Research Report

96-52

Modeling and Control of Multi-Phase Induction Machine with Structural Unbalance, Part 1: Machine Modeling and Multi-Dimensional Current Regulator

Y. Zhao*, T.A. Lipo

*GE Corporate R&D Center
1 River Road
Schenectady, NY 12301

Wisconsin Power Electronics Research Center
University of Wisconsin-Madison
Madison WI 53706-1691

Wisconsin
Electric
Machines &
Power
Electronics
Consortium

University of Wisconsin-Madison
College of Engineering
Wisconsin Power Electronics Research Center
2559D Engineering Hall
1415 Engineering Drive
Madison WI 53706-1691

IEEE Transactions on Energy Conversion, Vol. 11, No. 3, September 1996

MODELING AND CONTROL OF A MULTI-PHASE INDUCTION MACHINE WITH STRUCTURAL UNBALANCE

Part I. Machine Modeling and Multi-Dimensional Current Regulation

Yifan Zhao
GE Corporate R&D Center
1 River Road
Schenectady, NY 12301

Thomas A. Lipo
University of Wisconsin-Madison
1415 Johnson Drive
Madison, WI 53706

***Abstract*: The multiphase winding structure provides an induction machine with capabilities of starting and running even with one or more of its stator phases open-circuited. However, when operated with such a structurally unbalanced condition, the dynamic properties of the machine will change drastically from its balanced operation condition. For example, field-oriented control strategies developed for a balanced winding structure will no longer function properly and could lead to catastrophic consequences. In this paper, a unified approach to the modeling and field-oriented control of dual three phase induction machine with one phase open is presented. Using the concept of vector space decomposition, the proposed technique is established on the basis of the asymmetrical winding structure directly, and thus provides a precise, physically insightful tool to the modeling and control of induction machines with structural unbalance.**

***Key words*: multi-phase induction machine model, vector space decomposition, multi-dimensional current regulation.**

I. INTRODUCTION

The voltage source inverter fed induction machine drive systems have many advantages such as a rugged and low cost motor structure, capability of high waveform fidelity with PWM operation, reasonably high performance, etc. However, their applications are still limited to the lower end of the high power range due to the limitations on the ratings of the gate-turn-off type semiconductor power devices. To achieve high power ratings in such systems, multi-level inverters have been developed in the past decade as a promising approach. Another strong contender in achieving high power is the multi-phase inverter fed multi-phase induction machine drive system. In addition to enhancing power rating, a multi-phase system also has the merit of high reliability at the system level [2-4]. In particular, with loss of one or more of stator winding excitation sets, a multi-phase induction machine can continue to be operated with an asymmetrical winding structure and unbalanced excitation.

The most commonly used analytical tool for the analysis of unbalanced operation of electric machines has been the well known symmetrical component method [3-6]. In this method, a balanced structure is assumed even after the machine loses one or more of its phases. Although it has been used successfully in the steady state analysis on sinusoidal excitation, however, as far as the dynamics of the machine is concerned, the method loses its utility due to the fact that the interactions between the lost phases and the remainder of the machine windings no longer exist and thus drastically alter the dynamic behavior of the machine.

As the dynamic behavior of an electric machine is critical in a modern drive system, it is necessary to develop analytical tools which can handle the dynamics of electric machines under structurally unbalanced operation conditions. In this paper, a unified modeling and control approach for a dual three phase induction machine with one stator winding open is developed based upon the concept of vector space decomposition proposed in [1]. Instead of using the conventional symmetrical component method, the proposed technique is established on the basis of the asymmetrical winding structure directly, and thus provides a precise, physically insightful tool to the modeling and control of induction machine with various types of structural unbalance.

In a multi-phase induction machine drive, since more than two independent stator currents can flow in the general case, two dimensional (d-q) current regulation which has been used very extensively in normal three phase induction machine drives has been proved to be insufficient [1]. This paper will present the concept of multi-dimensional current regulation which is dedicated to the current regulation of induction machine with arbitrary phase numbers.

II. DECOMPOSITION TRANSFORMATION MATRICES

A. Stator Decomposition Transformation Matrix

Assume that a failure occurs in phase 6 of a drive system and has caused the cut out of that phase as shown in Fig. 1 a). The stator and rotor winding axes are shown in Fig 1 b) and c) respectively. Although the machine has five asymmetrical phases, as far as its electromechanical energy conversion property is concerned, it is still equivalent to a two dimensional d-q winding machine because of its cylindrical

96 WM 152-9 EC A paper recommended and approved by by the IEEE Electric Machinery Committee of the IEEE Power Engineering Society for presentation at the 1996 IEEE/PES Winter Meeting, January 21-25, 1996, Baltimore, MD. Manuscript submitted August 1, 1995; made available for printing November 13, 1995.

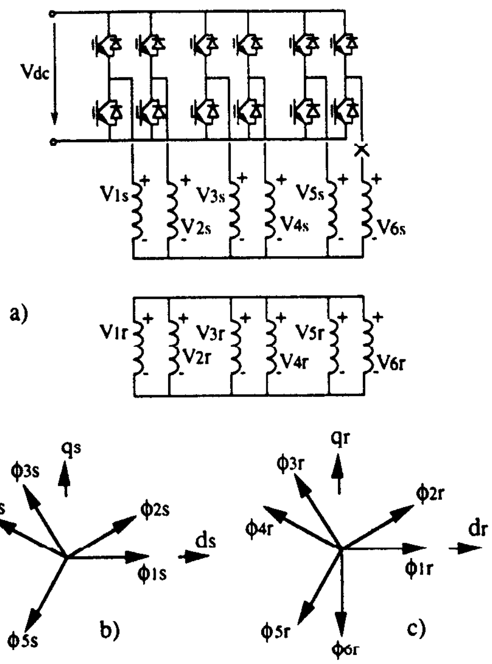

Fig. 1 a) Dual Three Phase Induction Machine Drive With One Phase Open
b) Stator Winding Flux Axes;
c) Rotor Winding Flux Axes.

structure. On the other hand, since five currents can flow independently in the general case, the electrical property of the machine is still characterized as a five dimensional system. This fact tells us that three extra dimensions which are non-electromechanical energy conversion related exist in the machine. In other words, the five dimensional space spanned by vectors of real machine variables can be expressed as the direct sum of two orthogonal subspaces with a two dimensional subspace representing the energy conversion property of the machine and the other, which is three dimensional, for the non-electromechanical energy conversion portion. Since the two different aspects of the machine property reside simultaneously in machine variables, it will be desirable to find a decomposition transformation to decouple them and thereby simplify the modeling and control of this asymmetrical five phase induction machine.

