Generic Torque-Maximizing Design Methodology of Permanent Magnet Vernier Machine

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Abstract - **A Permanent Magnet Vernier Machine has a toothed-pole structure, and its nonlinear relationship between the dimensions and the magnetic field makes its design cumbersome. This paper presents a novel generic design methodology of the machine which brings out the torque-maximizing structure in a convenient manner. Various suggestions concerning the design are also set forth.**

I. Introduction

A Permanent Magnet Vernier Machine (PMVM) has the feature of high torque at low speed, and therefore, is regarded to be suitable for direct drive applications such as robot arms or electric vehicles. The high torque feature is brought about by the so called 'magnetic gearing effect', which is based on its toothed-pole structure. In this structure, the relationship between the dimensions and the magnetic flux distribution becomes significantly nonlinear, and this feature makes the design optimization a time consuming process, which is basically a repetition of finite element method (FEM) analyses. This paper presents a novel generic design methodology of the PMVM, which realizes a torque maximizing structure in a convenient manner. The followings are the main topics.

1) Introduction of a general torque equation of the PMVM related to its basic component; the Elementary Domain (ED).
2) Magnetic field analyses of the ED with FEM, and an elaborate arrangement of the results to a generic form.
3) Proposal of a convenient design procedure based on the above results and a confirmation of its validity.

II. Torque Expression of PMVM

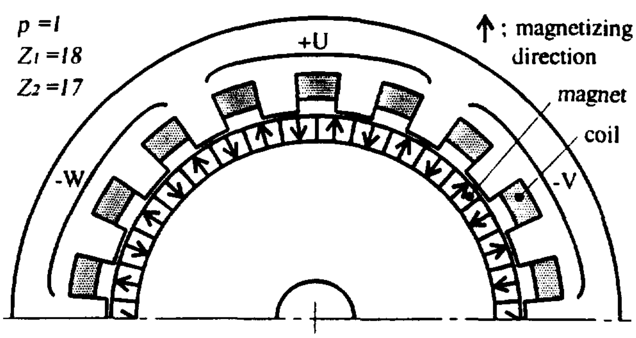

Fig. 1 Permanent Magnet Vernier Machine

Fig. 1 shows the cross section of one winding pole of a 3-phase PMVM. There exists a rule concerning Z_1, Z_2, and p as follows:

$$Z_2 = Z_1 \pm p \tag{1}$$

where p, Z_1, and Z_2 are the numbers of winding pole-pairs, stator teeth, and rotor pole-pairs, respectively. Now let us define ϕ as the magnetic flux linked to a concentrated full-pitch winding of one phase caused by rotor magnets. In the PMVM ϕ can be regarded as a sinusoidal function of the rotor position, which period depends on the magnet pole number. Hence, λ, the flux linkage of one phase winding can be expressed as

$$\lambda = k_w N\phi = k_w N\Phi \cos Z_2\theta \tag{2}$$

where k_w is the winding factor, N is the turn number of one phase winding, Φ is the amplitude of ϕ, and θ is the mechanical angle of the rotor. Magnet-flux-oriented torque of one phase in a PM machine (T_1) is expressed as (3) in general.

$$T_1 = i\frac{d\lambda}{d\theta} \tag{3}$$

where i is the current of the winding, prescribed as

$$i = \sqrt{2}I\cos(Z_2\theta + \alpha) \tag{4}$$

where I is the effective value of i, and α is the phase difference of i to λ. Substituting (2) and (4) to (3) results in

$$T_1 = \frac{1}{\sqrt{2}}Z_2 k_w N I\Phi\{\sin\alpha - \sin(2Z_2\theta + \alpha)\} \tag{5}$$

The torques of the other phases have the same amplitude to T_1 and ±120° phase differences in the '$\sin 2Z_2\theta$' term. Hence a general equation of the total torque T in a PMVM is obtained as (6).

$$T = \frac{3}{\sqrt{2}}Z_2 k_w N I\Phi\sin\alpha \tag{6}$$

In (6) T is proportional to Z_2, and that makes the PMVM a high-torque-at-low-speed machine. On the other hand, T is also proportional to Φ, and Φ will decrease monotonically with Z_2 because of leakage flux in teeth. Hence, in terms of pursuing the maximum torque, there is a tradeoff between Z_2 and Φ.

