

# VSI-PWM Rectifier/Inverter System with a Reduced Switch Count

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**Abstract** — A current controlled VSI-PWM rectifier and inverter with capacitor dc link is regarded as one of the most promising structures for three-phase to three-phase power conversion. This type of converter normally requires twelve switches for a rectifier and inverter composed of self turn-off switch such as a bi-polar transistor or IGBT with an anti-parallel diode. In this paper, a new three-phase to three-phase converter for ac motor drives is proposed. The proposed converter employs only eight switches and has the capability of delivering sinusoidal input currents with unity power factor and bidirectional power flow. This paper describes the feasibility and the operational limitations of the proposed structure. A mathematical model of the system is derived using generalized modulation theory and experimental results for steady state and dynamic behavior are presented to verify the developed model.

## I. INTRODUCTION

With the recent progress in power semiconductor device technology followed by advancements in power electronic control methods, variable frequency inverter-fed ac drives are being adopted for a wide variety of applications. Recently there has been growing interest in low cost ac drives to meet the needs for reducing cost. Improvements in power semiconductor switch technology have significantly reduced the cost and size of such drives and improved waveform quality. Little change, however, has been made to the basic structure of the conventional ac-ac converter, while significant advances have been made in soft switching techniques and control strategies such as field-oriented drives, sensor-less controls, and adaptive controls.

It is well known that a VSI-PWM rectifier and inverter with capacitor dc link (Fig. 1) is regarded as one of the most important structures for a three phase to three phase power converter. The leakage inductance of the induction motor is shown to emphasize the symmetric structure of the converter. This type of converter normally requires twelve switches for a rectifier and inverter composed of a self turn-off switch such as a power transistor or IGBT with an anti-parallel diode. Compared to a conventional thyristor converter, the distinguishing advantages of this structure are: 1) capability of unity or even leading power factor, 2) sinusoidal input current waveforms reducing harmonic pollution, 3) bidirectional power flow[1-2].

A number of low cost topologies have been suggested for fixed and variable speed drives in the low power range [3-8]. Van der Broeck et. al. discussed the harmonic effects of a two-phase PWM inverter reducing the number of switching devices at the output stage from six to four of the converter and a method of generating the three-phase waveforms with

two dc link voltages[3-4]. The modulation strategy suggested can produce three phase balanced sinusoidal waveforms at a reduced output voltage of 0.866 compared with the conventional six switch inverter. In another topology proposed by Enjeti et. al. (Fig. 2), the diode bridge rectifier is replaced by a single-phase current controlled rectifier employing two switches and two capacitors enabling input current and input power factor control[6-7]. In the proposed structure, only six power switches are required to provide single-phase to three-phase power conversion with the capability of bidirectional power flow with balanced three phase output voltages for ac motor drives. Covic et. al. proposed the new voltage control scheme with the same structure, which enables unity power factor and an improved dc link voltage control independent of input voltage fluctuations[8]. However, the single-phase rectifier has a limitation for high power applications because power flow is not constant, therefore, requires a much larger capacitance in the dc link, which makes system performance sluggish.

In this paper, a three-phase to three-phase VSI-PWM rectifier and inverter structure with eight switches is proposed. The proposed technique uses two identical inverters with four power switches both for a rectifier and for an inverter, and a two split capacitor dc high voltage link as shown in Fig. 3. The configuration incorporates an active rectifier structure that provides an active input current wave shaping feature and allows bidirectional power flow with four switches. A four switch inverter with the split capacitors in the dc link provides balanced three phase output to ac motor at adjustable voltage and frequency. An ac input is then connected to the two PWM voltages of rectifier and the center point of two split capacitors. A three phase ac motor is also connected to the output inverter and the same center point.

This paper describes the feasibility and the operational limitations of the proposed converter. A mathematical model of the system has been derived using generalized modulation theory and typical experimental results for steady state and dynamic responses are presented to illustrate important performance characteristics to verify the developed model.