The subspace corresponding to energy conversion can be determined by defining two axes, namely d_s-axis and q_s-axis, in the air gap flux plane of the machine. Referring to Fig. 1. b), the d_s-axis flux and q_s-axis flux can be written as:

$$\phi_{ds} = \phi_{1s} + \phi_{2s}\cos(30^o) + \phi_{3s}\cos(120^o) + \phi_{4s}\cos(150^o) + \phi_{5s}\cos(240^o)$$

or

$$\phi_{ds} = \begin{bmatrix} 1 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \phi_{1s} \\ \phi_{2s} \\ \phi_{3s} \\ \phi_{4s} \\ \phi_{5s} \end{bmatrix} \qquad (1)$$

and

$$\phi_{qs} = \phi_{2s}\sin(30^o) + \phi_{3s}\sin(120^o) + \phi_{4s}\sin(150^o) + \phi_{5s}\sin(240^o)$$

or:

$$\phi_{qs} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} \phi_{1s} \\ \phi_{2s} \\ \phi_{3s} \\ \phi_{4s} \\ \phi_{5s} \end{bmatrix} \qquad (2)$$

Eq. (1) states that the d_s-axis flux is the projection of the stator flux vector in a five dimensional space on another vector in that space. Therefore, the vector

$$\boldsymbol{d}: \quad \begin{bmatrix} 1 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}^T$$

represents the orientation of the d_s-axis in the five dimensional space.

Similarly, according to (2), the vector

$$\boldsymbol{q}: \quad \begin{bmatrix} 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}^T$$

represents the orientation of the q_s-axis. The subspace spanned by vectors $\boldsymbol{d}$ and $\boldsymbol{q}$ represents the energy conversion property of the machine.

The vectors which span the three dimensional non-electromechanical energy conversion subspace can be determined mathematically. Defining the three vectors as z_1, z_2, and z_3, the following relations exist because of the orthogonal relations:

$$d^T \cdot q = d^T \cdot z_1 = d^T \cdot z_2 = d^T \cdot z_3 = 0 \qquad (3)$$

$$q^T \cdot z_1 = q^T \cdot z_2 = q^T \cdot z_3 = 0 \qquad (4)$$

$$z_1^T \cdot z_2 = z_1^T \cdot z_3 = 0 \qquad (5)$$

$$z_2^T \cdot z_3 = 0 \qquad (6)$$

The vectors z_1, z_2, and z_3 can be solved from the above equations. Using these five vectors to form the new basis for the five dimensional space, the following normalized decomposition transformation results:

$$[T_s] = \begin{bmatrix} 0.5774 & 0.5000 & -0.2887 & -0.5000 & -0.2887 \\ 0.0000 & 0.3536 & 0.6124 & 0.3536 & -0.6124 \\ -0.4177 & 0.5706 & -0.5706 & 0.4177 & 0.0000 \\ -0.5196 & 0.4177 & 0.3804 & -0.5706 & 0.2921 \\ 0.4714 & 0.3536 & 0.2673 & 0.3536 & 0.6755 \end{bmatrix} \begin{matrix} d^T \\ q^T \\ z_1^T \\ z_2^T \\ z_3^T \end{matrix} \qquad (7)$$

In the following discussions the space spanned by the five vectors will be referred to as the space d-q-z_1-z_2-z_3. The transformation (7) transforms the coordinates of a vector in the original vector space to the new vector space and has the following properties:

i) Machine variable components which produce air gap flux or induced by air gap flux will be transformed to the d-q subspace. The d-q subspace, commonly referred to as the d-q plane, is electromechanical energy conversion related.

ii) Those components of machine variables which will not produce air gap penetrating flux will be mapped to the z1-z2-z3 subspace by (7). In this regard, these components can be classified as a new type of zero sequence component, and the z1-z2-z3 subspace will be called zero sequence subspace, or a zero sequence plane whenever only two out of the three axes are concerned.

B. Rotor Decomposition Transformation Matrix

The decomposition matrix for rotor variables of the machine remains the same as the transformation for balanced operation [1] because the rotor still maintains a balanced winding structure.

$$[T_r]=\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & -\frac{\sqrt{3}}{2} & -1 \\ 1 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{8}$$

III. MACHINE MODEL

A. Machine Model in Real Machine Variables

The following assumptions have been made in deriving the machine model:

1) machine windings are sinusoidally distributed;
2) flux path is linear;
3) mutual leakage inductances due to different phase windings in the same slots are neglected.

Under these assumptions the voltage equations using the real machine variables can be written as:

Stator voltage equation:

$$\begin{aligned}[V_s] &= [R_s]\cdot[i_s]+p\cdot([\lambda_s]) \\ &= [R_s]\cdot[i_s]+p\cdot([L_{ss}]\cdot[i_s]+[L_{sr}]\cdot[i_r])\end{aligned} \tag{9}$$

Rotor voltage equation:

$$\begin{aligned}[V_r] &= [R_r]\cdot[i_r]+p\cdot([\lambda_r]) \\ &= [R_r]\cdot[i_r]+p\cdot([L_{rr}]\cdot[i_r]+[L_{rs}]\cdot[i_s])\end{aligned} \tag{10}$$

where, in these equations, the voltage and current vectors are defined as:

$$[v_s]=\begin{bmatrix} v_{1s} \\ v_{2s} \\ v_{3s} \\ v_{4s} \\ v_{5s} \end{bmatrix},\ [i_s]=\begin{bmatrix} i_{1s} \\ i_{2s} \\ i_{3s} \\ i_{4s} \\ i_{5s} \end{bmatrix},\ [v_r]=\begin{bmatrix} v_{1r} \\ v_{2r} \\ v_{3r} \\ v_{4r} \\ v_{5r} \\ v_{6r} \end{bmatrix},\ [i_r]=\begin{bmatrix} i_{1r} \\ i_{2r} \\ i_{3r} \\ i_{4r} \\ i_{5r} \\ i_{6r} \end{bmatrix}.$$

The resistance and inductance matrices are defined as follows according to the machine structure:

$[R_s]$, $[R_r]$ - stator, rotor resistance matrices:

$$[R_s]_{5\times5}=r_s\cdot[I],\quad [R_r]_{6\times6}=r_r\cdot[I]$$

where $[I]$ is identity matrix.