All the parameters except Φ can be clearly specified in (6). The critical factor of its design is then the accurate estimation of Φ, which is explained in the following section.

III. Elementary Domain

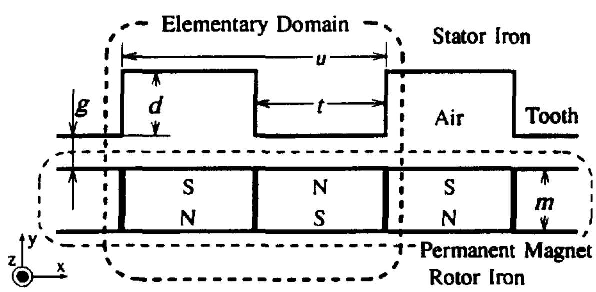

Fig. 2 'Elementary Domain; ED'

A. Elementary Domain

The PMVM is considered to have a basic component which has one tooth, one slot, and one magnet pole-pair. This is defined as an 'Elementary Domain; ED' in [1]. Fig. 2 shows ED, with the dimensions specifying the structure, which are g; airgap length, m; magnet thickness, d; slot length, and u; tooth pitch. The widths of tooth and slot can be different but are set to be equal at this time. ED is a 2D structure and uniform with z direction in the figure.

B. Reflection of ED to the torque expression

Let us define Φ_S as the amplitude of the total magnetic flux passing normally to the tooth pitch of one ED when the tooth and magnet are aligned. Φ_S can be expressed as (7), which also defines an 'effective-flux coefficient; k_F.'

$$\Phi_S = ulk_F B_r \tag{7}$$

where l is z direction thickness of ED, and B_r is the remanence of the magnet. In a PMVM, magnetic flux passing normally to the teeth contributes to the torque production. Hence k_F shows the effectiveness of the structure to exploit the potential of the magnet. Lining up EDs in x direction and short-circuiting the upper and lower yoke magnetically makes an ED array. In an ED array, each ED has the same magnetic field distribution. If the number of EDs in one array is n, the total effective flux Φ_T becomes (8).

$$\Phi_T = n\Phi_S = nulk_F B_r \tag{8}$$

In a PMVM, the number of teeth in a full-pitch coil is $Z_1/(2p)$. In this case Φ can be expressed as (9), taking account of the effect that the numbers of teeth and magnet pole-pairs in the coil pitch are different because of (1).

$$\Phi = V\frac{Z_1}{2p}ulk_F B_r \tag{9}$$

where V is defined as 'vernier coefficient', which corresponds to the effect just described. Substituting (9) to (6) yields (10), considering that $Z_1 u$ is the circumference of the stator inner radius R, and setting α to 90°.

$$T = \frac{3}{\sqrt{2}}\frac{\pi Rl}{p}k_w NIB_r Z_2 Vk_F \tag{10}$$

Usually, Z_1 is selected more than ten times as large as p. Thus it is a good approximation to assume $Z_1 \approx Z_2$ based on (1), whereupon

$$T \approx \frac{3}{\sqrt{2}}\frac{\pi Rl}{p}k_w NIB_r Z_1 Vk_F = 3\sqrt{2}\frac{(\pi R)^2 l}{p}k_w NIB_r V\frac{k_F}{u} \tag{11}$$

On the right side of (11), only $V\,k_F/u$ depends on the toothed-pole structure. Hence, $V\,k_F/u$ can be used as a torque-maximizing index with the toothed-pole dimensions as parameters.

k_F is a function of g, m, d, and u, and V is a function of these parameters and Z_1. These functions are apparently nonlinear, then to get the values of k_F and V should be done with FEM.

