

# A Novel Control Method for Input Output Harmonic Elimination of the PWM Boost Type Rectifier Under Unbalanced Operating Conditions

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**Abstract**-This paper presents a new control strategy to improve the performance of the PWM Boost Type Rectifier when operating under an unbalanced supply. An analytical solution for harmonic elimination under unbalanced input voltages is obtained resulting in a smooth (constant) power flow from ac to dc side in spite of the unbalanced voltage condition. Based on the analysis of the open loop configuration, a closed loop control solution is proposed. Simulation results show excellent response and stable operation of the new rectifier control algorithm. A laboratory prototype has been designed to verify the discussions and analyses done in this paper. Theoretical and experimental results show excellent agreement. Elimination of the possibility of low order ac and dc side harmonics due to unbalance is expected to materially affect the cost of dc link capacitor and ac side filter. The proposed method will be particularly useful in applications where the large second harmonic at the DC link may have a severe impact on system stability of multiply connected converters on a common link.

## I. INTRODUCTION

The Boost Type PWM Rectifier has been increasingly employed in recent years since it offers the possibility of a low distortion line current with unity power factor for any load condition [2] [6]. Another advantage over traditional phase-controlled thyristor rectifiers is its capability for nearly instantaneous reversal of power flow. Unfortunately, the features the PWM Boost Type Rectifier offers are fully realized only when the supply three phase input voltage source is balanced both in terms of its Thevenin equivalent voltage and impedance, which is often not the case in a real power system. It has been shown [1] that unbalanced input voltages or impedances cause an abnormal second harmonic at the dc bus which reflects back to the input causing a (non-zero sequence) third-order harmonic current to flow. Next, the third-order harmonic current causes a fourth-order harmonic voltage on the dc bus, and so on. This results in the appearance of even harmonics at the dc output and odd harmonics in the input currents. These additional components cause added losses in the dc link filter capacitor. They must also be considered in the filter design of these converters since they clearly add to the cost because they are of low frequency. In addition, ripple on the dc link is a known cause of interaction between current regulators which can even cause system instability [7]. The problem increases in severity as the number of converters connected to the link increases. While an attempt was made to reduce low order harmonics at the input and the output of the PWM Boost

Type Rectifier under unbalance [4], harmonics were only reduced but not eliminated. Enjeti and Choudhury [5] proposed a method for the harmonic elimination of the buck ac to dc converter under unbalanced conditions. However, the approach does not lend itself well to the more popular boost type rectifier so that an exact solution for this problem has never been addressed.

This paper proposes a completely new control strategy for harmonic elimination of the PWM Boost Type rectifier operating under unbalanced input voltages. Specifically, by PWM current regulation, the magnitudes and the phase angles of the three input currents are independently adjusted in order to maintain a smooth output dc voltage level entirely eliminating low-order harmonics. The proposed technique maintains a high quality sinusoidal current input and dc current output even though the input voltages remain unbalanced. However, the power factor cannot be adjusted in this case. This is not seen as a disadvantage particularly in situations where imbalance in the input supply is a temporary phenomenon. Simulation as well as experimental results is presented to confirm the proposed control strategy. The theoretical and experimental results show excellent agreement.

## II. THEORETICAL APPROACH

In Fig.1 it is assumed that the PWM rectifier is supplied by unbalanced input voltages but balanced input impedances. The assumptions used in the derivation are:

1. The system is lossless.
2. The switching functions used to represent switching action of the converter are unbalanced but contain no zero sequence.
3. Only fundamental components of switching functions and

$$SW_1, SW_2, SW_3$$

input currents are taken into account (PWM switching harmonics are not considered).

By using symmetrical component theory, line to neutral switching functions are given as:

$$\begin{matrix} SW_1 & 1 & 1 & 1 & 0 \\ SW_2 & = & 1 & a^2 & a & \cdot & S^+ \\ SW_3 & & 1 & a & a^2 & & S^- \end{matrix} \quad (1)$$

Since the zero sequence current is never present in this circuit, currents  $I_1$ ,  $I_2$  and  $I_3$  are given by,

$$\begin{aligned} I_1 &= 1 & 1 & 1 & 0 \\ I_2 &= 1 & a^2 & a & I^+ \\ I_3 &= 1 & a & a^2 & I^- \end{aligned} \quad (2)$$

where  $S^+$  and  $S^-$  are positive and negative sequence switching functions,  $I^+$  and  $I^-$  are positive and negative sequence currents and  $a = 1 \angle 20^\circ$