## II. CONFIGURATION OF PROPOSED STRUCTURE

Fig. 1 shows conventional rectifier/inverter fed induction motor drive system, employing two VSI-PWM inverters and

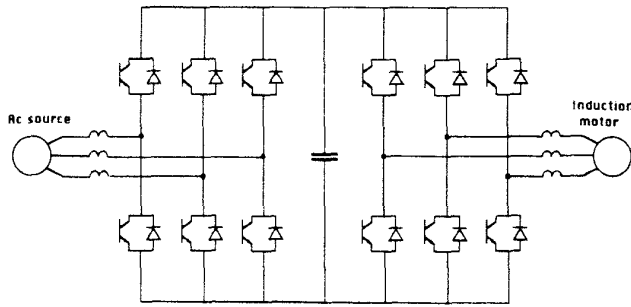


Fig. 1 Conventional VSI-PWM rectifier/inverter system

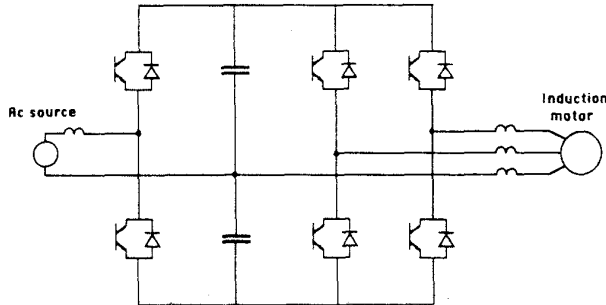


Fig. 2 Single phase to three phase converter

a capacitor dc link. An important feature of this structure are the placement of the inductances on the ac side for the boost operation of the rectifier and the placement of the capacitance in the dc link. Some ac line inductances are needed because  $Ldi/dt$  voltage form an essential element in forcing the currents track current references and a dc link capacitor acts as a dc voltage source and provides filtering operation. For proper operation, the dc link capacitor voltage must be sufficiently higher than the peak line-to-line voltage of ac main source and counter emf of ac motor for the back-bias of the anti-parallel diode and for the current controllability without current distortion.

The proposed circuit consists of replacing four switches for one input leg and one output leg and one capacitor in Fig. 1, by only two split capacitors without switches, as shown in Fig. 3. The split capacitor bank is charged through the diodes in  $S_{i1} - S_{i4}$ . The switches operate on a PWM pattern to shape the input currents to follow the current references. The filter inductors provide current filtering and voltage boost operation. The output inverter consists of a four switch inverter. The center point of the split capacitor link forms the third phase both for the rectifier and inverter. The output inverter can be operated either by voltage control or by current control.

In case of voltage control the output line-to-line voltage,

$$v_{13} = v_{o1} - v_{o3} \quad (1)$$

that is the voltage of the output phase 1 with the reference of the center point of the split capacitors can be defined by controlling  $S_{o1}$  and  $S_{o2}$  in a PWM fashion. In order to generate balanced three phase output voltages, the voltage

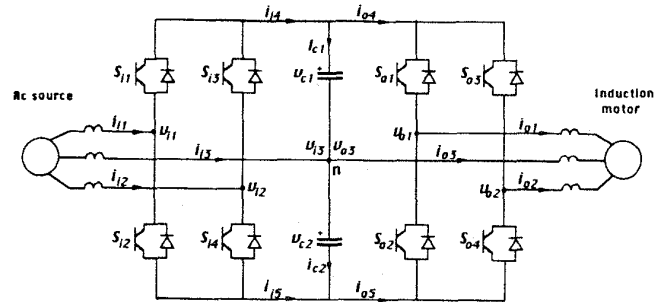


Fig. 3 Proposed rectifier/inverter system with a reduced switch count

$v_{23}$  should be shifted by  $+60^\circ$  or  $-60^\circ$  according to the output phase sequence. One can see that the line-to-line voltage  $v_{12}$  is a three-level PWM between  $+v_{c1}$ , 0, and  $-v_{c2}$ . On the other hand, the voltages  $v_{13}$  and  $v_{23}$  are two-level between  $+v_{c1}$  and  $-v_{c2}$ . However, the fundamental components in the three phase output voltages are same.

