

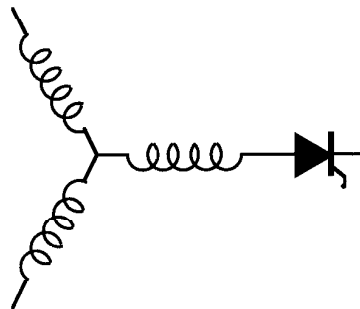
Research Report

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**An Approach to Modeling and Field-Oriented  
Control of a Three-Phase Induction Machine with  
Structural Unbalance**

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# AN APPROACH TO MODELING AND FIELD-ORIENTED CONTROL OF A THREE PHASE INDUCTION MACHINE WITH STRUCTURAL UNBALANCE

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**Abstract:** Structurally unbalanced operation of an induction machine typically occurs during emergency conditions. A unified modeling and control approach for a three phase induction machine drive with a particular structural unbalance (one phase open) is developed based directly on the resulting asymmetrical machine structure. With a new stator winding decoupling transformation developed in this paper, the machine (with one phase open) is made equivalent to a two phase machine with perpendicular  $d$ - $q$  stator windings having different numbers of turns. Based on this machine model, field-oriented control strategy is developed to guarantee the desired independent control of rotor flux and electromagnetic torque in spite of the unbalanced condition. The method employed may be extended to other type of structural unbalances.

## I. INTRODUCTION

The voltage source inverter fed three phase induction machine drives have found the largest application in the area of variable speed drives because of simplicity of its inverter configuration, economy and ruggedness of the motor structure, and high performance. However, as is common with most of the variable speed electrical machine drives, the reliability of this kind of system suffers mostly from the failure of semiconductor devices in the inverter. In most cases, the failure would result in the complete switch off of the drive system. However, in some applications where continuous operation is desirable or critical, such as in an electric vehicle or aircraft, a switch off of the entire drive system becomes unacceptable.

One common type of drive system faults is a failed short-circuited switch in the inverter as depicted in Fig. 1. In this case one of the motor phases is continuously connected to positive (or negative) side of the DC bus. As a consequence, there are only four switching modes that can be realized as shown in Fig. 2. Clearly, any switching in this case would result in considerable amount of DC current in the stator winding of the machine and thus may cause further damage to the rest of the inverter legs. Moreover, braking torque would be generated due to the flowing of the DC current in machine windings and thereby make the continuous driving of the load virtually impossible. Therefore, whenever a short-circuited switch fault occurs, it must be isolated before any measure can be taken to realize the "limp home mode" of operation.

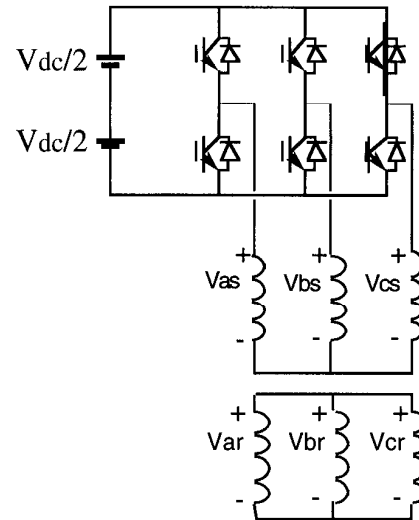


Fig. 1 Three Phase Induction Machine Drive System with One Switch Short-Circuited

Fig. 3 shows the topology that can solve the problem caused by a short-circuited switch. One fuse for each phase and a "triac" (inverse/parallel connected thyristors) connecting the stator winding neutral point to the DC bus center tap are required in this approach. The fault isolation device will be referred to as FISD in this paper. Whenever a short-circuited switch fault is detected, "off" gating signals are sent to all the switches in the inverter. In the mean time, "on" signal is sent to the FISD, a DC current will then find its way through the short-circuited switch, machine winding, and the FISD to blow out the fuse in the faulty phase and thus isolate the fault. The FISD will be kept at "on" state to provide another degree of freedom for the "limp home" operation. Since only one FISD is used in this topology, it will be referred to as single FISD topology latter in this report. The equivalent circuit topology of the drive system after the fault isolation is depicted in Fig. 4.