$[L_{ss}]$ - stator inductance matrix:

$$[L_{ss}]_{5\times5}=L_{ls}\cdot[I]+L_{ms}\begin{bmatrix} 1 & \sqrt{3}/2 & -1/2 & -\sqrt{3}/2 & -1/2 \\ \sqrt{3}/2 & 1 & 0 & -1/2 & -\sqrt{3}/2 \\ -1/2 & 0 & 1 & \sqrt{3}/2 & -1/2 \\ -\sqrt{3}/2 & -1/2 & \sqrt{3}/2 & 1 & 0 \\ -1/2 & -\sqrt{3}/2 & -1/2 & 0 & 1 \end{bmatrix}$$

where

L_{ls}, L_{ms} - stator leakage and magnetizing inductance.

$[L_{rr}]$ - rotor inductance matrix:

$$[L_{rr}]_{6\times6}=L_{lr}\cdot[I] + L_{ms}\begin{bmatrix} 1 & \sqrt{3}/2 & -1/2 & -\sqrt{3}/2 & -1/2 & 0 \\ \sqrt{3}/2 & 1 & 0 & -1/2 & -\sqrt{3}/2 & -1/2 \\ -1/2 & 0 & 1 & \sqrt{3}/2 & -1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 & \sqrt{3}/2 & 1 & 0 & -1/2 \\ -1/2 & -\sqrt{3}/2 & -1/2 & 0 & 1 & \sqrt{3}/2 \\ 0 & -1/2 & -\sqrt{3}/2 & -1/2 & \sqrt{3}/2 & 1 \end{bmatrix}$$

where

L_{lr} - rotor leakage inductance;

$[L_{sr}]$ - stator-rotor mutual inductance matrix:

$$[L_{sr}]_{5\times6}=L_{ms}\cdot \begin{bmatrix} c(\theta_r) & c(\frac{\pi}{6}+\theta_r) & c(\frac{4\pi}{6}+\theta_r) & c(\frac{5\pi}{6}+\theta_r) & c(\frac{8\pi}{6}+\theta_r) & c(\frac{9\pi}{6}+\theta_r) \\ c(\frac{11\pi}{6}+\theta_r) & c(\theta_r) & c(\frac{3\pi}{6}+\theta_r) & c(\frac{4\pi}{6}+\theta_r) & c(\frac{7\pi}{6}+\theta_r) & c(\frac{8\pi}{6}+\theta_r) \\ c(\frac{8\pi}{6}+\theta_r) & c(\frac{9\pi}{6}+\theta_r) & c(\theta_r) & c(\frac{\pi}{6}+\theta_r) & c(\frac{4\pi}{6}+\theta_r) & c(\frac{5\pi}{6}+\theta_r) \\ c(\frac{7\pi}{6}+\theta_r) & c(\frac{8\pi}{6}+\theta_r) & c(\frac{11\pi}{6}+\theta_r) & c(\theta_r) & c(\frac{3\pi}{6}+\theta_r) & c(\frac{4\pi}{6}+\theta_r) \\ c(\frac{4\pi}{6}+\theta_r) & c(\frac{5\pi}{6}+\theta_r) & c(\frac{8\pi}{6}+\theta_r) & c(\frac{9\pi}{6}+\theta_r) & c(\theta_r) & c(\frac{\pi}{6}+\theta_r) \end{bmatrix}$$

where

c - abbreviation for cos;
θ_r - rotor angular position.

$[L_{rs}]$ - rotor-stator mutual inductance matrix:

$$[L_{rs}]_{6\times 5} = L_{ms}\cdot$$
$$\begin{bmatrix} c(\theta_r) & c(\frac{\pi}{6}-\theta_r) & c(\frac{4\pi}{6}-\theta_r) & c(\frac{5\pi}{6}-\theta_r) & c(\frac{8\pi}{6}-\theta_r) \\ c(\frac{11\pi}{6}-\theta_r) & c(\theta_r) & c(\frac{3\pi}{6}-\theta_r) & c(\frac{4\pi}{6}-\theta_r) & c(\frac{7\pi}{6}-\theta_r) \\ c(\frac{8\pi}{6}-\theta_r) & c(\frac{9\pi}{6}-\theta_r) & c(\theta_r) & c(\frac{\pi}{6}-\theta_r) & c(\frac{4\pi}{6}-\theta_r) \\ c(\frac{7\pi}{6}-\theta_r) & c(\frac{8\pi}{6}-\theta_r) & c(\frac{11\pi}{6}-\theta_r) & c(\theta_r) & c(\frac{3\pi}{6}-\theta_r) \\ c(\frac{4\pi}{6}-\theta_r) & c(\frac{5\pi}{6}-\theta_r) & c(\frac{8\pi}{6}-\theta_r) & c(\frac{9\pi}{6}-\theta_r) & c(\theta_r) \\ c(\frac{3\pi}{6}-\theta_r) & c(\frac{4\pi}{6}-\theta_r) & c(\frac{7\pi}{6}-\theta_r) & c(\frac{8\pi}{6}-\theta_r) & c(\frac{11\pi}{6}-\theta_r) \end{bmatrix}$$

B. Machine Model In The d-q-z_1-z_2-z_3 Vector Space

Applying the transformations $[T_s]$ and $[T_r]$ to the voltage equations (9) and (10) yields:

Stator voltage equation:

$$[T_s]\cdot[V_s] = [T_s]\cdot[R_s]\cdot[T_s]^{-1}\cdot[T_s]\cdot[i_s] + p\cdot\left([T_s]\cdot[L_{ss}]\cdot[T_s]^{-1}\cdot[T_s]\cdot[i_s] + [T_s]\cdot[L_{sr}]\cdot[T_r]^{-1}\cdot[T_r]\cdot[i_r]\right) \quad (11)$$

Rotor voltage equation:

$$[T_r]\cdot[V_r] = [T_r]\cdot[R_r]\cdot[T_r]^{-1}\cdot[T_r]\cdot[i_r] + p\cdot\left([T_r]\cdot[L_{rr}]\cdot[T_r]^{-1}\cdot[T_r]\cdot[i_r] + [T_r]\cdot[L_{rs}]\cdot[T_s]^{-1}\cdot[T_s]\cdot[i_s]\right) \quad (12)$$

One can define the machine variables in the d-q-z_1-z_2-z_3 space as:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \\ v_{z1}^s \\ v_{z2}^s \\ v_{z3}^s \end{bmatrix} = [T_s]\cdot[V_s], \quad \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{z1}^s \\ i_{z2}^s \\ i_{z3}^s \end{bmatrix} = [T_s]\cdot[i_s],$$

$$\begin{bmatrix} v_{dr}^r \\ v_{qr}^r \\ v_{z1r}^r \\ v_{z2r}^r \\ v_{z3r}^r \\ v_{z4r}^r \end{bmatrix} = [T_r]\cdot[V_r] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} i_{dr}^r \\ i_{qr}^r \\ i_{z1r}^r \\ i_{z2r}^r \\ i_{z3r}^r \\ i_{z4r}^r \end{bmatrix} = [T_r]\cdot[i_r].$$

The convention used here for variable expression is:

x_α^β

x — variable (voltage, current, flux, etc.);

α — axis (ds, qs, dr, qr, z_{1s}, etc.);

β — frame (stator, rotor, synchronous, etc.).

From (11) and (12), the following equations describing the dynamics of the machine in the d-q-z_1-z_2-z_3 vector space can be derived.

Machine model in the d-q subspace:

Stator voltage equation:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix}\cdot\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \frac{d}{dt}\left\{\begin{bmatrix} L_{ls}+3L_{ms} & 0 \\ 0 & L_{ls}+2L_{ms} \end{bmatrix}\cdot\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + L_{ms}\begin{bmatrix} 3\cos(\theta_r) & -3\sin(\theta_r) \\ \sqrt{6}\sin(\theta_r) & \sqrt{6}\cos(\theta_r) \end{bmatrix}\cdot\begin{bmatrix} i_{dr}^r \\ i_{qr}^r \end{bmatrix}\right\} \quad (13)$$

Rotor voltage equation:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix}\cdot\begin{bmatrix} i_{dr}^r \\ i_{qr}^r \end{bmatrix} + \frac{d}{dt}\left\{\begin{bmatrix} L_{lr}+3L_{ms} & 0 \\ 0 & L_{lr}+3L_{ms} \end{bmatrix}\cdot\begin{bmatrix} i_{dr}^r \\ i_{qr}^r \end{bmatrix} + L_{ms}\begin{bmatrix} 3\cos(\theta_r) & \sqrt{6}\sin(\theta_r) \\ -3\sin(\theta_r) & \sqrt{6}\cos(\theta_r) \end{bmatrix}\cdot\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix}\right\} \quad (14)$$

Machine model in the zero sequence subspace:

Stator voltage equation:

$$\frac{d}{dt}\begin{bmatrix} i_{z1s}^s \\ i_{z2s}^s \\ i_{z3s}^s \end{bmatrix} = \begin{bmatrix} -\frac{r_s}{L_{ls}} & 0 & 0 \\ 0 & -\frac{r_s}{L_{ls}} & 0 \\ 0 & 0 & -\frac{r_s}{L_{ls}} \end{bmatrix}\cdot\begin{bmatrix} i_{z1s}^s \\ i_{z2s}^s \\ i_{z3s}^s \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{ls}} & 0 & 0 \\ 0 & \frac{1}{L_{ls}} & 0 \\ 0 & 0 & \frac{1}{L_{ls}} \end{bmatrix}\cdot\begin{bmatrix} v_{z1s}^s \\ v_{z2s}^s \\ v_{z3s}^s \end{bmatrix} \quad (15)$$

Rotor voltage equation:

$$\frac{d}{dt}\begin{bmatrix} i_{z1r}^r \\ i_{z2r}^r \\ i_{z3r}^r \\ i_{z4r}^r \end{bmatrix} = \begin{bmatrix} -\frac{r_r}{L_{lr}} & 0 & 0 & 0 \\ 0 & -\frac{r_r}{L_{lr}} & 0 & 0 \\ 0 & 0 & -\frac{r_r}{L_{lr}} & 0 \\ 0 & 0 & 0 & -\frac{r_r}{L_{lr}} \end{bmatrix}\cdot\begin{bmatrix} i_{z1r}^r \\ i_{z2r}^r \\ i_{z3r}^r \\ i_{z4r}^r \end{bmatrix} \quad (16)$$

It is observed that there are no excitation terms in (16). Therefore, this portion of the machine dynamics can never be excited and, hence, will not be discussed further in this paper.