Output current  $I_0$  is given by,

$$\begin{aligned} I_0 &= SW_1 I_1 + SW_2 I_2 + SW_3 I_3 = (S^+ + S^-)(I^+ + I^-) \\ &+ (a^2 S^+ + a S^-)(a^2 I^+ + a I^-) + (a S^+ + a^2 S^-)(a I^+ + a^2 I^-) \\ &= 3S^+ I^- + 3S^- I^+ \end{aligned} \quad (3)$$

In the time domain, the pulsating component of the output current is expressed as:

$$\begin{aligned} I_0(t) &= 3 \operatorname{Re} \left\{ \left| S^+ \right| e^{j\omega t} e^{j\theta_s^+} \right\} \operatorname{Re} \left\{ \left| I^- \right| e^{j\omega t} e^{j\theta_i^-} \right\} \\ &+ 3 \operatorname{Re} \left\{ \left| S^- \right| e^{j\omega t} e^{j\theta_s^-} \right\} \operatorname{Re} \left\{ \left| I^+ \right| e^{j\omega t} e^{j\theta_i^+} \right\} \end{aligned} \quad (4)$$

By using trigonometric identity:

$$\cos(\alpha)\cos(\beta) = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

one arrives at the result:

$$\begin{aligned} I_o(2\omega t) &= \frac{3}{2} \left| S^+ \right| \left| I^- \right| \cos(2\omega t + \theta_s^+ + \theta_i^-) \\ &+ \frac{3}{2} \left| S^- \right| \left| I^+ \right| \cos(2\omega t + \theta_s^- + \theta_i^+) \end{aligned} \quad (5)$$

The above equation shows the presence of the second-order harmonic at the output of the rectifier. It also indicates that the second-order harmonic could be eliminated under the following conditions.

$$\begin{aligned} 1. \left| S^+ \right| \left| I^- \right| &= \left| S^- \right| \left| I^+ \right| \\ 2. \theta_s^- + \theta_i^+ &= \pi + \theta_s^+ + \theta_i^- \end{aligned} \quad (6)$$

where  $S^+$ ,  $S^-$ ,  $\theta_s^+$  and  $\theta_s^-$  are magnitudes and phase shifts of positive and negative switching functions.  $I^+$ ,  $I^-$ ,  $\theta_i^+$  and  $\theta_i^-$  are magnitudes and phase shifts of input currents.

The per-phase equivalent circuit under unbalanced input voltages and unbalanced synthesized voltages at the input of the rectifier are shown in Fig. 2. From Fig.2, two additional equations can be obtained and given by,

From the power equation, the following relationship can be obtained:

$$ZI^+ = U^+ - V_s^+ \quad (7)$$

$$ZI^- = U^- - V_s^- \quad (8)$$

where  $V_s^+$  and  $V_s^-$  are the positive and negative sequence voltages at the rectifier input and  $U^+$  and  $U^-$  are the positive and negative sequence input voltages.

$$V_s^+ = V_{dc} S^+ / 2\sqrt{2}, V_s^- = V_{dc} S^- / 2\sqrt{2}$$

where  $V_{dc}$  is the output dc voltage.

$$P_{dc} = \operatorname{Real}(3U^+ I^+ + 3U^- I^-) \quad (9)$$

where  $P_{dc}$  is the average output power.

The solution for harmonic elimination is obtained by combining (6), (7), (8) and (9).

The solution is given by the following set of equations:

$$1. \frac{\left| S^+ \right|}{\left| S^- \right|} = \frac{\left| U^+ \right|}{\left| U^- \right|} \quad (10)$$

$$2. \theta_u^- - \theta_s^- = \theta_s^+ - \theta_u^+ \quad (11)$$

$$3. \left| S^+ \right| = \frac{2\sqrt{2} \left| U^+ \right|}{V_{dc}} \cos(\theta_u^+ - \theta_s^+) \quad (12)$$

$$4. \sin(2\theta_u^+ - \theta_s^+) = \frac{2P_{dc} Z}{3 \left( \left| U^+ \right|^2 - \left| U^- \right|^2 \right)} \quad (13)$$

The set of four equations shown above represents the steady-state solution for input-output harmonic elimination of the PWM Boost Type Rectifier under unbalanced input voltages.