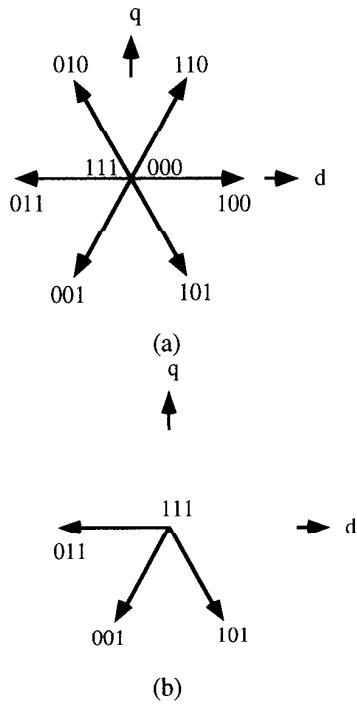


Fig. 2 Realizable Voltage Vectors (a) Before And (b) After The Short-Circuited Switch Fault

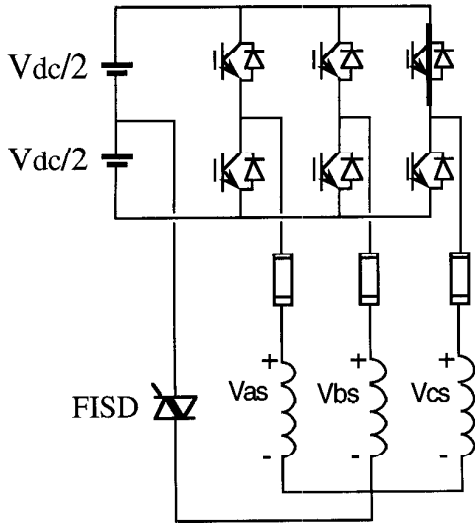


Fig. 3 Single FISD Topology for Isolating Short-Circuited Switch Fault

With one phase open-circuited, an induction machine can continue to be operated with an asymmetrical winding structure and unbalanced excitation. However, the loss of one phase will drastically change the dynamic behavior of the machine because the interactions between the lost phase and the rest of the machine windings due to mutual coupling no longer exist. As the transient response of an electric

machine is critical in a modern drive system, it is necessary to develop analytical tools which can handle the dynamics of electric machines under structurally unbalanced operating conditions. In this paper, a unified modeling and control approach for a three phase induction machine with one phase open is proposed. Instead of using the conventional symmetrical component method, the proposed technique is established on the basis of the asymmetrical winding structure directly, and thus provides a precise, insightful tool to the modeling and control of induction machine with structural unbalance.

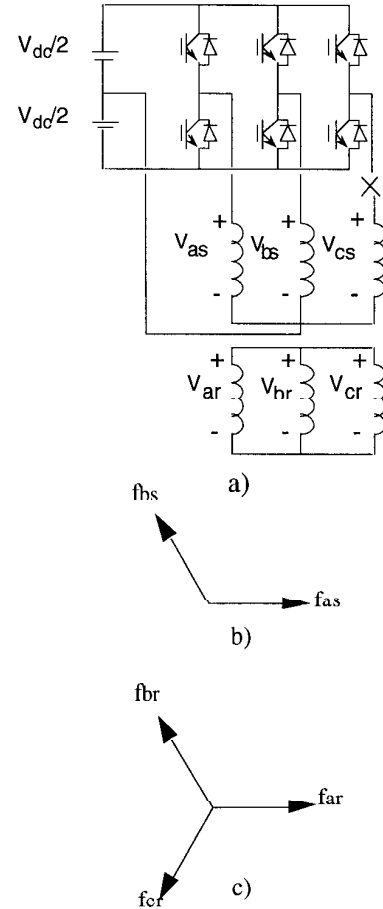


Fig. 4 a) Three Phase Induction Machine Drive System with One Phase Open, b) Stator winding axes, c) Rotor winding

## II. MACHINE MODELLING

With one phase open (assume phase c), the voltage equations of the machine written in real machine variables (phase variables) can be found as follows.