In the case of current control the template waveforms for output current references are usually taken from the field-oriented controller of the induction motor. Those for input current reference are taken from the ac mains through voltage transformers. The phase voltages of ac mains are introduced for unity power factor input current reference and multiplied by the output of the outer loop dc link voltage controller (magnitude of input current is determined to maintain constant the dc link voltage) to form the current reference template waveforms. The controlled current of the PWM using a hysteresis current regulator consists of monitoring the current of each phase and using the ON-OFF switching to ensure that the real current tracks the template waveform within a narrow hysteresis band. With the current control of the currents  $i_1$  and  $i_2$  are controlled in a PWM manner with a phase shift of  $120^\circ$  while  $i_3$  is automatically quasi-sinusoidal ( $i_3 = -(i_1 + i_2)$ ), because there is no connection of the neutral point. One sees that the ac source and induction motor are not completely decoupled because one input leg and one output leg are connected directly with two dc link capacitors.

The inverter or rectifier operates in one of four nonzero voltage vectors, as shown in Fig. 4. The voltage vector is defined by the following complex vector expression:

$$\vec{V} = \frac{2}{3} (v_1 + \bar{a}v_2 + a^2v_3) \quad (2)$$

where,

$$\bar{a} = e^{-j2\pi/3}$$

If balanced three phase sinusoidal waveforms are required, the voltage vector should be controlled in a circular manner. Therefore, the maximum circular locus results in 0.866 compared than the conventional six switch inverter. By the technique introducing the third harmonic, the ratio can be increased.

A dc bus center point is assumed to be the ground reference. Each location corresponds to one switching state of the

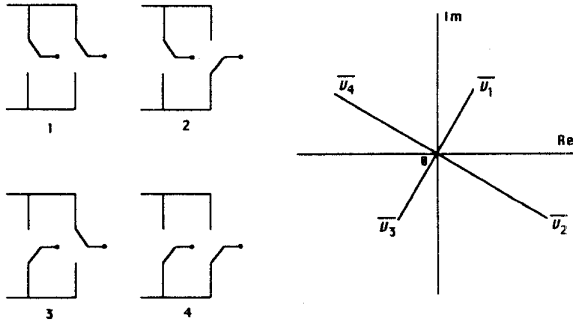


Fig. 4 Inverter switching states and voltage vectors

converter. The line to ground voltages are uniquely determined by the PWM inverter switching according to the voltage or current controller. There is no zero voltage vector and one can expect that the switching frequency of the proposed circuit will be somewhat higher than that of the six switch inverter, in which six nonzero voltage vector and two zero vectors are available. Both of the dc link capacitor voltages must be higher than the peak value of input and output line-to-line voltages for current controllability, respectively. Thus, the overall dc link voltage of the proposed circuit must be twice as high as that of the six switch inverter.

### III. ANALYSIS OF SYSTEM

For the proposed current controlled inverter or rectifier, the switching requirement can be stated as follows. Given a desired set of three phase voltages and a set of three phase currents for the output inverter:

$$\begin{aligned}
 v_{o1} &= V_o \sin(\omega_o t) \\
 v_{o2} &= V_o \sin(\omega_o t - 2\pi/3) \\
 v_{o3} &= V_o \sin(\omega_o t + 2\pi/3) \\
 i_{o1} &= I_o \sin(\omega_o t - \theta_o) \\
 i_{o2} &= I_o \sin(\omega_o t - \theta_o - 2\pi/3) \\
 i_{o3} &= I_o \sin(\omega_o t - \theta_o + 2\pi/3)
 \end{aligned} \quad (3)$$

where,  $V_o$  and  $I_o$  are the magnitudes of the output voltages and currents, respectively. Determine the switching function [S] that will produce a desired set of line-to-ground voltages

$$\begin{bmatrix} v_{o1n} \\ v_{o2n} \end{bmatrix} = \begin{bmatrix} S_{o1} & S_{o2} \\ S_{o3} & S_{o4} \end{bmatrix} \begin{bmatrix} V_{dc} \\ -V_{dc} \end{bmatrix} \quad (4)$$

$$\begin{aligned}
 S_{o1} + S_{o2} &= 1, & S_{o3} + S_{o4} &= 1 \\
 0 \leq S_{on} &\leq 1, & n &= 1 \dots 4
 \end{aligned}$$

where,

$$\begin{aligned}
 v_{o1n} &= v_{o1} - v_{o3} = \sqrt{3} V_{ac} \sin(\omega_o t - \pi/6) \\
 v_{o2n} &= v_{o2} - v_{o3} = \sqrt{3} V_{ac} \sin(\omega_o t - \pi/2)
 \end{aligned} \quad (5)$$

