A Direct Approach to Electrical Machine Performance Evaluation: Torque Density Assessment and Sizing Optimization

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***Abstract* – Electrical machine design should be approached as a system optimization, more than a simple machine sizing when the machines are fed by a power converter. A great variety of electrical machines are available to accomplish this goal, and the task of comparing different options can be very difficult. A general sizing equation would be a very attractive tool which can easily be applied to every radial flux machines (RFM), axial flux machines (AFM) and transverse flux machines (TFM) and which takes different waveforms and machine characteristics into account. In this paper, a general approach is presented to summarize and interpret such an equation for radial, axial and transverse motors. Radial flux surface mounted PM machine and non-slotted TORUS surface mounted PM machine are used as sample applications. Optimum machine design for high power/torque density, high efficiency are achieved and illustrated in the paper for the non-slotted TORUS machines. Furthermore, the sizing equations are developed to compare six different 200 HP 6 pole 1200 rpm radial and axial flux machines in terms of torque/power densities, efficiencies, utilization, heat dissipation and weight.**

1. Introduction

Comparison of various electrical machines is a very challenging activity since many variables exist for each machine type. One of the methods to compare the electrical machines is the $D_g^2 L_e$ sizing equation that utilizes machine power based on the airgap volume. $D_o^3 L_e$ sizing equations developed by Honsinger [1] use machine outer diameter since it is directly related to the volume, consequently cost and size of the machine. This approach put the emphasis on choosing the correct electrical loading for the machines. However, it provides reasonable results only for small pole numbers.

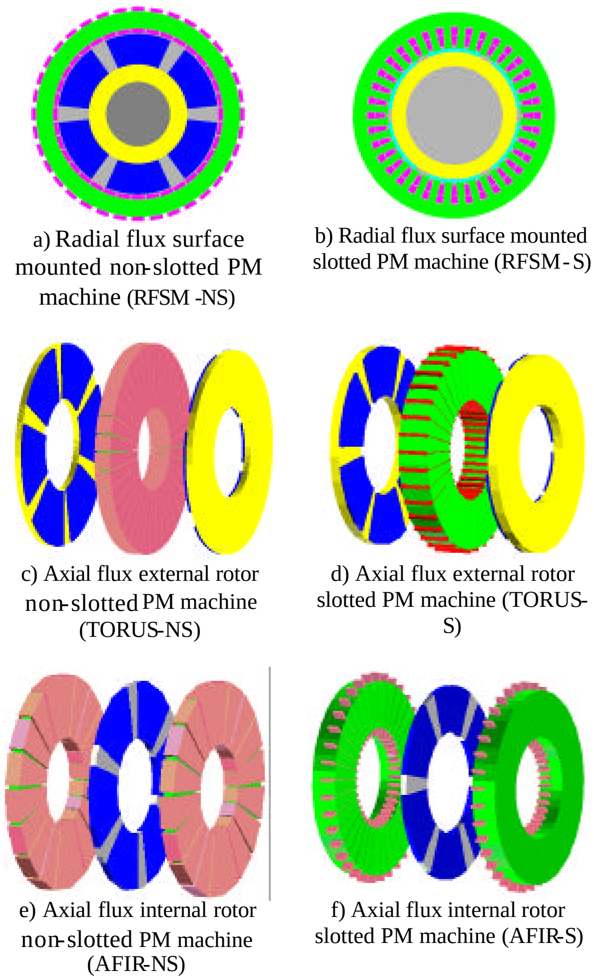
The aim of this paper is to develop and perfect the principles, analysis and calculation methods for both slotted and non-slotted permanent magnet motors. For this purpose, a generalized sizing approach [2-4] for all RFM, AFM and TFM machines are developed so as to eliminate the drawbacks of the traditional sizing equations mentioned earlier. Special factors accounting for the effect of non-sinusoidal current and back EMF waveforms are introduced in this paper. Applications of the developed sizing equations are utilized to analyze the sizing of both radial flux surface mounted PM machine and axial flux surface mounted TORUS type PM disc machine. Optimization of the slotless TORUS machine is provided for maximum power density and maximum efficiency points. Finally, the generalized sizing equations are used to compare six different slotted and non-slotted radial and axial flux surface mounted PM machines in terms of torque/power density, efficiency, utilization, heat dissipation and weight using generalized sizing equations [5-6]. Traditional radial flux surface mounted PM machine has been chosen as a reference machine. Six RFM and AFM and their acronyms are selected for detailed analysis and tabulated in Table I.