In the analysis just completed, d-q reference frames were attached to the stator and the rotor separately. To transform the rotor variables to the stationary reference frame and thus eliminate the *sin* and *cos* terms in the above equations, the following rotation transformation is appropriate:

$$[T_r^s] = \begin{bmatrix} \cos(\theta_r) & -\sin(\theta_r) \\ \sin(\theta_r) & \cos(\theta_r) \end{bmatrix} \quad (17)$$

With this transformation, the d-q stator and rotor equations can be expressed as:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} r_s + L_{ds}p & 0 \\ 0 & r_s + L_{qs}p \end{bmatrix}\cdot\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \begin{bmatrix} M_d p & 0 \\ 0 & M_q p \end{bmatrix}\cdot\begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r + L_r p & 0 \\ 0 & r_r + L_r p \end{bmatrix}\cdot\begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} + \begin{bmatrix} M_d p & \omega_r M_q \\ -\omega_r M_d & M_q p \end{bmatrix}\cdot\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (19)$$

where:

$p = \frac{d}{dt}$

$L_{ds} = L_{ls} + 3L_{ms}$, $L_{qs} = L_{ls} + 2L_{ms}$, $L_r = L_{lr} + 3L_{ms}$

$M_d = 3L_{ms}$, $M_q = \sqrt{6}L_{ms}$

Eqs. (18) and (19) suggest that the machine model in the d-q plane is a two phase machine with asymmetrical d-q windings.

If stator and rotor flux linkages are used as state variables, (18) and (19) can also be written as:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{ds}^s \\ \lambda_{qs}^s \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix} \cdot \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} + \begin{bmatrix} 0 & -\omega_r \\ \omega_r & 0 \end{bmatrix} \cdot \begin{bmatrix} \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} \tag{21}$$

where:

$$\begin{bmatrix} \lambda_{ds}^s \\ \lambda_{qs}^s \end{bmatrix} = \begin{bmatrix} L_{ds} & 0 \\ 0 & L_{qs} \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \begin{bmatrix} M_d & 0 \\ 0 & M_q \end{bmatrix} \cdot \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} = \begin{bmatrix} M_d & 0 \\ 0 & M_q \end{bmatrix} \cdot \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \begin{bmatrix} L_r & 0 \\ 0 & L_r \end{bmatrix} \cdot \begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \tag{23}$$

C. Electromagnetic Torque

The electromagnetic torque of electrical machine is expressed as:

$$T_e = \frac{1}{2}[i]^T \cdot \left(\frac{\partial}{\partial \theta_m} [L] \right) \cdot [i] \tag{24}$$

As the d-q plane is electromechanical energy conversion related, the current vector in (24) should include d-q plane currents only, i.e.

$$[i] = \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ i_{dr}^r \\ i_{qr}^r \end{bmatrix} \tag{25}$$

For the same reason, the d-q plane inductance matrix should be used for the inductance matrix $[L]$ in (24). It can be determined as:

$$[L] = \begin{bmatrix} L_{ds,ds} & L_{ds,qs} & L_{ds,dr} & L_{ds,qr} \\ L_{qs,ds} & L_{qs,qs} & L_{qs,dr} & L_{qs,qr} \\ L_{dr,ds} & L_{dr,qs} & L_{dr,dr} & L_{dr,qr} \\ L_{qr,ds} & L_{qr,qs} & L_{qr,dr} & L_{qr,qr} \end{bmatrix} = \begin{bmatrix} L_{ds} & 0 & M_d\cos(\theta_r) & -M_d\sin(\theta_r) \\ 0 & L_{qs} & M_q\sin(\theta_r) & M_q\cos(\theta_r) \\ M_d\cos(\theta_r) & M_q\sin(\theta_r) & L_r & 0 \\ -M_d\sin(\theta_r) & M_q\cos(\theta_r) & 0 & L_r \end{bmatrix} \tag{26}$$

The electromagnetic torque of the machine is obtained by substituting (25) and (26) into (24), which yields:

$$T_e = \frac{P}{2}(M_q i_{qs}^s i_{dr}^s - M_d i_{ds}^s i_{qr}^s) \tag{27}$$

Another expression for electromagnetic torque which is extremely useful for the purpose of torque control is the form expressing the rotor flux linkage interacting with the stator current. This form can be obtained by solving $\begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix}$ from (23) and inserting the result in (27). The expression is:

$$T_e = \frac{P}{2}\frac{1}{L_r}(M_q i_{qs}^s \lambda_{dr}^s - M_d i_{ds}^s \lambda_{qr}^s) \tag{28}$$

IV. DOUBLE-PLANE CURRENT REGULATION

At this juncture the development of the dynamic model of the machine in the d-q-z_1-z_2-z_3 space has been completed. It is expected that the control of the machine will also be simplified due to the decoupled nature of the machine model in the new space. However, the control of the machine in the d-q-z1-z2-z3 space can not be implemented until the inverter model is transformed to the new space as well. This problem will now be solved.

A. Line Voltage to Phase Voltage Transformation

Most drive systems are configured without a machine neutral wire connection. In this case, a switching mode of the inverter only gives the machine's line to line voltages and a transformation from line voltage to phase voltage is generally required for purpose of machine current regulation. For machines operated under balanced conditions, this transformation is simply a constant matrix and can easily be solved from the loop equations and the node equation at the terminals of the machine. However, for the unbalanced operation investigated in this paper, a constant matrix for the line voltage to phase voltage transformation does not exist. The transformation from line voltage to phase voltage remains to be identified. Generally, this transformation would have the form:

$$[v_{phase}] = F([i_{phase}],[v_{line}],[\eta],\omega_r) \tag{29}$$

$[\eta]$—machine parameter vector.

Since instantaneous machine currents and the shaft speed are involved in the transformation, the phase voltages corresponding to each switching mode must be calculated on-line, and is thus impractical due to the complexity of the relation. Although an accurate off-line transformation is impossible, it will be shown in the following that a constant transformation which is a very close approximation of (29) exists.