#### A. The Physical Meaning of the Proposed Solution in d-q Stationary Frame

The proposed solution for harmonic elimination can be explained in the d-q stationary frame, where cancellation of input

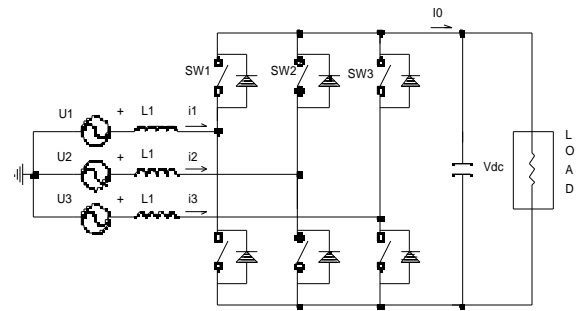


Fig.1 The PWM Boost Type Rectifier

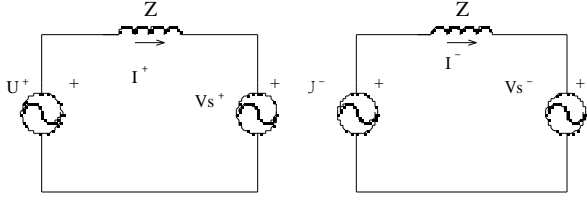


Fig.2 Per-phase positive and negative sequence equivalent circuit

pulsating power becomes more obvious. The positive and negative sequence components of the input voltages and currents are instantaneous quantities and are shown on Fig.3. The instantaneous power input,  $p(t)$  is given by:

$$p(t) = p_0(t) + p_1(t) + p_2(t) + p_3(t) \quad (14)$$

(14)

where,

$$p_0(t) = 3/2(u_{pq}i_{pq} + u_{pd}i_{pd} + u_{nq}i_{nq} + u_{nd}i_{nd}) \quad (15)$$

(15)

$$p_1(t) = 3/2(u_{pq}i_{nq} + u_{nq}i_{pq} + u_{pd}i_{nd} + u_{nd}i_{pd}) \quad (16)$$

(16)

$$p_2(t) = 3/2(u_{pd}i_{pq} - u_{pq}i_{pd}) \quad (17)$$

(17)

$$p_3(t) = 3/2(u_{nd}i_{nq} - u_{nq}i_{nd}) \quad (18)$$

(18)

The first term,  $p_0(t)$ , represents the constant portion of input pulsating power, since all its components are products of in-phase q and d quantities of the positive and negative sequence input voltages and currents. This term also exists in a balanced three-phase system.

The second term,  $p_1(t)$ , represents the pulsating portion, since all its components are products of the positive and negative in-phase components of input currents and voltages. It basically represents a mutual interaction between positive and negative sequence quantities, which normally does not exist in a balanced three-phase system.

The third term,  $p_2(t)$ , represents the interaction between d and q positive sequence quantities. This term exists in a balanced three-phase system.

The fourth term,  $p_3(t)$ , represents the interaction between d and q negative sequence quantities. This term does not normally exist in a balanced three-phase system.

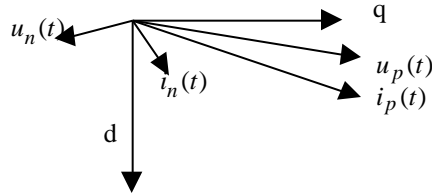


Fig. 3 The positive and negative sequence input voltages and currents in d-q stationary frame

Similarly, instantaneous power on the inductor,  $p_l(t)$ , consists of three parts and can be expressed as:

$$p_{1l}(t) = 3/2 \omega L (2i_{pd}i_{nq} - 2i_{pq}i_{nd}) \quad (19)$$

(19)

$$p_{2l}(t) = 3/2 \omega L (i_{pq}i_{pq} + i_{pd}i_{pd}) \quad (20)$$

(20)

$$p_{3l}(t) = 3/2 \omega L (i_{nq}i_{nq} + i_{nd}i_{nd}) \quad (21)$$

(21)

The solution for second harmonic elimination assumes the following form:

$$1. p_1(t) = p_{1l}(t) \quad (22)$$

(22)

$$2. p_2(t) = p_{2l}(t) \quad (23)$$

(23)

$$3. p_3(t) = p_{3l}(t) \quad (24)$$

(24)

The power going into the converter is given by,

$$p_c(t) = p(t) - p_l(t) \quad (25)$$

(25)

The power going into the converter, as shown in Fig.4, is constant since all the other components are shown to cancel out.