Stator voltage equation:

$$[V_s] = [R_s] \cdot [i_s] + p \cdot ([\lambda_s])$$

$$- [R_s] \cdot [i_s] + p \cdot ([L_{ss}] \cdot [i_s] + [L_{sr}] \cdot [i_r]) \quad (1)$$

Rotor voltage equation:

$$\begin{aligned} [V_r] &= [R_r] \cdot [i_r] + p \cdot ([\lambda_r]) \\ &= [R_r] \cdot [i_r] + p \cdot ([L_{rr}] \cdot [i_r] + [L_{rs}] \cdot [i_s]) \quad (2) \end{aligned}$$

The resistance and inductance matrices in (1) and (2) are given as follows according to the asymmetrical machine winding structure.

$[R_s]$ ,  $[R_r]$  - stator, rotor resistance matrices:

$$[R_s] = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix}, [R_r] = \begin{bmatrix} r_r & 0 & 0 \\ 0 & r_r & 0 \\ 0 & 0 & r_r \end{bmatrix}$$

$[L_{ss}]$  - stator self inductance matrix:

$$[L_{ss}] = L_{ls} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + L_{ms} \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

where

$L_{ls}$ ,  $L_{ms}$  - stator leakage and magnetizing inductance

$[L_{rr}]$  - rotor self inductance matrix:

$$[L_{rr}] = L_{lr} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + L_{ms} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$$

where

$L_{lr}$  - rotor leakage inductance;

$[L_{sr}]$  - stator-rotor mutual inductance matrix:

$$[L_{sr}] = L_{ms}$$

$$\begin{bmatrix} \cos(\theta_r) & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r) & \cos(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

where

$\theta_r$  - rotor angular position.

$[L_{rs}]$  - rotor-stator mutual inductance matrix:

$$[L_{rs}] = L_{ms} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

To represent the dynamics of the machine by a  $d$ - $q$  machine model, a  $d$ - $q$  transformation is generally required. However, for the machine investigated now, the well known

$d$ - $q$ -0 transformation will not only apply to the asymmetrical stator winding structure. The  $d$ - $q$  transformation for stator should be redefined.

It is not difficult to show that the eigenvalues of the stator magnetizing inductance matrix are:

$$\sigma_1 = \frac{3}{2} L_{ms}, \quad \sigma_2 = \frac{1}{2} L_{ms},$$

indicating that a time invariant transformation of variables remains possible.

The eigenvectors corresponding to the two eigenvalues are:

Using the two eigenvectors as the  $d$  and  $q$  axes the following stator variable transformation matrix results:

$$[T_s] = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{matrix} d^T \\ q^T \end{matrix} \quad (3)$$

As far as the rotor windings are concerned, the  $d$ - $q$ -0 transformation for a balanced three phase system is still applicable since the rotor maintains a symmetrical structure. The transformation is:

$$[T_r] = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix},$$

$$[T_r]^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (4)$$

Applying the transformations  $[T_s]$  and  $[T_r]$  to the voltage equations (1.1) and (1.2), the following  $d$ - $q$  plane voltage equations can be obtained. The convention used here for variable expression is:

$x_\alpha^\beta$

$x$  - variable (voltage, current, flux, etc.);

$\alpha$  - axis (ds, qs, dr, qr, etc.);

$\beta$  - frame (stator, rotor, synchronous, etc.).

Stator voltage equation:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \frac{d}{dt} \left\{ \begin{bmatrix} L_{ls} + \frac{3}{2} L_{ms} & 0 \\ 0 & L_{ls} + \frac{1}{2} L_{ms} \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \right. \\ \left. + L_{ms} \begin{bmatrix} \frac{3}{2} \cos(\theta_r + \frac{\pi}{6}) & -\frac{3}{2} \sin(\theta_r + \frac{\pi}{6}) \\ \frac{\sqrt{3}}{2} \sin(\theta_r + \frac{\pi}{6}) & \frac{\sqrt{3}}{2} \cos(\theta_r + \frac{\pi}{6}) \end{bmatrix} \cdot \begin{bmatrix} i_{dr}^r \\ i_{qr}^r \end{bmatrix} \right\} \quad (5)$$