TABLE I
RADIAL AND AXIAL FLUX SURFACE MOUNTED PM MACHINES ANALYZED

Abbr.	**Radial and Axial Flux Surface Magnet PM Machine Types**
RFSM-NS	Radial flux surface mounted Non-slotted PM machine
RFSM-S	Radial flux surface mounted slotted PM machine (or conventional PM machine)
TORUS-NS	Axial flux non-slotted external rotor internal stator PM machine
TORUS-S	Axial flux slotted external rotor internal stator PM machine
AFIR-NS	Axial flux non-slotted internal rotor external stator PM machine
AFIR-S	Axial flux slotted internal rotor external stator PM machine

2. Analyzed Machine Structures

Six different 200 HP, 6 pole, 1200 rpm radial and axial flux permanent magnet machines have been analyzed in a recent study [7-9]. Two radial flux machines, namely radial flux surface mounted non-slotted PM machine (RFSM-NS) and radial flux surface mounted slotted PM machine (RFSM-S) were investigated. The rest of the machine types were axial flux surface mounted non-slotted and slotted one-stator-two-rotor (TORUS) and two-stator-one-rotor (AFIR) type PM machines. In general, axial flux disc motors have N stators and N+1 rotors (N≥1) for external rotor and internal stator disc motor (TORUS) types and N+1 stators and N rotors (N≥1) for internal rotor and external stator disc motor (AFIR) types. The topologies used in the study are illustrated in Figure 1.



a) Radial flux surface mounted non-slotted PM machine (RFSM -NS)

b) Radial flux surface mounted slotted PM machine (RFSM - S)

c) Axial flux external rotor non-slotted PM machine (TORUS-NS)

d) Axial flux external rotor slotted PM machine (TORUS-S)

e) Axial flux internal rotor non-slotted PM machine (AFIR-NS)

f) Axial flux internal rotor slotted PM machine (AFIR-S)

Fig. 1. Radial and axial flux surface mounted PM machines analyzed in this study

3. Sizing Equation Analysis and Torque Density

In general, if stator leakage inductance and resistance are neglected, the output power for any electrical machine can be expressed as

$$P_R = \eta \frac{m}{T}\int_0^T e(t)\, i(t)\, dt = \eta m K_p E_{PK} I_{PK} \qquad (1)$$

where $e(t)$ and E_{pk} are phase air gap EMF and its peak value, $i(t)$ and I_{pk} are phase current and the peak phase current, h is machine efficiency, m is number of phases of the machine and T is period of one cycle of the EMF.

The quantity K_p is termed the electrical power waveform factor and defined as

$$K_p = \frac{1}{T}\int_0^T \frac{e(t)\times i(t)}{E_{pk}\times I_{pk}}\, dt = \frac{1}{T}\int_0^T f_e(t)\, f_i(t)\, dt \qquad (2)$$

where $f_e(t)=e(t)/E_{pk}$ and $f_i(t)=i(t)/I_{pk}$ are the expressions for the normalized EMF and current waveforms. In order to indicate the effect of the current waveform, a definition for current waveform factor, K_i, is also useful,

$$K_i = \frac{I_{PK}}{I_{rms}} = \left[\frac{1}{T}\int_0^T \left(\frac{i(t)}{I_{PK}}\right)^2 dt\right]^{-1/2} \qquad (3)$$