### III. CONTROL METHOD

Based on the analysis of the open loop configuration presented above, a feed-forward control method is proposed. In order to control the output dc voltage and eliminate harmonics at the input and output of the PWM Boost Type Rectifier under unbalance, not only current magnitudes but also their phase angles have to be controlled. The DC bus error is used to synthesize the magnitudes and the phase angles of the positive and negative sequence reference currents. The sequence components are then transformed into three phase (abc) quantities which become reference signals for the hysteresis controller [3].

The relationship between input currents and input voltages in the open loop configuration for harmonic elimination is obtained by combining (7), (8), (10) and (11) and given by,

$$\frac{|U^+|}{|U^-|} = \frac{|I^+|}{|I^-|} \quad (26)$$

(26)

$$\theta_u^+ - \theta_i^+ = \theta_i^- - \theta_u^- - \pi \quad (27)$$

(27)

However, it is always true that average power input is equal to average power output and given by the following equation,

$$P_{dc} = V_{dc} I_{dc} = 3|U^+||I^+|\cos(\theta_u^+ - \theta_i^+) + 3|U^-||I^-|\cos(\theta_u^- - \theta_i^-) \quad (28)$$

(28)

The output DC voltage is proportional to the positive and negative sequence currents (magnitudes), so is the error signal ( $V_{ref} - V_{dc}$ ). In order to satisfy (26), the condition for second harmonic elimination, the positive and negative sequence commands for current magnitudes should satisfy (29) and (30).

$$|I^+| = K_p |U^+| (V_{ref} - V_{dc})$$

(29)

$$|I^-| = K_p |U^-| (V_{ref} - V_{dc})$$

(30)

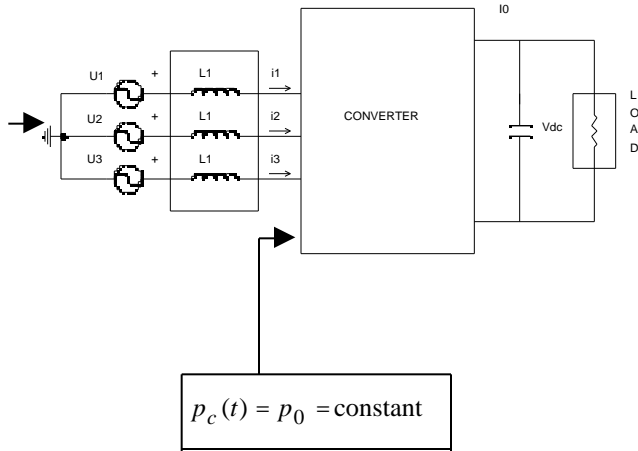
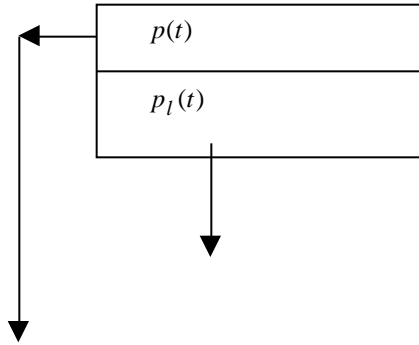


Figure 4. Cancellation of the instantaneous pulsating power

Equations (29) and (30) represent positive and negative sequence magnitude commands. The positive and negative sequence commands for phase angles are derived from (27) and (28). Therefore, the phase angle commands are given by,

$$\theta_i^+ = \theta_u^+ - K_{p1} a \cos(V_{ref} - V_{dc})$$

(31)

$$\theta_i^- = \theta_u^- + \pi + K_{p1} a \cos(V_{ref} - V_{dc})$$

(32)

where acos is an inverse cosine function.