Rotor voltage equation:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix} \cdot \begin{bmatrix} i_{dr}^r \\ i_{qr}^r \end{bmatrix}$$

$$\begin{aligned}
& + \frac{d}{dt} \begin{bmatrix} L_{lr} + \frac{3}{2}L_{ms} & 0 \\ 0 & L_{lr} + \frac{3}{2}L_{ms} \end{bmatrix} \begin{bmatrix} i_{dr}^r \\ i_{qr}^r \end{bmatrix} \\
& + L_{ms} \begin{bmatrix} \frac{1}{2} \cos(\theta_r + \frac{\pi}{6}) & \frac{\sqrt{3}}{2} \sin(\theta_r + \frac{\pi}{6}) \\ -\frac{1}{2} \sin(\theta_r + \frac{\pi}{6}) & \frac{\sqrt{3}}{2} \cos(\theta_r + \frac{\pi}{6}) \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (6)
\end{aligned}$$

To eliminate the sin and cos terms in above equations the following rotor frame to stationary frame transformation is appropriate:

$$[T_r^s] = \begin{bmatrix} \cos(\theta_r + \frac{\pi}{6}) & -\sin(\theta_r + \frac{\pi}{6}) \\ \sin(\theta_r + \frac{\pi}{6}) & \cos(\theta_r + \frac{\pi}{6}) \end{bmatrix} \quad (7)$$

Applying the rotation transformation to rotor variables in (1.5) and (1.6) the following stator and rotor combined voltage equation results:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + L_{ds}p & 0 & M_d p & 0 \\ 0 & r_s + L_{qs}p & 0 & M_q p \\ M_d p & \omega_r M_q & r_r + L_r p & \omega_r L_r \\ -\omega_r M_d & M_q p & -\omega_r L_r & r_r + L_r p \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (8)$$

where:

$$p = \frac{d}{dt}$$

$$L_{ds} = L_{ls} + \frac{3}{2}L_{ms}, \quad L_{qs} = L_{ls} + \frac{1}{2}L_{ms}, \quad L_r = L_{lr} + \frac{3}{2}L_{ms}$$

$$M_d = \frac{3}{2}L_{ms}, \quad M_q = \frac{\sqrt{3}}{2}L_{ms}$$

The electromagnetic torque of the machine is expressed as:

$$T_e = \frac{1}{2} [i]^T \cdot \left( \frac{\partial}{\partial \theta_m} [L] \right) \cdot [i] \quad (9)$$

where  $\theta_m$  is the rotor mechanical angular position,  $\theta_m = \frac{2\theta_r}{P}$ , and P is the number of poles.

In (9) the current vector and the inductance matrix can be determined as:

$$[i] = \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (10)$$

$$[L] = \begin{bmatrix} L_{ds,ds} & L_{ds,qs} & L_{ds,dr} & L_{ds,qr} \\ L_{qs,ds} & L_{qs,qs} & L_{qs,dr} & L_{qs,qr} \\ L_{dr,ds} & L_{dr,qs} & L_{dr,dr} & L_{dr,qr} \\ L_{qr,ds} & L_{qr,qs} & L_{qr,dr} & L_{qr,qr} \end{bmatrix} \quad 4$$

$$= \begin{bmatrix} L_{ds} & 0 & M_d \cos(\theta_r + \frac{\pi}{6}) & -M_d \sin(\theta_r + \frac{\pi}{6}) \\ 0 & L_{qs} & M_q \sin(\theta_r + \frac{\pi}{6}) & M_q \cos(\theta_r + \frac{\pi}{6}) \\ M_d \cos(\theta_r + \frac{\pi}{6}) & M_q \sin(\theta_r + \frac{\pi}{6}) & L_r & 0 \\ -M_d \sin(\theta_r + \frac{\pi}{6}) & M_q \cos(\theta_r + \frac{\pi}{6}) & 0 & L_r \end{bmatrix} \quad (11)$$

The expression of electromagnetic torque can be obtained by substituting (10) and (11) into (9), which gives:

$$T = \frac{P}{2} (M_q i_{qs}^s i_{dr}^s - M_d i_{ds}^s i_{qr}^s) \quad (12)$$

It is observed from the analysis above that after one phase of a three phase induction machine is open-circuited, the machine model is equivalent to a two phase asymmetrical stator winding machine as shown in Fig. 5.