where I_{rms} is the rms value of the phase current. The peak value of the phase airgap EMF in (1) is given by

$$E_{PK} = \begin{cases} K_e N_t B_g \dfrac{f}{p}\lambda_o D_o L_e & \text{for RFM} \\ K_e N_t B_g \dfrac{f}{p}(1-\lambda^2) D_o^2 & \text{for AFM} \\ K_e N_t B_g \dfrac{f}{p}\lambda_o D_o L_e & \text{for TFM} \end{cases} \qquad (4)$$

where K_e is the EMF factor which incorporates the winding distribution factor K_w and the per unit portion of the total air gap area spanned by the salient poles of the machine (if any), N_t is the number of turn per phase, B_g is the flux density in the airgap, f is the converter frequency, p is the machine pole pairs, λ_o is the diameter ratio for RFM defined as D_g/D_o, λ is the diameter ratio for AFM defined as D_i/D_o, D_o is the diameter of the machine outer surface, D_g is the diameter of the machine airgap surface, D_i is the diameter of the machine inner surface and L_e is the effective stack length of the machine.

The peak phase current in (1) is given by

$$I_{PK} = \begin{cases} \dfrac{1}{1+K_\phi} K_i A \pi \lambda_o \dfrac{D_o}{2 m_1 N_t} & \text{for RFM} \\ \dfrac{1}{1+K_\phi} K_i A \pi \dfrac{1+\lambda}{2} \dfrac{D_o}{2 m_1 N_t} & \text{for AFM} \\ \dfrac{1}{1+K_\phi} K_i A \dfrac{L_e}{2 N_t} & \text{for TFM} \end{cases} \qquad (5)$$

where A is the total electrical loading in. $K_\phi = A_r / A_s$ is the ratio of electrical loading on rotor and stator. In a machine topology without a rotor winding, $K_\phi = 0$. In general case, the total electrical loading, A, should include both the stator electrical loading A_s and rotor electrical loading A_r.

Combining (1) through (5), the general purpose sizing equations take the following form for RFM, AFM and TFM.

For RFM,

$$P_R = \begin{cases} \dfrac{1}{1+K_\phi}\dfrac{m}{m_1}\dfrac{\pi}{2} K_e K_i K_p \eta B_g A \dfrac{f}{p}\lambda_o^2 D_o^2 L_e \\ \dfrac{1}{1+K_\phi}\dfrac{m}{m_1}\dfrac{\pi}{2} K_e K_i K_p K_l \eta B_g A \dfrac{f}{p}\lambda_o^2 D_o^3 \end{cases} \qquad (6)$$

for AFM,

$$P_R = \begin{cases} \frac{1}{1+K_\phi}\frac{m}{m_1}\frac{\pi}{2}K_e K_i K_p K_L \eta B_g A \frac{f}{p}(1-\lambda^2)\frac{1+\lambda}{2}D_o^2 L_e \\ \frac{1}{1+K_\phi}\frac{m}{m_1}\frac{\pi}{2}K_e K_i K_p \eta B_g A \frac{f}{p}(1-\lambda^2)\frac{1+\lambda}{2}D_o^3 \end{cases} \quad (7)$$

and for TFM,

$$P_R = \begin{cases} \frac{1}{1+K_\phi}\frac{m}{2}K_e K_i K_p K_l \eta B_g A \frac{f}{p}\lambda_o^2 D_o^2 L_e \\ \frac{1}{1+K_\phi}\frac{m}{2}K_e K_i K_p K_l^2 \eta B_g A \frac{f}{p}\lambda_o^3 D_o^3 \end{cases} \quad (8)$$

where $K_l = L_e / D_g$ is the aspect ratio coefficient for the RFM and the TFM and $K_L = D_o / L_e$ is the aspect ratio coefficient for AFM.