It can be proved that with the winding resistances ignored and the rotor locked, the differential equation of the machine with line to neutral voltages as inputs is:

$$[L_1 \;\; L_2]\cdot\frac{d}{dt}\begin{bmatrix} i_{1s} \\ i_{2s} \\ i_{3s} \\ i_{4s} \\ i^s_{dr} \\ i^s_{qr} \end{bmatrix} = \begin{bmatrix} v_{1s} \\ v_{2s} \\ v_{3s} \\ v_{4s} \\ v_{5s} \end{bmatrix} \tag{30}$$

where:

$$[L_1] = \begin{bmatrix} L_{ls}+\frac{3}{2}L_{ms} & \frac{1+\sqrt{3}}{2}L_{ms} & 0 & \frac{1-\sqrt{3}}{2}L_{ms} \\ \sqrt{3}L_{ms} & L_{ls}+\frac{2+\sqrt{3}}{2}L_{ms} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}-1}{2}L_{ms} \\ 0 & \frac{1}{2}L_{ms} & L_{ls}+\frac{3}{2}L_{ms} & \frac{1+\sqrt{3}}{2}L_{ms} \\ -\frac{\sqrt{3}}{2}L_{ms} & -\frac{1}{2}L_{ms} & \frac{\sqrt{3}}{2}L_{ms} & L_{ls}+L_{ms} \\ -L_{ls}-\frac{3}{2}L_{ms} & -L_{ls}-\frac{2+\sqrt{3}}{2}L_{ms} & -L_{ls}-\frac{3}{2}L_{ms} & -L_{ls}-L_{ms} \end{bmatrix}$$

$$[L_2] = \begin{bmatrix} \sqrt{3}L_{ms} & 0 \\ \frac{3}{2}L_{ms} & \frac{\sqrt{3}}{2}L_{ms} \\ -\frac{\sqrt{3}}{2}L_{ms} & \frac{3}{2}L_{ms} \\ -\frac{3}{2}L_{ms} & \frac{\sqrt{3}}{2}L_{ms} \\ -\frac{\sqrt{3}}{2}L_{ms} & -\frac{3}{2}L_{ms} \end{bmatrix}$$

The differential equation can also be expressed with line to line voltages as inputs:

$$\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}\cdot\frac{d}{dt}\begin{bmatrix} i_{1s} \\ i_{2s} \\ i_{3s} \\ i_{4s} \\ i^s_{dr} \\ i^s_{qr} \end{bmatrix} = \begin{bmatrix} v_{12s} \\ v_{23s} \\ v_{34s} \\ v_{45s} \\ 0 \\ 0 \end{bmatrix} \tag{31}$$

where:

$$L_{11} = \begin{bmatrix} L_{ls}+\frac{3-2\sqrt{3}}{2}L_{ms} & -L_{ls}-\frac{1}{2}L_{ms} & -\frac{\sqrt{3}}{2}L_{ms} & -(\sqrt{3}-1)L_{ms} \\ \sqrt{3}L_{ms} & L_{ls}+\frac{1+\sqrt{3}}{2}L_{ms} & -L_{ls}-\frac{3-\sqrt{3}}{2}L_{ms} & -L_{ms} \\ \frac{\sqrt{3}}{2}L_{ms} & L_{ms} & L_{ls}+\frac{3-\sqrt{3}}{2}L_{ms} & -L_{ls}+\frac{\sqrt{3}-1}{2}L_{ms} \\ L_{ls}+\frac{3-\sqrt{3}}{2}L_{ms} & L_{ls}+\frac{1+\sqrt{3}}{2}L_{ms} & L_{ls}+\frac{\sqrt{3}+3}{2}L_{ms} & 2L_{ls}+2L_{ms} \end{bmatrix}$$

$$L_{12} = \begin{bmatrix} \frac{2\sqrt{3}-3}{2}L_{ms} & -\frac{\sqrt{3}}{2}L_{ms} \\ \frac{3+\sqrt{3}}{2}L_{ms} & \frac{\sqrt{3}-3}{2}L_{ms} \\ \frac{3-\sqrt{3}}{2}L_{ms} & \frac{3-\sqrt{3}}{2}L_{ms} \\ \frac{\sqrt{3}-3}{2}L_{ms} & \frac{\sqrt{3}+3}{2}L_{ms} \end{bmatrix}$$

$$L_{21} = \sqrt{3}L_{ms}\begin{bmatrix} \frac{3\sqrt{3}}{2}L_{ms} & \frac{\sqrt{3}+3}{2}L_{ms} & 0 & \frac{\sqrt{3}-3}{2}L_{ms} \\ \frac{3}{2}L_{ms} & \frac{\sqrt{3}+3}{2}L_{ms} & 3L_{ms} & \frac{\sqrt{3}+3}{2}L_{ms} \end{bmatrix}$$

$$L_{22} = \begin{bmatrix} L_{lr}+3L_{ms} & 0 \\ 0 & L_{lr}+3L_{ms} \end{bmatrix}$$

The inductance matrix in (31) is a square matrix and can be shown to have full rank. Therefore, the derivative of the current vector can be solved from (31) as:

$$\frac{d}{dt}\begin{bmatrix} i_{1s} \\ i_{2s} \\ i_{3s} \\ i_{4s} \\ i^s_{dr} \\ i^s_{qr} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}^{-1}\cdot\begin{bmatrix} v_{12s} \\ v_{23s} \\ v_{34s} \\ v_{45s} \\ 0 \\ 0 \end{bmatrix} \tag{32}$$

Substituting (32) into (30), the following line voltage to phase voltage transformation results:

$$\begin{bmatrix} v_{1s} \\ v_{2s} \\ v_{3s} \\ v_{4s} \\ v_{5s} \end{bmatrix} = [T_f^p]\cdot\begin{bmatrix} v_{12s} \\ v_{23s} \\ v_{34s} \\ v_{45s} \\ 0 \\ 0 \end{bmatrix} \tag{33}$$

where:

$$[T_f^p] = [L_1 \;\; L_2]\cdot\begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}^{-1} \tag{34}$$

Since the last two elements of the voltage vector on the right side of (33) are zeros and hence do not contribute any thing to phase voltages, the last two columns of $[T_f^p]$ can be ignored. The resultant transformation is a 5 by 4 matrix with its elements being only the functions of machine parameters and thus can be readily calculated off-line. For the given machine parameters this transformation is

$$\begin{bmatrix} v_{1s} \\ v_{2s} \\ v_{3s} \\ v_{4s} \\ v_{5s} \end{bmatrix} = \begin{bmatrix} 0.7921 & 0.6039 & 0.4302 & 0.2421 \\ -0.2079 & 0.6039 & 0.4302 & 0.2421 \\ -0.2079 & -0.3961 & 0.4302 & 0.2421 \\ -0.2079 & -0.3961 & -0.5698 & 0.2421 \\ -0.2079 & -0.3961 & -0.5698 & -0.9579 \end{bmatrix}\cdot\begin{bmatrix} v_{12s} \\ v_{23s} \\ v_{34s} \\ v_{45s} \end{bmatrix} \tag{35}$$