where,  $n$  is the dc bus center point assumed to be ground. Here, the dc capacitor voltages  $v_{c1}$  and  $v_{c2}$  are assumed to be  $V_{dc}$ . One can confirm that the phase difference is  $60^\circ$ . The above equation can be solved as follows

$$S_{o1} = 0.5[1 + a_o \sin(\omega_o t - \pi/6)] \quad (6)$$

$$S_{o2} = 0.5[1 - a_o \sin(\omega_o t - \pi/6)]$$

$$S_{o3} = 0.5[1 + a_o \sin(\omega_o t - \pi/2)]$$

$$S_{o4} = 0.5[1 - a_o \sin(\omega_o t - \pi/2)]$$

where,  $a_o = \sqrt{3} \frac{V_o}{V_{dc}}$ ,  $|a_o| \leq 1$

The dc link currents will be

$$\begin{bmatrix} i_{o4} \\ i_{o5} \end{bmatrix} = \begin{bmatrix} S_{o1} & S_{o2} \\ S_{o3} & S_{o4} \end{bmatrix}^T \begin{bmatrix} i_{o1} \\ i_{o2} \end{bmatrix} \quad (7)$$

$$\begin{aligned}
 i_{o4} &= \frac{I_o}{2} \sin(\omega_o t - \theta_o - \pi/3) + \frac{\sqrt{3}}{4} a_o I_o \cos \theta_o \\
 &= -\frac{1}{2} i_{o3} + \frac{1}{2} I_{op}
 \end{aligned} \quad (8)$$

$$\begin{aligned}
 i_{o5} &= \frac{I_o}{2} \sin(\omega_o t - \theta_o - \pi/3) - \frac{\sqrt{3}}{4} a_o I_o \cos \theta_o \\
 &= -\frac{1}{2} i_{o3} - \frac{1}{2} I_{op}
 \end{aligned}$$

where,  $I_{op} = \frac{\sqrt{3}}{2} a_o I_o \cos \theta_o$

For the rectifier the same procedure (3)-(8) can be applied

$$i_{i4} = -\frac{1}{2} i_{i3} + \frac{1}{2} I_{ip} \quad i_{i5} = -\frac{1}{2} i_{i3} - \frac{1}{2} I_{ip} \quad (9)$$

where,  $I_{ip} = \frac{\sqrt{3}}{2} a_i I_{ip} \cos \theta_i$

Here,  $I_{ip}$  and  $I_{op}$  are the power components of input current and output current, respectively. In the case of a rectifier operated at unity power factor ( $\theta_i = \pi/2$ ), the power component of input current  $I_{ip}$  equals the required magnitude of input current. It can be noted that dc link currents have the dc power components and ac fluctuating current components. From the above results, Kirchhoff's Current Law is satisfied, that is,

$$i_{o3} + i_{o4} + i_{o5} = 0 \quad (10)$$

$$i_{i3} + i_{i4} + i_{i5} = 0$$

The the power calculated on the ac side matches that on the dc side, i.e. for the inverter side

$$P_{ac} = \frac{3}{2} V_o I_o \cos \theta_o \quad (11)$$

$$P_{dc} = V_o (i_{o4} - i_{o5}) = V_o I_{op} = \frac{3}{2} V_o I_o \cos \theta_o \quad (12)$$

and the same for the rectifier side.  
The capacitor charging currents will be

$$i_{c1} = i_{i4} - i_{o4} = -\frac{1}{2}(i_{i3} - i_{o3}) + \frac{1}{2}(I_{ip} - I_{op}) \quad (13)$$

$$i_{c2} = i_{i5} - i_{o5} = +\frac{1}{2}(i_{i3} - i_{o3}) + \frac{1}{2}(I_{ip} - I_{op})$$

$$i_{c1} + i_{c2} = I_{ip} - I_{op} \quad (14)$$

The capacitor votages will be

$$v_{c1} = \int i_{c1} dt, \quad v_{c2} = \int i_{c2} dt \quad (15)$$

The voltage fluctuations of two capacitors have two components of two frequencies with the following magnitudes, respectively.

$$\Delta v_{c1} = \frac{I_i}{2\omega_1 C} \quad \Delta v_{c2} = \frac{I_o}{2\omega_o C} \quad (16)$$

The overall dc link voltage will be

$$v = v_{c1} + v_{c2} = \int (i_{c1} + i_{c2}) dt = \int (I_{ip} - I_{op}) dt \quad (17)$$

The overall dc link voltage can be maintained constant by the dc link voltage controller which makes the sum of two charging currents zero by controlling the magnitude of input current  $I_{ip}$ , to equal  $I_{op}$ , but individual capacitor voltages fluctuate and depend on the operating frequency, capacitance, and the magnitude of current.

