} & R'_{22} & R'_{23} \\ R'_{31} & R'_{32} & R'_{33} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_{loop} \end{bmatrix} = N \begin{bmatrix} I_1 \\ 0 \\ 0 \end{bmatrix} \quad (13)$$

Note that pre-multiplying Eq. 9 by the matrix 12 results in a non-symmetric reluctance matrix.

Using linear algebra, Φ_{loop} can be expressed in terms of Φ_1 and Φ_2 , and Φ_2 can be expressed in terms of Φ_1 .

$$\Phi_{loop} = -R'_{33}{}^{-1} R'_{31} \Phi_1 - R'_{33}{}^{-1} R'_{32} \Phi_2 \quad (14)$$

$$\Phi_2 = -R_1^{-1} R_2 \Phi_1 \quad (15)$$

where

$$R_1 = R'_{22} - R'_{23} R'_{33}{}^{-1} R'_{32} \quad (16)$$

$$R_2 = R'_{21} - R'_{23} R'_{33}{}^{-1} R'_{31} \quad (17)$$

Assuming the resistance, r , of each phase is the same, the vector of terminal voltages is given by

$$V = rI + \frac{d\Phi_{phase}}{dt} \quad (18)$$

The end winding flux linkages are assumed to be entirely in air. Therefore, they are linear with respect to flux linkages and current. The end winding flux linkages are given by

$$\Phi_{ph.e} = L_{end} I \quad (19)$$

where L_{end} is the end winding leakage inductances.

The flux linking only the bore is given by

$$\Phi_{ph.b} = \Phi_{phase} - \Phi_{ph.e} \quad (20)$$

Substituting Eq. 17 into Eq. 10 yields

$$\Phi_{ph.b} = [U_{m \times m} - WR_1^{-1} R_2] \Phi_1 \quad (21)$$

Given $\Phi_{ph.b}$, Φ_1 can be obtained. Using Eq. 15, Φ_2 can be determined. Eq. 14 can be used to determine Φ_{loop} . An algorithm for a simulation based on the equations just developed is presented in Fig. 6.

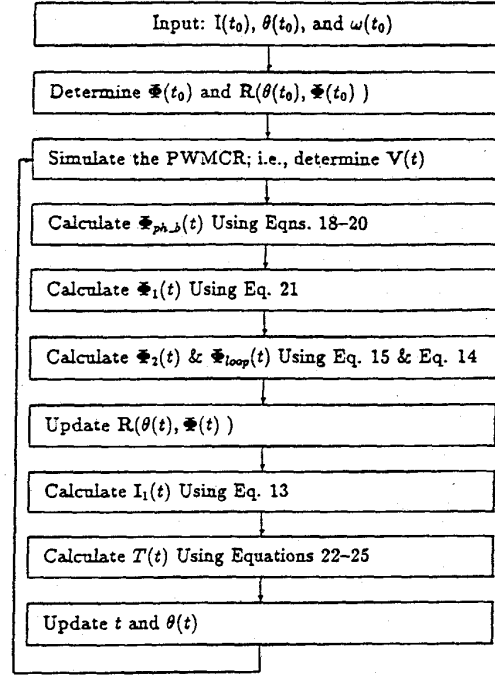


Figure 6: Algorithm for simulation of a field regulated reluctance machine based on the magnetic circuit model.

VI. MCM TORQUE CALCULATIONS

The expression for the electrical torque used in the magnetic circuit model can be developed neglecting the change in T over a mechanical angle of $\Delta\theta_m$,

$$T d\theta_m = dW_{em} \Rightarrow T = \frac{\Delta W_{em}}{\Delta\theta_m} \quad (22)$$

where T is the electrical torque, $d\theta_m$ is the differential mechanical angle, and dW_{em} is the differential energy converted from electrical to mechanical form.

The energy converted from electrical to mechanical form is given by

$$\Delta W_{em} = \Delta W_{el} - \Delta W_{fld} \quad (23)$$

where ΔW_{el} is the energy input from the electrical supply minus the copper losses, and ΔW_{fld} is the increase in the magnetic energy stored over $\Delta\theta$.

The energy input from the electrical supply minus copper losses is given by

$$\Delta W_{el} = \sum_{k=1}^{\#of\ phases} \Delta \Phi_k i_k \quad (24)$$

The stored magnetic energy is calculated using

$$\Delta W_{fld} = \frac{1}{2} \sum_{j=1}^{\#airprimitives} Vol_j \frac{B_j^2}{\mu_0} + \sum_{l=1}^{\#ironprimitives} Vol_l w_{fld,l} \quad (25)$$

where Vol is the volume of the primitive reluctance and w_{fld} is the magnetic energy density. The magnetic energy density is determined from look up tables.

VII. MCM INDUCTANCE CALCULATIONS

Eq. 13 can be rewritten as

$$\begin{bmatrix} R''_{11} & R''_{12} \\ R''_{21} & R''_{22} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = N \begin{bmatrix} I_1 \\ 0 \end{bmatrix} \quad (26)$$

Inverting the reluctance matrix and partitioning the resulting matrix yields

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} N \begin{bmatrix} I_1 \\ 0 \end{bmatrix} \quad (27)$$

Multiplying Eq. 27 by

$$[\mathbf{U}_{m \times m} \quad \mathbf{W}] \quad (28)$$

results in

$$\Phi_{ph,b} = \Phi_1 + \mathbf{W}\Phi_2 = N[\rho_{11} + \mathbf{W}\rho_{21}] I_1. \quad (29)$$

From $\lambda = N\phi = LI$, the bore inductance matrix is given by

$$\mathbf{L}_b = N^2 [\rho_{11} + \mathbf{W}\rho_{21}]. \quad (30)$$

VIII. SUMMARY

The development of a new magnetic circuit model for a field regulated reluctance machine based on loop equations is presented in this paper. It is shown that loop equations can accommodate current and flux constraints in a simpler manner than when using node equations. The loop equation approach to magnetic circuit modeling permits a solution to be obtained by analyzing the behavior of the machine over only one pole. The model is extended to include multiple coils per phase belt. Expressions are developed for the self and mutual inductances in terms of primitive reluctances, the number of turns per phase, and the winding matrix. Equations describing the model provide the basis for simulation of an FRRM. A simulation technique based on lumped magnetic circuit reluctances is developed. Part II of this paper presents inclusion of magnetic saturation in the MCM and comparison of MCM with experimental results.

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