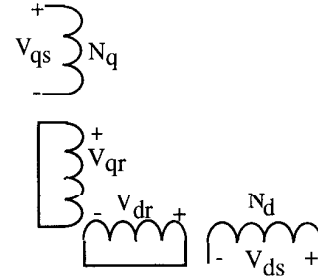


Fig. 5 Asymmetrical Stator Winding Two Phase Machine

When flux linkages are used as state variables, the machine model can be expressed as:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{ds}^s \\ \lambda_{qs}^s \end{bmatrix} \quad (13)$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix} \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} + \begin{bmatrix} 0 & -\omega_r \\ \omega_r & 0 \end{bmatrix} \begin{bmatrix} \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} \quad (14)$$

$$T_e = \frac{P}{2} \frac{1}{L_r} (M_q i_{qs}^s \lambda_{dr}^s - M_d i_{ds}^s \lambda_{qr}^s) \quad (15)$$

where:

$$\begin{bmatrix} \lambda_{ds}^s \\ \lambda_{qs}^s \end{bmatrix} = \begin{bmatrix} L_{ds} & 0 \\ 0 & L_{qs} \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \begin{bmatrix} M_d & 0 \\ 0 & M_q \end{bmatrix} \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} = \begin{bmatrix} M_d & 0 \\ 0 & M_q \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \begin{bmatrix} L_r & 0 \\ 0 & L_r \end{bmatrix} \cdot \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (17)$$

### III. FIELD ORIENTED CONTROL

Typical field-oriented control of an induction machine is accomplished by locking a synchronous reference frame  $d^e$ - $q^e$  to the rotor flux vector. The rotation transformation from the stationary reference frame to the synchronous reference frame for an induction machine operating under balanced condition is:

$$[T(\theta_e)] = \begin{bmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{bmatrix} \quad (18)$$

For the unbalanced operation situation investigated now, this transformation is still applicable to the rotor variables. However, to overcome the effect of an asymmetrical stator winding structure and thereby to obtain a non-pulsating torque, an unbalanced stator excitation must be provided to the machine. As a consequence, the transformation of stator variables using the balanced transformation (18) would result in AC components in the synchronous frame, which is certainly not desirable for the purpose of field-oriented control. In this regard, it is necessary to redefine the rotation transformation for stator variables.

Consider the stator MMF produced by the asymmetrical stator windings in Fig. 5. Since the rotor is maintained as a balanced structure, it can be inferred that for the machine to produce non-pulsating electromagnetic torque, the stator MMF must remain balanced as well. Therefore, the rotation transformation (18) continues to be applicable to the stator MMF vector, that is:

$$\begin{bmatrix} f_{ds}^e \\ f_{qs}^e \end{bmatrix} = \begin{bmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{bmatrix} \cdot \begin{bmatrix} f_{ds}^s \\ f_{qs}^s \end{bmatrix} \quad (19)$$

where  $f$  denotes the MMF.

As the numbers of turns of the stator windings in the equivalent machine model in Fig. 5 are  $N_d$  and  $N_q$  respectively, the stator MMF vector can be expressed as:

$$\begin{bmatrix} f_{ds}^s \\ f_{qs}^s \end{bmatrix} = \begin{bmatrix} N_d & 0 \\ 0 & N_q \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (20)$$

Substituting (20) into (19) we have:

$$\begin{bmatrix} f_{ds}^e \\ f_{qs}^e \end{bmatrix} = \begin{bmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{bmatrix} \cdot \begin{bmatrix} N_d & 0 \\ 0 & N_q \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix}$$

$$= \begin{bmatrix} N_d \cos(\theta_e) & N_q \sin(\theta_e) \\ -N_d \sin(\theta_e) & N_q \cos(\theta_e) \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad 5$$

$$= \sqrt{N_d N_q} \begin{bmatrix} \sqrt{\frac{N_d}{N_q}} \cos(\theta_e) & \sqrt{\frac{N_q}{N_d}} \sin(\theta_e) \\ -\sqrt{\frac{N_d}{N_q}} \sin(\theta_e) & \sqrt{\frac{N_q}{N_d}} \cos(\theta_e) \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (21)$$