Equations (31) and (32) are approximated by the following two equations:

$$\theta_i^+ = \theta_u^+ - K_{p1} (V_{ref} - V_{dc})$$

(33)

$$\theta_i^- = \theta_u^- + \pi + K_{p1} (V_{ref} - V_{dc})$$

(34)

Positive and negative sequence current commands are transformed to abc quantities and used as references for hysteresis controller. The transformation is given by,

$$i_{1ref}(t) = |I^+| \sin(\omega t + \theta_i^+) + |I^-| \sin(\omega t + \theta_i^-)$$

(35)

$$i_{2ref}(t) = |I^-| \sin(\omega t + \theta_i^+ - \frac{2\pi}{3}) + |I^-| \sin(\omega t + \theta_i^- + \frac{2\pi}{3})$$

(36)

$$i_{3ref}(t) = |I^+| \sin(\omega t + \theta_i^+ + \frac{2\pi}{3}) + |I^-| \sin(\omega t + \theta_i^- - \frac{2\pi}{3})$$

(37)

Only two proportional controllers are utilized (one for the magnitude, the other for phase angle control) which has been determined to be sufficient for good regulation. An integral controller can be added to further reduce the steady state error. The proposed control method is shown in more detail in Figure 5.

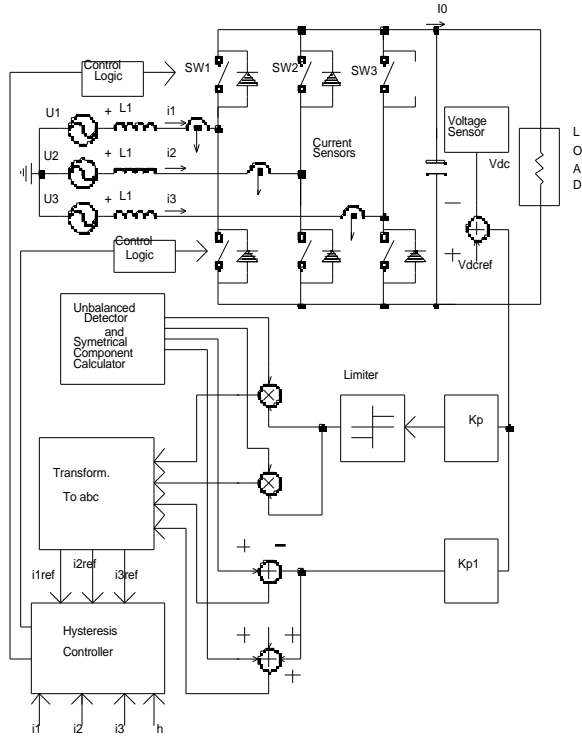


Figure 5. The proposed control loop solution under unbalanced input voltages.

#### IV SIMULATION AND EXPERIMENTAL RESULTS

In order to demonstrate feasibility, the system shown in Fig. 5 has been simulated in SABER. The parameters used for the simulation are shown in Table I.

TABLE I  
Parameters used in simulation

Parameter	Value	Parameter	Value
Input supply peak voltages	$U_1 = 100V$	DC-link capacitor, $C$	$100\mu F$
	$U_2 = 105V$		
	$U_3 = 104V$		

DC-link voltage, $V_{dc}$	270V	Output resistive load, $R$	100
Input, $L$ inductances	1mH	Fundamental frequency, $f$	50Hz

The plot in Fig.6 shows the controlled output dc voltage under unbalanced input voltages. It also gives the current commands for the hysteresis controller.

Good control of the dc voltage is evident in spite of the phase unbalance. A laboratory prototype was built to confirm the theoretical results. The bridge consists of six IGBT switches and six diodes connected in parallel. The control circuit consists of: voltage sensor board, current sensor board,

hysteresis board, gate driver board and DSP56000 development system, as shown in Fig.7. CPU runs at 27MHz and is connected to an IBM personal computer. The zero voltage detector along with one counter (counter (1)) which runs at 2MHz, measures the frequency of the system and synchronizes the control functions (current commands) for D/A converter.

The output of the zero voltage detector, as shown in Fig.7, is connected to the interrupt line (priority level 1) and to the gate of the counter. On the falling edge of the input signal, the output of the zero crossing detector, counter (1) stops counting. Since the program jumps into interrupt subroutine at that point (IRQ1), the second counter (counter (2)), which runs at 8Mhz, gets loaded with the value measured by counter (1) divided by 255. In other words, the second counter is loaded by  $T/255$  which represents the sampling time of the system. Since the frequency of the network may vary, the appearance of the second interrupt may vary accordingly. The hysteresis board consists of three comparators and three inverters. The inputs to the three comparators, as shown in Fig.7, come from the current sensor board and the outputs of the D/A board. The software has been written in assembly language and consists of a main loop and two levels of interruption loops.