B. Inverter Voltage Vector Transformation

The voltage source inverter depicted in Fig. 1 has five working phases remaining. The number of switching modes of this five phase inverter is 32. For each of the 32 switching modes, a line to line voltage vector applied to the machine by the inverter can be uniquely determined. To obtain the projections of this voltage vector on the d-q and the z_1-z_2-z_3 subspaces, one can use transformation (35) to obtain the corresponding phase voltage vector first, and then transform the phase voltage vector to the d-q and z_1-z_2-z_3 subspaces using the decomposition transformation (7). The two transformations can be cascaded to form a one-step transformation:

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \\ v_{z1s}^s \\ v_{z2s}^s \\ v_{z3s}^s \end{bmatrix} = \begin{bmatrix} 0.5774 & 1.0774 & 0.7887 & 0.2887 \\ -0.1470 & 0.0735 & 0.5631 & 0.7836 \\ -0.4177 & 0.1529 & -0.4177 & 0.0000 \\ -0.5196 & -0.1019 & 0.2785 & -0.2921 \\ 0.0304 & -0.0152 & -0.1164 & -0.1620 \end{bmatrix} \cdot \begin{bmatrix} v_{12s} \\ v_{23s} \\ v_{34s} \\ v_{45s} \end{bmatrix} \quad (36)$$

With this transformation, the 32 line to line voltage vectors corresponding to the 32 switching modes of the inverter can be transformed to the d-q plane and the z_1-z_2 zero sequence plane respectively as shown in Fig. 2 a) and b). The projections of the voltage vectors on the z_3-axis are not shown due to the assumption that the stator windings of the machine are tied to a single neutral. In this case, the z_3-axis is linearly dependent to the rest of the axes, and the drive system is then, in actuality, a four dimensional system.

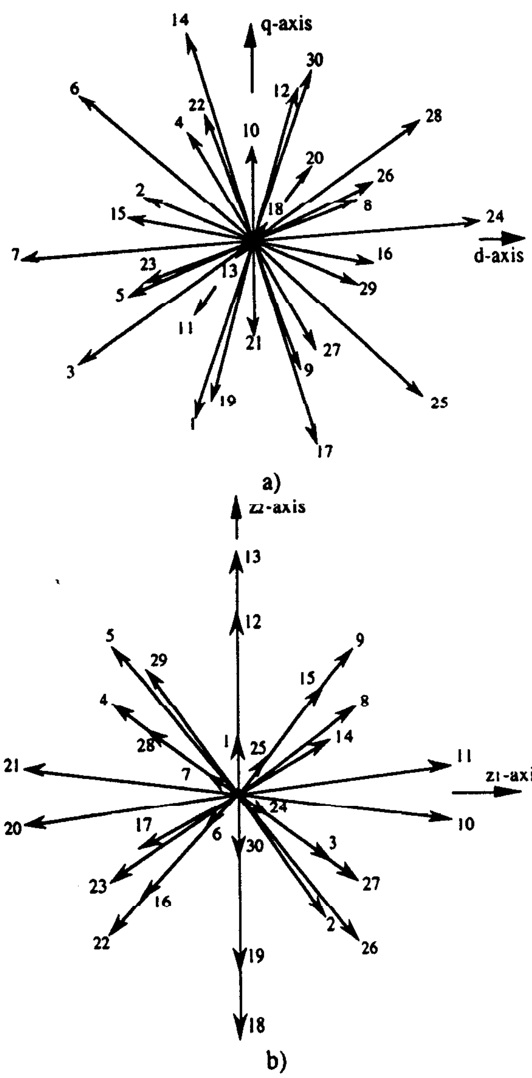

Fig. 2 a) Inverter Output Voltage Vectors Projected on d-q Plane;
b) Inverter Output Voltage Vectors Projected on z1-z2 plane.

C. Double-Plane Current Regulation Scheme

The four dimensional property of the system requires that the current regulation be performed on a double-plane basis. The two planes are named d-q and z1-z2 as suggested by the previous discussions. The principle of the double-plane current regulation scheme is illustrated as follows.

On the d-q plane, if ignoring r_s and r_r, the following voltage equation can be derived from (20) through (23):

$$\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} L_{ds}' & 0 \\ 0 & L_{qs}' \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \begin{bmatrix} 0 & -\omega_r \frac{M_d}{L_r} \\ \omega_r \frac{M_q}{L_r} & 0 \end{bmatrix} \cdot \begin{bmatrix} \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} \quad (37)$$

where $L_{ds}' = L_{ds} - \frac{L_{ds} M_d}{L_r}$, $L_{qs}' = L_{qs} - \frac{L_{qs} M_q}{L_r}$.

Letting

$$\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} = \begin{bmatrix} i_{ds}^{s*} \\ i_{qs}^{s*} \end{bmatrix} - \begin{bmatrix} \Delta i_{ds}^s \\ \Delta i_{qs}^s \end{bmatrix} \quad (38)$$

in (37) and reorganizing the resultant equation we have:

$$\begin{bmatrix} L_{ds}' & 0 \\ 0 & L_{qs}' \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} \Delta i_{ds}^s \\ \Delta i_{qs}^s \end{bmatrix} = \begin{bmatrix} e_{ds}^s \\ e_{qs}^s \end{bmatrix} - \begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} \quad (39)$$

where $\begin{bmatrix} i_{ds}^{s*} \\ i_{qs}^{s*} \end{bmatrix}$ and $\begin{bmatrix} \Delta i_{ds}^s \\ \Delta i_{qs}^s \end{bmatrix}$ are the current reference and the current error vectors in the d-q plane, and:

$$\begin{bmatrix} e_{ds}^s \\ e_{qs}^s \end{bmatrix} \triangleq E = \begin{bmatrix} L_{ds}' & 0 \\ 0 & L_{qs}' \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} i_{ds}^{s*} \\ i_{qs}^{s*} \end{bmatrix} + \begin{bmatrix} 0 & -\omega_r \frac{M_d}{L_r} \\ \omega_r \frac{M_q}{L_r} & 0 \end{bmatrix} \cdot \begin{bmatrix} \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} \quad (40)$$