Assuming the pole arcs under the two stator windings are equal, then:

$$\frac{N_d}{N_q} = \frac{M_d}{M_q} \quad (22)$$

and

$$\begin{bmatrix} f_{ds}^e \\ f_{qs}^e \end{bmatrix} = \sqrt{N_d N_q} \begin{bmatrix} \sqrt{\frac{M_d}{M_q}} \cos(\theta_e) & \sqrt{\frac{M_q}{M_d}} \sin(\theta_e) \\ -\sqrt{\frac{M_d}{M_q}} \sin(\theta_e) & \sqrt{\frac{M_q}{M_d}} \cos(\theta_e) \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (23)$$

or

$$\begin{bmatrix} \frac{f_{ds}^e}{\sqrt{N_d N_q}} \\ \frac{f_{qs}^e}{\sqrt{N_d N_q}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{M_d}{M_q}} \cos(\theta_e) & \sqrt{\frac{M_q}{M_d}} \sin(\theta_e) \\ -\sqrt{\frac{M_d}{M_q}} \sin(\theta_e) & \sqrt{\frac{M_q}{M_d}} \cos(\theta_e) \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix}$$

It is useful to define

$$\begin{bmatrix} \frac{f_{ds}^e}{\sqrt{N_d N_q}} \\ \frac{f_{qs}^e}{\sqrt{N_d N_q}} \end{bmatrix} = \begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} \quad (25)$$

We finally have, from (24):

$$\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{M_d}{M_q}} \cos(\theta_e) & \sqrt{\frac{M_q}{M_d}} \sin(\theta_e) \\ -\sqrt{\frac{M_d}{M_q}} \sin(\theta_e) & \sqrt{\frac{M_q}{M_d}} \cos(\theta_e) \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (26)$$

Eq. (26) is the  $d$ - $q$  plane stator current rotation transformation we have been looking for. As is suggested by (25), with this transformation, the stator windings in the synchronous reference frame will be made equivalent to a pair of balanced  $d$ - $q$  windings as depicted in Fig. 6.

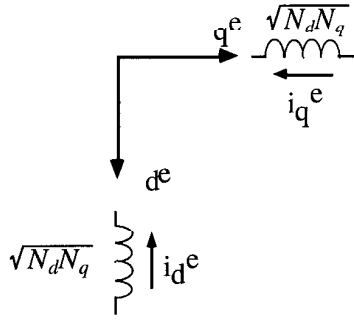


Fig. 6 Synchronous Reference Frame Equivalent Stator Windings

Assume that the machine is fed by a current regulated source. Using the rotation transformations (18) and (26) and aligning  $d^e$ -axis to rotor flux vector, the following set of equations governing the dynamic behavior of the machine under field-oriented control can be derived from Eqs. (13) through (17).

$$0 = r_r i_{dr}^e + \frac{d}{dt} \lambda_{dr}^e \quad (27)$$

$$0 = r_r i_{qr}^e + (\omega_e - \omega_r) \lambda_{dr}^e \quad (28)$$

$$\lambda_{dr}^e = \sqrt{M_d M_q} i_{ds}^e + L_r i_{dr}^e \quad (29)$$

$$\lambda_{qr}^e = \sqrt{M_d M_q} i_{qs}^e + L_r i_{qr}^e = 0 \quad (30)$$

$$T_e = \frac{P}{2} \frac{\sqrt{M_d M_q}}{L_r} i_{qs}^e \lambda_{dr}^e \quad (31)$$

The block diagram of the machine under rotor flux oriented control is sketched in Fig. 7.

It is observed that Eqs. (27) through (31) are identical in form to the rotor flux-oriented dynamic equations of a balanced three phase induction machine. Therefore, techniques of field oriented-control developed for balanced operation can be used for the unbalanced operation case discussed here as long as the stator current rotation transformation is replaced by the unbalanced transformation (26) and the air gap magnetizing inductance modified to  $\sqrt{M_d M_q}$ .