The main structure of the program is shown in Fig.8. The first level of interruption loop (IRQ1) occurs once in a cycle. It detects the zero points of the input voltage. This subroutine sets the value in counter (2) to be  $T/255$ . It also reads the instantaneous values of the three input voltages and calculates variables necessary for calculating the sequence components of the three input voltages. At the end of this subroutine current commands are sent to the D/A converter. Fig.9 shows the simplified flowchart of the loop IRQ1. The second level of the interruption loop (IRQ2) occurs 255 times a cycle. It measures the input voltages and calculates variables necessary for calculating the sequence components. At the end of this subroutine, the commands for three input currents are sent to D/A converters. Fig.10 shows the simplified flowchart of the loop IRQ2.

The main loop (MAIN) occurs once a cycle and calculates the sequence components of the three input voltages based on measurements and calculations performed in two interruption loops. It also measures the output dc voltage and calculates the error between its set and measured value. Finally, it calculates the sequence commands for the three input currents. Fig.11 shows the simplified flowchart of the loop MAIN.

Experimental results are shown on Fig.12, Fig.13, and Fig.14. Fig.12 shows input currents in phase 1 and phase under unbalanced input voltages. The currents are unbalanced, as expected, to cancel the pulsating power coming from the input. The currents are sinusoidal and do not contain low-order harmonics. Fig. 13 shows the output dc voltage of the PWM Boost Type Rectifier under unbalanced supply voltages. Fig.14 shows the frequency spectrum of the output DC voltage. Phase 2 is significantly unbalanced (by 50%) with respect to other two phases. In spite of the level of unbalance, the output voltage is smooth and contains no low order harmonics.

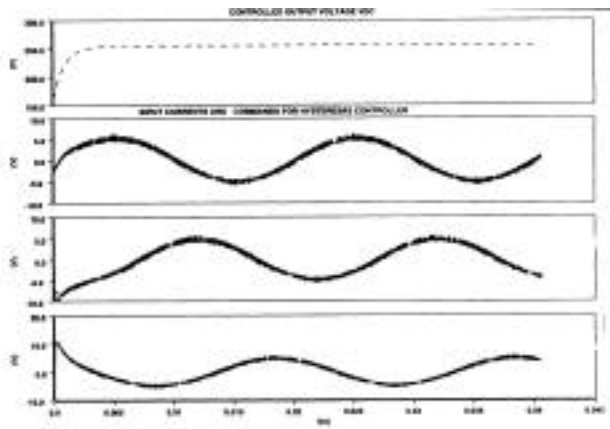


Fig. 6. Controlled output voltage, input currents and commands for the hysteresis controllers (simulation results).