The machine torque density and power density for the total volume can be defined as

$$T_{den.} = \frac{T_R}{\frac{p}{4}D_{tot}^2 L_{tot}} = \frac{P_R}{\Omega_r \frac{p}{4}D_{tot}^2 L_{tot}} \quad (9)$$

$$P_{den.} = \frac{P_R}{\frac{p}{4}D_{tot}^2 L_{tot}} \quad (10)$$

where T_R is the rated torque of the machine, D_{tot} is the total machine outer diameter including the stack outer diameter and the protrusion of the end winding from the iron stack in the radial direction, L_{tot} is the total length of the machine including the stack length and the protrusion of the end winding from the iron stack in the axial direction, Ω_r is the rotor angular speed.

4. Application of Sizing Equation for Radial and Axial Flux Surface Mounted PM Machines

A. Application of Sizing Equations for Slotted Radial Flux Surface Mounted PM Machines

Generalized sizing equation approach can directly be applied to radial flux surface mounted PM machines. The outer surface diameter D_o can be written as

$$D_o = (P_R / \frac{1}{1+K_\phi}\frac{m}{m_1}\frac{\pi}{2}K_e K_i K_p K_l \eta B_g A \frac{f}{p}\lambda_o^2)^{1/3} \quad (11)$$

The total length of the machine is

$$L_{tot} = L_e + L_{ext} \quad (12)$$

where L_{ext} is the protrusion of the end winding from the iron stack in axial direction. The aspect ratio coefficient can be defined as

$$K_l = \frac{L_e}{D_g} \quad (13)$$

should be chosen based upon practical requirement of the application which has significant effect on the characteristics of the radial flux machine. Hence, in order to optimize the machine performance, the aspect ratio coefficient K_l has to be chosen carefully. To realize the required $D_o^2 L_e$ sizing equation, it is useful to understand the ratio

$$\lambda_o = \frac{D_g}{D_o} = f(d_{ss}, d_{cs}, D_g, p) \quad (14)$$

where d_{ss} is the depth of the stator slot, d_{cs} is the depth of the stator core. In practice, the depth d_{ss} and d_{cs} depend on the stator electrical loading A_s, the current density J_s, the slot fill factor K_{cu}, and the flux densities in the iron core and teeth, and the airgap surface diameter D_g is determined by the aspect ratio coefficient K_l.

B. Application of Sizing Equations for Slotless TORUS Type PM Disc Machines

The concept of the axial flux permanent magnet TORUS type machine was presented in previous studies [10-12]. The generalized sizing equation approach can easily be applied to axial flux permanent magnet non-slotted TORUS type machine (TORUS-NS). The outer surface diameter D_o can be written as

$$D_o = (P_R / \frac{1}{1+K_\phi}\frac{m}{m_1}\frac{\pi}{2}K_e K_i K_p \eta B_g A \frac{f}{p}(1-\lambda^2)\frac{1+\lambda}{2})^{1/3} \quad (15)$$

The machine total outer diameter D_t for the TORUS-NS machine is given by

$$D_t = D_o + 2W_{cu} \quad (16)$$

where W_{cu} is the protrusion of the end winding from the iron stack in the radial direction. For the back-to-back wrapped winding, protrusions exist toward the axis of the machine as well as towards the outsides and can be calculated as

$$W_{cu} = \frac{D_i - \sqrt{\left(D_i^2 - 2A_s D_g\right)/K_{cu} J_s}}{2} \quad (17)$$

where J_s is the current density and K_{cu} is the copper fill factor.

The axial length of the machine L_e is given by

$$L_e = L_s + 2L_r + 2g \quad (18)$$

where L_s is axial length of the stator, L_r is axial length of the rotor and g is the air gap length. The axial length of the stator L_s is

$$L_s = L_{cs} + 2\,W_{cu} \quad (19)$$

The axial length of the stator core L_{cs} can be written as

$$L_{cs} = \frac{B_g \alpha_p \pi D_o (1+\lambda)}{B_{cs}\, 4p} \quad (20)$$

where B_{cs} is the flux density in the stator core and α_p is the ratio of average airgap flux density to peak airgap flux density.