Eq. (39) can also be written as:

$$\begin{bmatrix} L_{ds}' & 0 \\ 0 & L_{qs}' \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} \Delta i_{ds}^s \\ \Delta i_{qs}^s \end{bmatrix} = E - \begin{bmatrix} v_{ds}^s(k) \\ v_{qs}^s(k) \end{bmatrix} \quad (41)$$

Although (39) and (41) are essentially the same, the latter states explicitly that the machine is fed by a switched power supply with k denotes the kth switching mode of the inverter.

Eq. (41) expresses the fact that with the position of the vector E known, one can choose a switching mode to determine the derivative of the current error vector and thereby to control the direction in which the current vector would change under that switching mode. With the switching mode being properly selected, the current error can always be decreased. The position of the vector E can be determined from (40) assuming the rotor flux vector is known, which is true when the machine is operated with direct field-oriented control.

As far as the current regulation on the z_1-z_2 plane is concerned, the same concept discussed above can be employed but with a much simpler expression for the current error behavior. Eq. (42) describes the counterpart of (41) on z_1-z_2 plane. It is derived from (15) by ignoring r_s and considering the fact that the current error on z_1-z_2 plane is essentially equal to the negative of real current since the current reference vector on this plane is always zero.

$$\frac{d}{dt}\begin{bmatrix}\Delta i^s_{z1s}\\ \Delta i^s_{z2s}\end{bmatrix} = -\begin{bmatrix}\frac{1}{L_{ls}} & 0\\ 0 & \frac{1}{L_{ls}}\end{bmatrix}\cdot\begin{bmatrix}v^s_{z1s}(k)\\ v^s_{z2s}(k)\end{bmatrix} \tag{42}$$

Eq. (42) states that the direction of the derivative of the current error vector on the z_1-z_2 plane is opposite to the voltage vector transformed from the inverter output voltage vector to this plane.

At this point, the double-plane current regulation scheme for the machine operated with one phase open can be summarized as:

i) Search for a inverter switching mode according to the positions of the current error vector and the vector E on d-q voltage vector plane (Fig. 2. a) such that the dot product of the current error vector and the derivative of the current error vector calculated from (41) is negative.

ii) On the z1-z2 voltage vector plane (Fig. 2. b), the dot product of the current error vector and the current error derivative vector, which is produced by the same switching mode and calculated by (42), should also be negative.

iii) If ii) is not satisfied, repeat from i).

V. CONCLUSIONS

The modeling of asymmetrical multi-phase induction machine based directly on the asymmetrical winding structure and the regulation of multi-phase current on a multi-plane basis have been conceptualized in this paper.

The proposed analytical modeling approach includes the development of a decomposition transformation to deal with the asymmetrical stator winding structure. With this transformation, the dynamics of the machine can be decomposed with a d-q plane machine model, which is an equivalent two phase induction machine with asymmetrical stator windings, to represent the electromechanical energy conversion property of the machine, and a zero sequence plane machine model describe the behavior of non-electromechanical energy conversion related components.

The double-plane current regulation, or so called multi-dimensional current regulation in the general case, regulates machine currents on the d-q plane and the zero sequence plane separately. The technique not only solves the problem of multi-phase current regulation, but also makes the problem conceptually clear. That is, the torque control of induction machine with arbitrary phase numbers can always be performed on a d-q plane, and any current components other than d-q will be nonetheless minimized on the planes orthogonal to the d-q plane if the efficiency of the machine is a major concern.

VI. REFERENCES

[1] Y. Zhao and T. A. Lipo, "Space Vector PWM Control of Dual Three Phase Induction Machine Using Vector Space Decomposition," *IEEE Trans. on Ind. Appl.*, vol. 31, No. 5, Sept./Oct. 1995, pp. 1100-1109.

[2] T. M. Jahns, "Improved Reliability in Solid-State AC Drives by Means of Multiple Independent Phase-Drive Units," *IEEE Trans. on Ind. Appl.*, vol. IA-16, No. 3, May/June 1980, pp. 321-331.

[3] E. A. Klingshirn, "High Phase Order Induction Motors, Part I-Description and Theoretical Considerations," *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-102, No. 1, January, 1983, pp. 47-53.

[4] E. A. Klingshirn, "High Phase Order Induction Motors, Part II-Experimental Results," *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-102, No. 1, January, 1983, pp. 54-59.

[5] C. L. Fortescue, "Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks", *AIEE Trans.*, vol. 37, 1918, pp. 1027-1115.

[6] D. White and H. Woodson, *Electromechanical Energy Conversion* , New York, Wiley, 1959.

[7] T. A. Lipo and D. W. Novotny, *Dynamics and Control of AC Drives*, course notes for ECE711, University of Wisconsin-Madison, 1990.

[8] E. E. Ward, and H. Harer, "Preliminary Investigation of an Inverter Fed 5-Phase Induction Motor", *IEE Proc., Vol. 116 (B)*, No. 6, June 1969, pp. 980-984.

[9] R. H. Nelson, P. C. Krause, "Induction Machine Analysis for Arbitrary Displacement Between Multiple Winding Sets," *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-93, May/June 1974, pp. 841-848.

[10] J-R Fu and T. A. Lipo, "A Strategy to Isolate The Switching Device Fault of a Current Regulated Motor Drive." *IEEE Ind. Appl. Soc. Ann. Meeting*, Toronto, Canada, Oct. 2-8, 1993. pp. 1015-1020.

Yifan Zhao (S'91), Thomas A. Lipo (M'64—SM'71—F'87), for biographies, please see companion paper, this issue.