#### IV. VOLTAGE CONTROLLER

From the above analysis, it can be observed that in case of zero or very low output frequency operation the voltage fluctuations of capacitance are too high to satisfy the requirement that each capacitor voltage must be greater than the peak line-to-line voltage even if the overall dc link voltage can be maintained constant. Thus the lower limit of output frequency must be bounded and the transient duration must be kept as short as possible during starting.

Fig. 5 shows the output voltage controller using sine triangle PWM. The wanted set of three phase balanced output voltages is transformed to the set of two line-to-ground voltage and then the effect of fluctuating dc link voltage due to the ac component of dc link current is compensated.

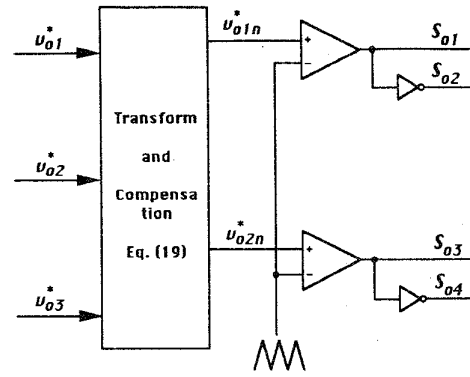


Fig. 5 Sine triangle PWM for inverter control

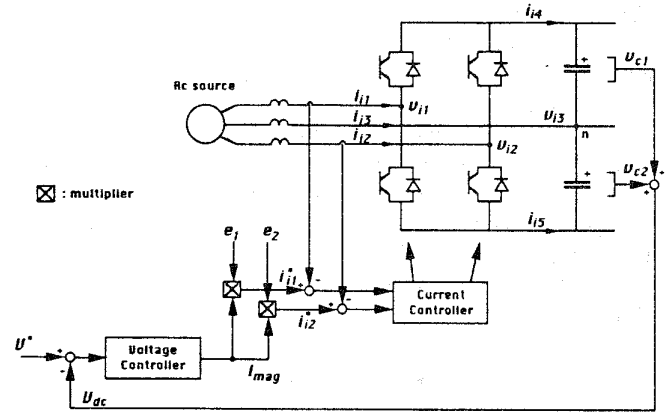


Fig. 6 The dc link voltage controller of rectifier

$$v_{o1n}^* = (v_{o1}^* - v_{o3}^*) - (v_{c1} - v_{c2})/2 \quad (18)$$

$$v_{o2n}^* = (v_{o2}^* - v_{o3}^*) - (v_{c1} - v_{c2})/2$$

These voltage references are compared by a triangular modulation signal to generate the gating signals for two legs of inverter.

Fig. 6 shows the suggested dc link controller, in which PI controller controls the sum of capacitor voltage, in that, the overall dc link voltage. The feed-forward compensation from the power measurement can be added to PI controller output to improve the performance of the controller. The compensated controller output  $I_{mag}$  is multiplied by the ac main phase voltages through a transformer to generate the input current references. Because the input references are in phase with the input phase voltages, the PI controller output is, in effect, the magnitude of power component of current, the required power by the capacitors to maintain the dc link voltage constant. Reactive current components can be added to the current references to control the input power factor.

Various current controllers can be applied to the control of this system. For the ramp comparison method to control the switching frequency, the current reference is compared with the actual measured current to calculate the current error, which are passed through the current controller, usually PI controller, to generate the required PWM voltages at the

rectifier output. In this case the same voltage control scheme used in output inverter control shown in Fig. 5 can also be applied. The controlled current of the PWM using a hysteresis current regulator can also be applied.

In the transient state, for example, during motor starting, the capacitor voltage may vary so much as to drop below the controllability limit because of low output frequency. In order to prevent voltage drop during transients, a function block is added to the controller which controls the difference of two capacitor voltages. This function block is nothing but a P controller which adds compensation term to the output frequency to the input current references. However, if this function block always operates, a current with output frequency will flow in input rectifier side which is not acceptable because of the saturation of input transformer. In the steady state the function block should not operate by introducing the dead band of the function block. This dead band enables the function block operate only in the transient state.