The axial length of rotor L_r becomes

$$L_r = L_{cr} + L_{PM} \tag{21}$$

Also, the axial length of the rotor core L_{cr} is

$$L_{cr} = \frac{B_u \, \pi \, D_o (1+\lambda)}{B_{cr} \, 8p} \tag{22}$$

where B_{cr} is the flux density in the rotor disc core, and B_u is the attainable flux density on the surface of the PM.

The PM length L_{PM} can be calculated as

$$L_{PM} = \frac{\mu_r B_g}{B_r - \frac{K_f}{K_d} B_u} (g + W_{cu}) \tag{23}$$

where μ_r is the recoil relative permeability of the magnet, B_r is the residual flux density of the PM material, K_d is the leakage flux factor and $K_f = B_{gpk}/B_g$ is the peak value corrected factor of air-gap flux density in radial direction of the disc motor. These factors can be obtained using finite element analysis. Figure 2 shows how the airgap flux density of the slotless TORUS machine changes over one pole as the diameter varies from inner diameter D_i to outer diameter D_o.

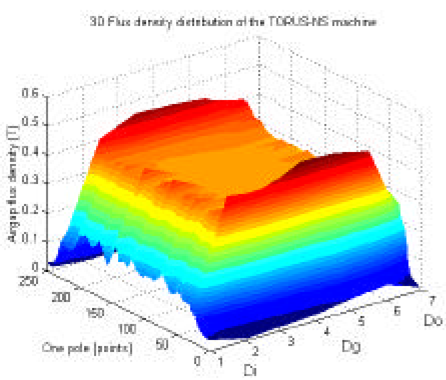


Fig. 2. No load airgap flux density distribution of the TORUS-NS machine obtained from FEA (at D_i, D_g and D_o)

Combining (17) through (23), the axial length L_e becomes

$$L_e = \frac{\pi D_o (1+\lambda)}{4p} \left(\frac{\alpha_p B_g}{B_{cs}} + \frac{B_u}{B_{cr}} \right) + (2W_{cu} + 2g) \times \left(1 + \frac{\mu_r B_g}{B_r - \frac{K_f}{K_d} B_u} \right) \tag{24}$$

In order to optimize the performance of the axial flux machines, the ratio λ and the airgap flux density must be chosen carefully since the diameter ratio, λ, and airgap flux density are design parameters which have significant effect on the characteristics of the machine. In Figure 3, power density is shown as a function of airgap flux density and the ratio λ for TORUS-NS machine.

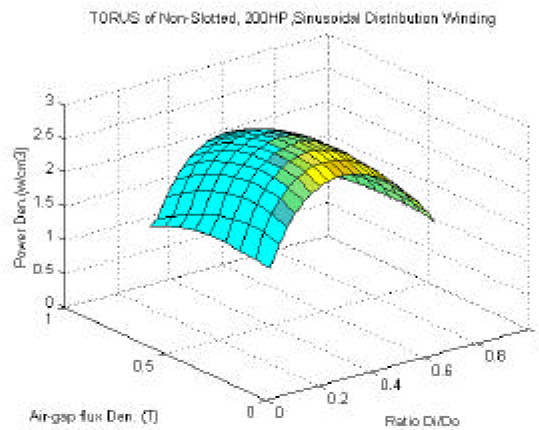


Fig. 3. Power density of TORUS-NS machine as functions of airgap flux density (B_g) and diameter ratio (λ), P_R=200 *HP*, n_s=1200 *rpm*, p=3, A=600 *A/cm*, J_s=9.0 *A/mm*^{2}