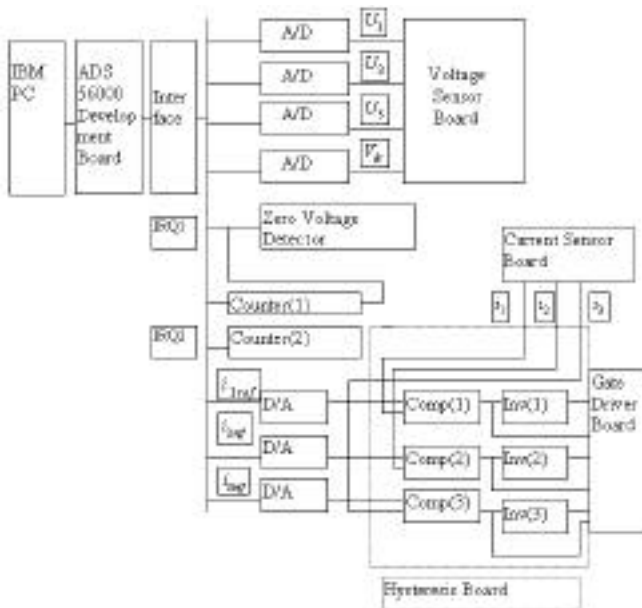


Figure 7. Hardware configuration

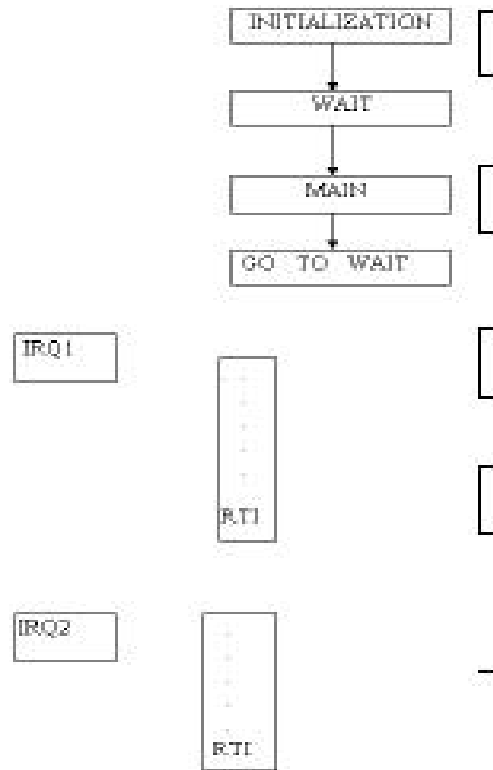


Figure 8. The main structure of the program

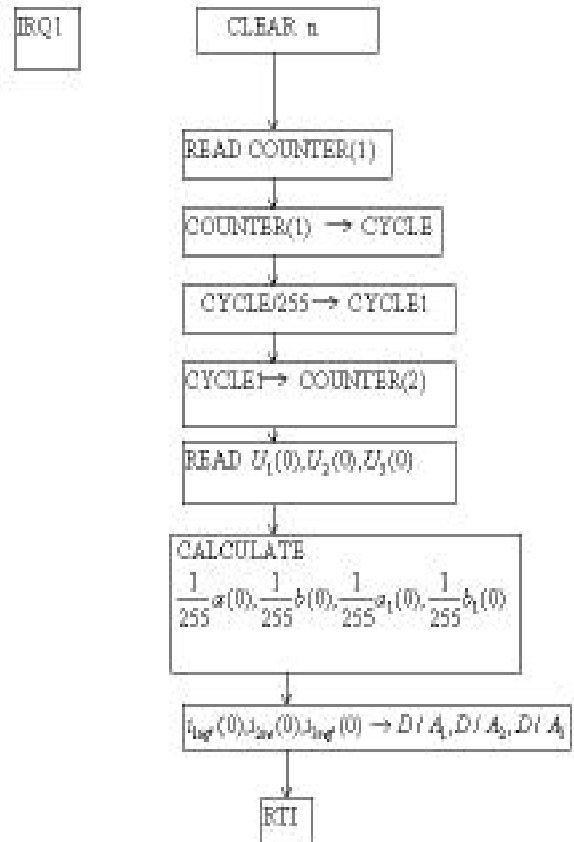


Figure 9. Flowchart of interruption loop (IRQ1)

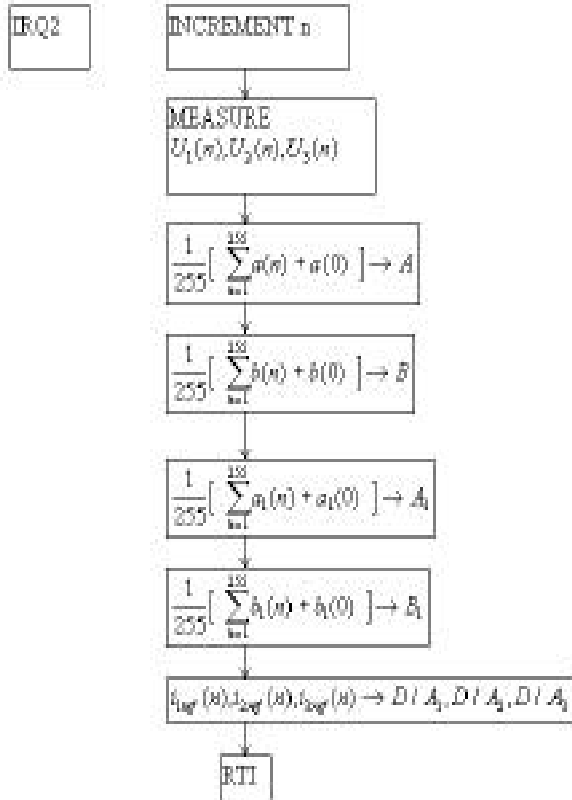


Figure 10. Flowchart of interruption loop (IRQ2)