## V. EXPERIMENTAL RESULTS

A 2kVA prototype was constructed using 400V, 30A MOSFET module, two 2000 $\mu$ F capacitors, and three 5mH inductors. A set of reduced input voltages was used by a input transformer. A 2kW induction motor was used as a load of the inverter. The output inverter was operated by the sine triangle PWM and the rectifier was operated by the current hysteresis control. Motorola MC56001 DSP processor was used for the control of the overall system. The sampling time for the voltage control and the current hysteresis control was 50 $\mu$ sec. The dc link capacitance voltages are charged initially to the voltage of a three phase full bridge diode rectifier (if no switch is activated, the anti-parallel diodes comprise diode rectifier).

Figs. 7-11 show the experimental waveforms of voltages and currents of the proposed rectifier/inverter system illustrating the steady state and transient response. Fig. 7 shows the steady state input current waveforms of rectifier, which are controlled by the hysteresis current control. The current waveform  $i_1$  shows that it is controlled sinusoidally and that of  $i_3$ , which is the phase not controlled shows that it is also controlled automatically but has a somewhat different waveform compared with that of  $i_1$ .

Fig. 8 illustrates the transient waveforms of the dc link voltage and input current for the step change of the reference voltage of dc link from 100V, which is the voltage of the diode rectifier, to 160V. One sees that the magnitude of input current, which is the output of the voltage PI controller, changes according to the step change of the reference voltage, and that the dc link voltage follows the voltage command. It is seen that the dc link voltage is self supporting through the charge of the filter capacitance. After the dc link voltage is stabilized, the converter has good current controllability.

Fig. 9 shows the same waveforms according to the change of the load. One sees that the magnitude of input current increases and then is maintained constant.

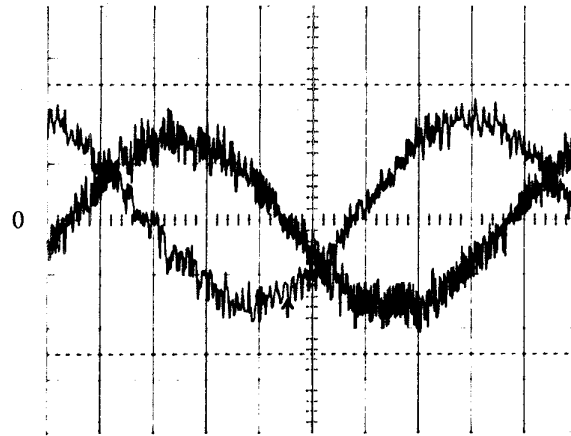


Fig. 7 Steady state waveforms of input currents  
scale: 2A/div., time: 2msec/div.

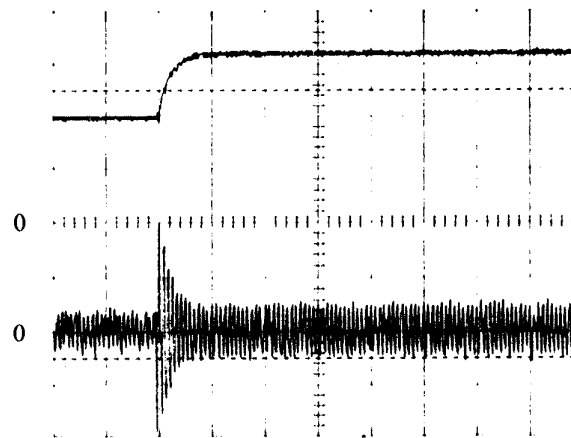


Fig. 8 Transient response to step change in dc link reference voltage  
(a) dc link voltage (upper trace) (b) input current (lower trace)  
scale: 50V/div., 5A/div., respectively, time: 200msec/div.