IV. FEM Analyses of Elementary Domain

A. Preparation

FEM analyses of the ED have been done by the Maxwell 2D Field Simulator Ver. 6.5.04(Ansoft Co.). Fig. 3 shows the analyzed model of an ED array. Therein three EDs and an adjacent symmetry boundary function as six EDs. Table 1 shows the material settings of the model. By assigning a high permeance to the yoke, magnetic field of each ED becomes the same. k_F is obtained from the flux passing through the line a (Φ_a) with using (12).

$$k_F = \frac{\Phi_a}{3ulB_r} \tag{12}$$

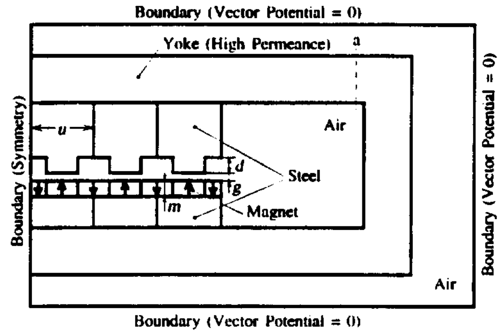

Fig. 3. Analyzed model of an ED array

Table 1 Material settings of the analyzed model

Magnet	Br=1.23T, Hc=-8.9×10^{5}A/m (NdFeB)
Steel	μ_r=2200, saturation over about 1.5T
Yoke	μ_r=10^7, non-saturation

From the preliminary analyses, several important facts were uncovered:

1) The smaller the value of g is, the larger k_F becomes.
2) Similar but different size EDs show the same k_F, i.e., it is sufficient to analyze with one parameter of g, m, d, and u to be constant. The airgap length is the most suitable for the fixed parameter. Normalized parameters can now be introduced as
$m'=m/g,\ d'=d/g,\ u'=u/g$ (13)
It is also preferable to modify the torque equation with the normalized parameters as (14) from (11).

$$T = 3\sqrt{2}\frac{(\pi R)^2 l}{pg}k_w NIB_r V\frac{k_F}{u'} \tag{14}$$

3) Better characteristics, including high torque, is obtained when $Z_2=Z_1-p$ than when $Z_2=Z_1+p$. Hence, the following discussions are carried out on the former condition.

B. Investigation of k_F

Fig. 4 shows some examples of k_F - u' curves with various m' and d'. Ranges of the parameters are chosen over practical values. From the acquired curves, the following facts are revealed.

1) In the practical range the maximum value of k_F is roughly 0.25.
2) With constant m' and u', k_F for d'=0.4u' is over 98% of k_F for d'=0.5u' in any cases. Hence setting d'=0.4u' is sufficient.

Fig. 5 shows k_F/u'-u' curves with various m' and d'. The following points are perceived from the curves.

1) There exists a k_F/u' maximizing u' (u_m') in each case.
2) u_m' increases as m' increases.

3) As m' increases, the dependency of k_F/u' to d' increases. This suggests that a deep slot is effective when the magnet is thick.
4) The selection of u' becomes critical when m' is small because of the steep slope of k_F/u'-u' curve.

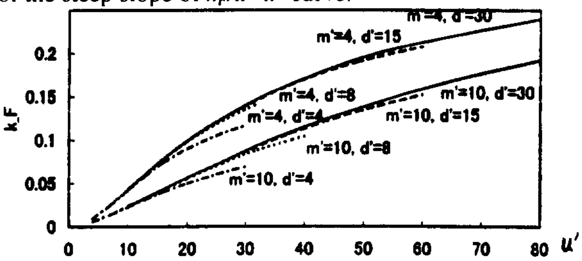

Fig. 4 k_F-u' curves

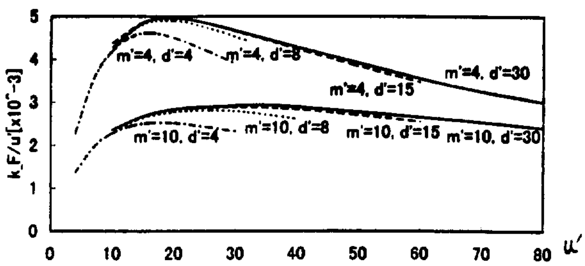

Fig. 5 k_F/u'-u' curves

C. Investigation of V

Fig. 6 shows the model analyzed to investigate the value of V. This corresponds to half of the winding pole pitch of PMVM with the magnet poles aligned to a tooth at one side and unaligned at the other side. The magnetic flux passing through the line v, Φ_V, which corresponds to the maximum magnetic flux linked to a full-pitch coil, is measured. V is calculated from (15).

$$V = \frac{\Phi_V}{qlk_F B_r} \tag{15}$$

where $q(=Z_1 u/4)$ is the horizontal length of the model. k_F is obtained from the k_F-u' curves. Fig. 7 shows the relationship between V and Z_1 with various values of q. It is noted that V is almost constant with Z_1. Similar tendencies are observed with other values of m and d. Therefore, V can be treated to be independent of Z_1, and that brings a remarkable benefit: u_m' indicates the torque maximizing tooth pitch regardless of Z_1.