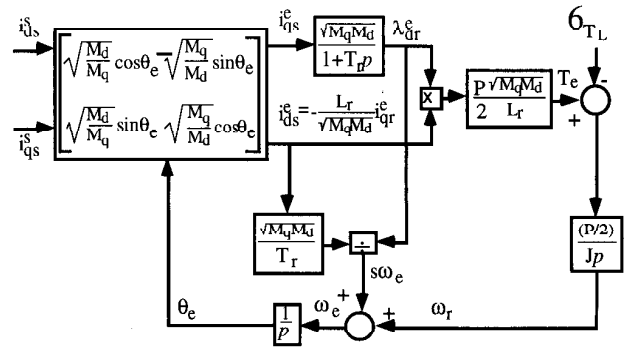


Fig. 7 Block Diagram of Rotor Flux Oriented Machine

There are two basic means to achieve rotor flux field orientation: indirect and direct. Direct field-orientation, which relying directly upon the knowledge of the rotor flux linkage vector and thus being physically insightful, will be adopted for the purpose of testing the theoretical results.

The implementation of a direct field-oriented controller requires that the position and the amplitude of the rotor flux linkage vector to be known. Usually, this is accomplished by employment of a rotor flux estimator. The implementation of the rotor flux linkage estimator is illustrated as follows.

The  $d$ - $q$  plane rotor voltage equation and flux in rotor reference frame can be expressed as:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix} \cdot \begin{bmatrix} i_{dr}^r \\ i_{qr}^r \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{dr}^r \\ \lambda_{qr}^r \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} \lambda_{dr}^r \\ \lambda_{qr}^r \end{bmatrix} = \begin{bmatrix} \sqrt{M_d M_q} & 0 \\ 0 & \sqrt{M_d M_q} \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^r \\ i_{qs}^r \end{bmatrix} + \begin{bmatrix} L_r & 0 \\ 0 & L_r \end{bmatrix} \cdot \begin{bmatrix} i_{dr}^r \\ i_{qr}^r \end{bmatrix} \quad (33)$$

Upon solving the rotor current vector from (33) and inserting the result into (32), a current model rotor flux estimator equation can be obtained:

$$\frac{d}{dt} \begin{bmatrix} \lambda_{dr}^r \\ \lambda_{qr}^r \end{bmatrix} + \frac{r_r}{L_r} \begin{bmatrix} \lambda_{dr}^r \\ \lambda_{qr}^r \end{bmatrix} = \begin{bmatrix} \frac{r_r}{L_r} \sqrt{M_d M_q} & 0 \\ 0 & \frac{r_r}{L_r} \sqrt{M_d M_q} \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^r \\ i_{qs}^r \end{bmatrix} \quad (34)$$

The advantage of this implementation is that it is linear and has real eigenvalues. Moreover, it requires relative rotor position rather than rotor velocity as is required by an implementation in the stationary reference frame. The block diagrams of the direct field-oriented controller and the flux estimator are shown in Fig. 8 and Fig. 9.