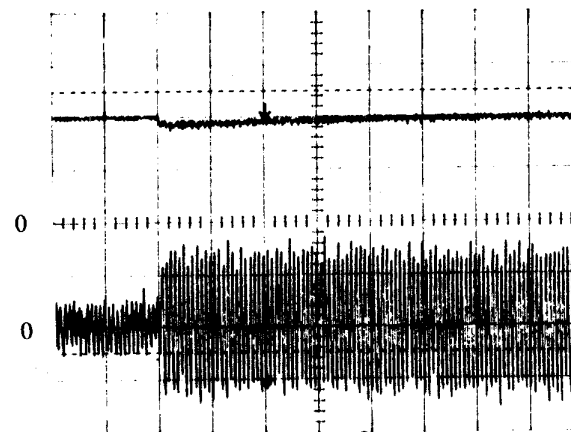


Fig. 9 Transient response under load change  
(a) dc link voltage (upper trace) (b) input current (lower trace)  
scale: 50V/div., 2A/div., respectively, time: 200msec/div.

Figs. 10 and 11 show the characteristics of the induction motor starting. Fig. 10 shows that the dc link voltage  $v_{cl}$  and

$v_{c2}$  start to fluctuate with the frequency of output voltage. Fig. 11 shows the ac waveforms of the overall dc link voltage  $v_{c1} + v_{c2}$ , and the difference of two capacitor voltages  $v_{c1} - v_{c2}$  to exaggerate the fluctuation. One can note that as the motor starts, the overall dc link voltage is constant even if the individual capacitor voltages fluctuate, as expected from the previous analytical results.

## VI. CONCLUSION

In this paper, a modified current controlled VSI-PWM rectifier and inverter structure with reduced switch number is proposed. The proposed structure requires only eight switches and two split capacitors in the dc link for both rectifier and inverter and has the capability of requiring only sinusoidal input currents with unity power factor. The circuit is fully capable of bidirectional power flow. This paper describes the feasibility and the operational limitations of the proposed structure. A dc link voltage controller and the methods for inverter voltage and current controller is suggested. A mathematical model of the system has been derived using generalized modulation theory and verified by the experimental results for steady state and dynamic response. Under the some limitations, the proposed structure appears to be a promising and far more economical topology than the conventional VSI-PWM system with six switch rectifiers and inverters.

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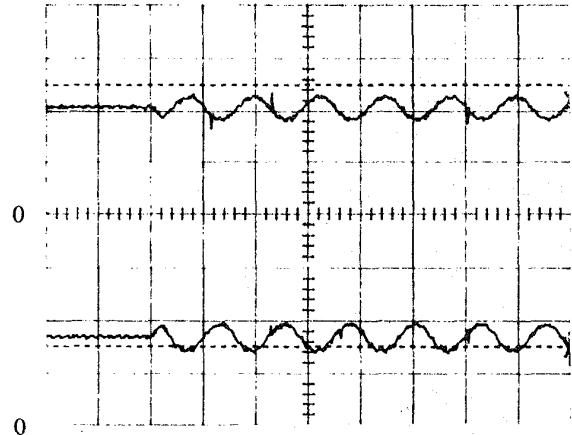


Fig. 10 Transient response during motor starting  
(a) voltage of the upper capacitor  $v_{c1}$  (upper trace)  
(b) voltage of the lower capacitor  $v_{c2}$  (lower trace)  
scale: 50V/div., time: 200msec/div.

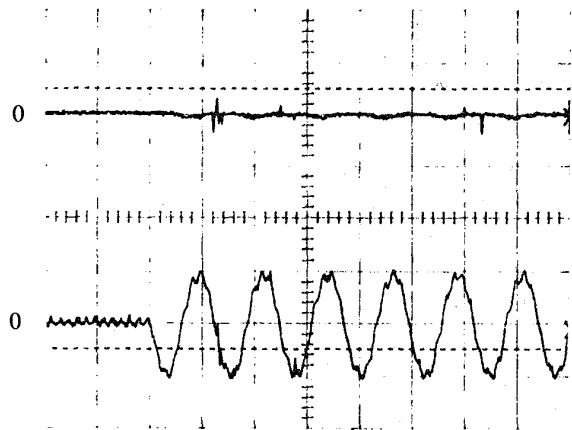


Fig. 11 Transient response during motor starting (ac components)  
(a) sum of the two capacitor voltage  $v_{c1} + v_{c2}$  (upper trace)  
(b) difference of the two capacitor voltage  $v_{c1} - v_{c2}$  (lower trace)  
scale: 20V/div., time: 200msec/div.