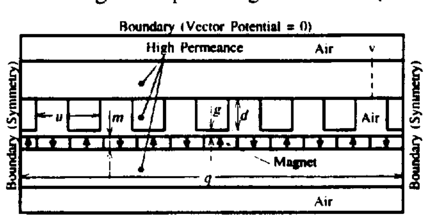

Fig. 6 Analyzed model for measuring V

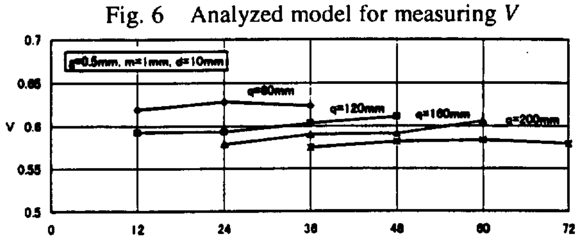

Fig. 7 V-Z_1 curves

V. Design Methodology of PMVM

A. Design Procedure

Based on the above discussion, a torque maximizing design of PMVM can be achieved through a simple procedure as follows.

1) Set the initial condition (machine diameter, stator inner radius, airgap length, coil specs, magnet material, etc.)
2) Set the magnet thickness from the condition to avoid the irreversible demagnetization of the magnet by coil excitation.
3) Set the slot length temporarily and identify the torque maximizing tooth pitch from the k_F/u'-u' curves. Check the slot length being appropriate compared to an index; d=0.4u.
4) Choose Z_1 so that the tooth pitch becomes the closest to the torque maximizing value. Set Z_2 by (1).

B. Case Study

To check the validity of the proposed method, a case study has been carried out. Table 2 is the initial condition of the case study.

Table 2 Initial conditions of the case study

stator	R=44, l=60, g=0.4, outer radius; 60 [mm]
magnet	Br=1.23T, Hc=-8.9×10^{5}A/m
coil	I=4.4A(max. 8.8A), N=100turn

The magnet thickness is set to 1.6mm from the coil specs, and the slot length is chosen as 6mm, i.e., m'=4.0 and d'=15.0. In that case u_m' is about 20 from Fig. 5, and the slot length is sufficiently large. Therefore, the parameters can be determined as follows: Z_1=30, Z_2=29, $u=2\pi R/Z_1$=9.2mm, d=6mm.

Fig. 8 shows Φ and the torque with various values of Z_1 in the PMVM for the case study, acquired both by FEM analyses with machine models and by the calculations with (9) and (14). The calculated values agree very well to the FEM-based ones, and it is noted that the torque becomes the maximum when Z_1=30.

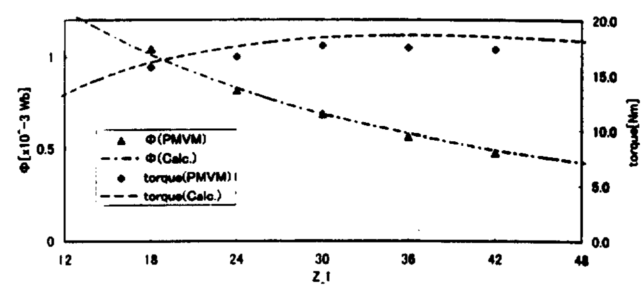

Fig. 8 Results of the case study

VI. Conclusion

A novel torque maximizing design methodology for a PMVM has been presented. Compared to the conventional way, the design policy is clear and the design effort is greatly reduced.

References

[1] J-F. Llibre, D. Matt, "High Performance Vernier Reluctance Magnet Machine. Application to Electric Vehicle," in Proc. EPE '95, 1995, pp.2.889-2.894.