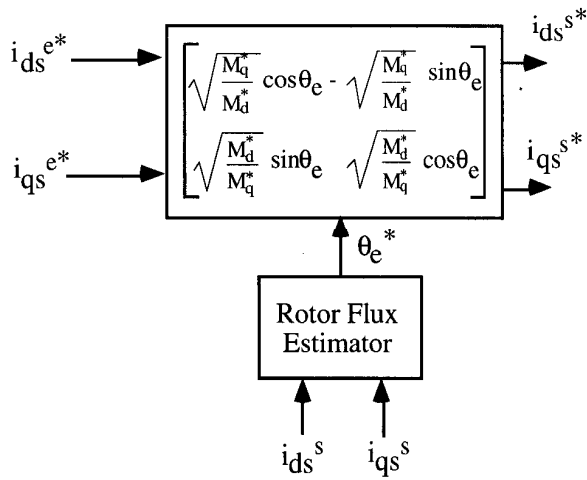


Fig. 8 Block Diagram of Direct Field-Oriented Control

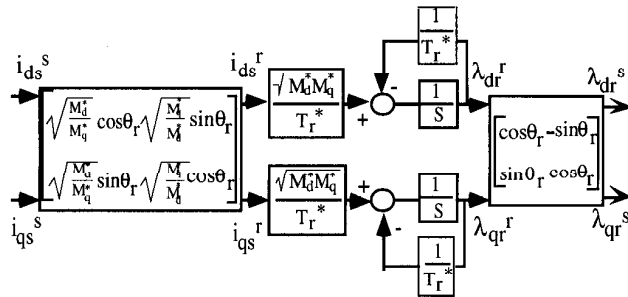


Fig. 1.9 Rotor Flux Estimator

#### IV. SIMULATION RESULTS AND SUMMARY

Fig. 10 illustrates the simulation results of direct field-oriented control using the technique proposed in this paper. With a constant  $i_{ds}^e$  and a step change in  $i_{qs}^e$ , the machine torque response is proportional to  $i_{qs}^e$  while the amplitude of the rotor flux remains unaffected as is normally the case for field oriented control. Smooth, non-pulsating torque can also be observed even though the machine is powered from an asymmetrical two phase supply. From the results shown, it is assured that field-oriented control of the machine with an asymmetrical stator winding structure has been attained.

#### V. CONCLUSION

In summary, a unified modeling and control approach for a three phase induction machine drive with one phase open has been proposed in this paper. It includes the development of a stator winding decoupling transformation to deal with the asymmetrical stator winding structure. Using this transformation, the original three phase machine (with one phase open) can be made equivalent to a two phase machine with perpendicular  $d$ - $q$  stator windings of different numbers of turns. Based on this machine, field-oriented control strategy is developed to guarantee desired

independent control of rotor flux and electromagnetic torque in spite of the unbalanced condition.

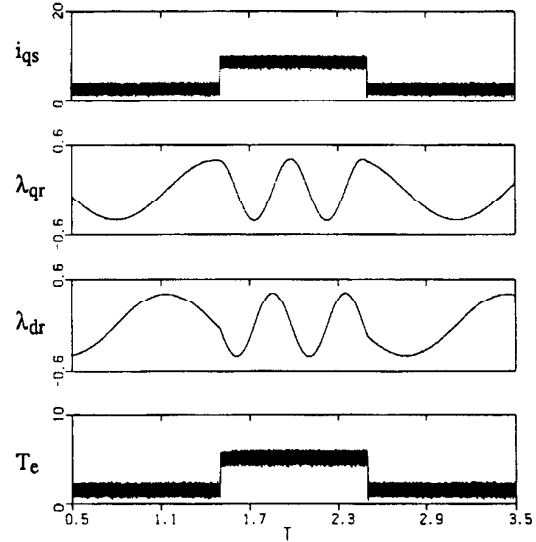


Fig. 10 Simulation Result: Demonstration of Field-Oriented Control. Trace 1: q-axis current (synchronous frame); Trace 2&3: q&d axes rotor flux (rotor frame); Trace 4: electromagnetic torque.

#### VI. REFERENCES

- [1] T-H Liu, J-R Fu and T. A. Lipo, "A Strategy for Improving Reliability of Field-Oriented Controlled Induction Motor Drives." IEEE Trans. on Ind. Appl., Vol. 29, No. 5, Sept./Oct. 1993, pp. 910-918.
- [2] J-R Fu and T. A. Lipo, "A Strategy to Isolate The Switching Device Fault of a Current Regulated Motor Drive." IEEE Ind. Appl. Soc. Ann. Meeting, Toronto, Canada, Oct. 2-8, 1993. pp. 1015-1020.
- [3] J.-R. Fu and T. A. Lipo, "Disturbance Free Operation of A Multiphase Current Regulated Motor Drive with an Opened Phase", IEEE Trans. on Ind. Appl., Vol. 30, No. 5, Sept./Oct. 1994, pp. 1267-1274.
- [4] T. A. Lipo and D. W. Novotny, Dynamics and Control of AC Drives, Course Notes for ECE711, University of Wisconsin-Madison, 1990.
- [5] T. M. Jahns, "Improved Reliability in Solid-State AC Drives by Means of Multiple Independent Phase-Drive Units," IEEE Trans. on Ind. Appl., Vol. IA-16, No. 3, May/June 1980, pp. 321-331.
- [6] S.G. Bosga, J.J.A. v.d. Burgt, J.L. Duarte, A.J.A. Vandenput, "Unsymmetrical Operation of a Voltage-Controlled Field-Oriented Electrical Drive System." AEGEAN Conference on Electrical Machines and Power Electronics, Kusadasi, Turkey, June 5-7, 1995, pp. 549-554.