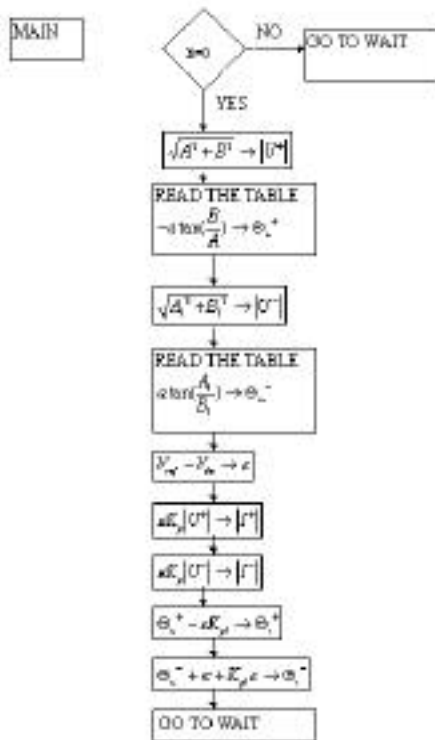


Figure 11. Flowchart of the main loop (MAIN)

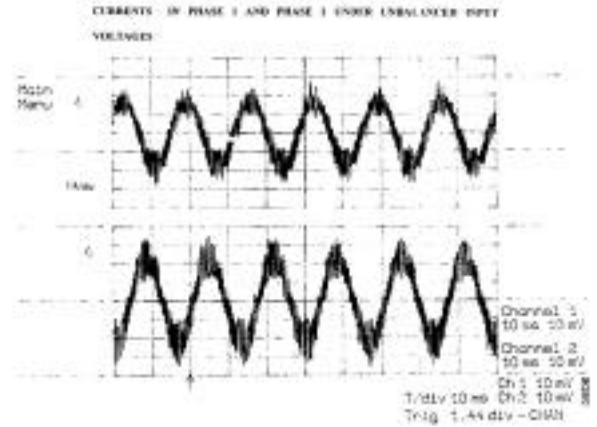


Figure 12. Currents in phase 1 and 2 under unbalanced input voltages (experimental results)

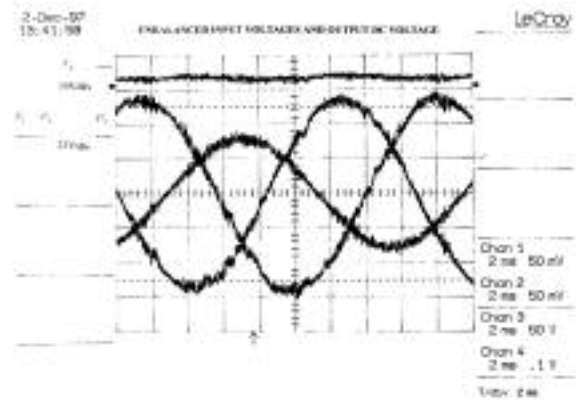


Figure 13. Controlled output dc voltage and three input voltages (experimental results)

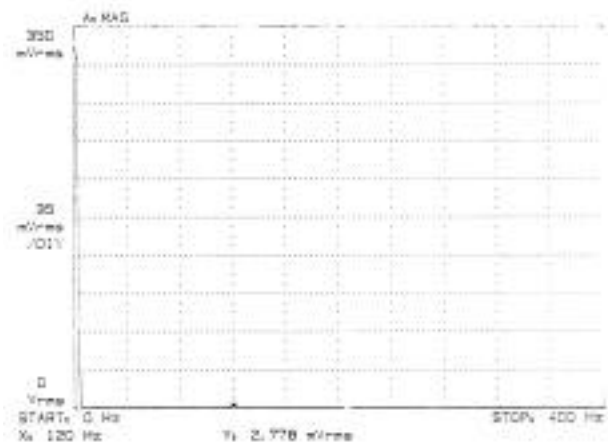


Figure 14. Frequency spectrum of the output dc voltage (experimental results)

## V. CONCLUSIONS

This paper proposes a new control strategy to improve the performance of the PWM Boost Type Rectifier under unbalanced operating conditions. An analytical solution for

harmonic elimination under unbalanced input voltages and balanced input impedances is obtained. Based on analysis of the open loop configuration, a closed loop solution is proposed. With the proposed technique, high quality input and output waveforms are maintained under unbalanced input voltages and balanced input impedances. The simulation results show the excellent response and stable operation of the PWM Boost Type Rectifier under unbalanced operating conditions when this method is applied. The laboratory prototype has been designed to verify the discussions and analyses done in this paper. An excellent agreement occurs between the theoretical and experimental results.

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