From this plot, the maximum power density occurs at an airgap flux density of 0.43 T and the diameter ratio of λ=0.454. For that maximum power density point, the machine efficiency is 95.1% and close to the maximum efficiency point, namely 95.6%. It should be mentioned that the machine efficiency does not change significantly as the ratio λ changes. Furthermore, optimization of the TORUS-NS machine for maximum efficiency point is shown in Figure 4. It shows the maximum efficiency plot as a function of the airgap flux density and the ratio λ. From this plot, the maximum efficiency, which is 95.6%, occurs at a power density of 2.359 W/cm^3 and a ratio λ of 0.517. The airgap flux density at the maximum efficiency point is 0.80 T. It should be mentioned that copper loss, stator core loss and rotor mechanical loss are included in the efficiency calculation. The results obtained for both maximum power density and maximum efficiency points are tabulated in Tables II and III.

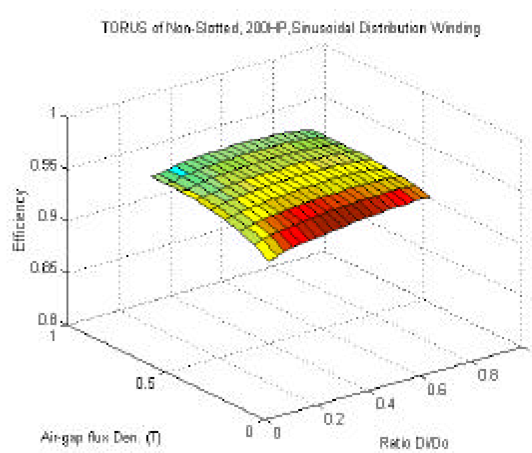


Fig. 4. Efficiency of TORUS-NS machine as functions of airgap flux density (B_g) and diameter ratio (λ), P_R=200 *HP*, n_s=1200 *rpm*, p=3, A=600 *A/cm*, J_s=9.0 *A/mm*^{2}

TABLE II
OPTIMIZATION OF TORUS-NS MACHINE FOR MAXIMUM POWER DENSITY POINT

Maximum power density (MPD):	$P_{d\ max}$= 2.77 W/cm^3
Diameter ratio (λ) at MPD point:	D_i/D_o=0.454
Airgap flux density at MPD point:	B_g=0.43 T
Efficiency at MPD point:	η = 95.1%

TABLE III
OPTIMIZATION OF TORUS-NS MACHINE FOR MAXIMUM EFFICIENCY POINT

Maximum Efficiency (ME): Power density at ME point:	η_{max} = 95.6% P_{den} = 2.359 W/cm^3
Diameter ratio (λ) at ME point:	D_i/D_o=0.517
Airgap flux density at ME point:	B_g=0.80 T

5. Comparison of Radial and Axial Flux Surface Mounted PM Machines

Comparison of six different radial and axial flux PM machines in terms of power/torque density, efficiency, heat dissipation, weight and utilization factor is accomplished for 200 HP, 1200 rpm, 6 pole PM drives. Conventional radial flux surface mounted PM machine has been chosen as a reference machine. For comparison purposes, the density ratio is defined as

$$Densityratio=\frac{T_{den.}}{T_{den.-RFSM}}=\frac{P_{den.}}{P_{den.-RFSM}} \qquad (25)$$

where $T_{den.\text{-}RFSM}$ and $P_{den.\text{-}RFSM}$ are the torque density and power density of the RFSM machine respectively.

Also, the utilization factor is given by,

$$C_W=\frac{P_R(kW)\,\eta}{W_{eight}\,(Kg)}(kW/Kg) \qquad (26)$$

To consider the temperature rise, one can introduce a "heat dissipation" factor defined as

$$\text{Heat\_diss}=\begin{cases}\dfrac{\text{Copper\_loss}+\text{Iron\_loss}}{\pi D_o L_{tot}} & \text{for RFSM}\\[2ex] \dfrac{\text{Copper\_loss}+\text{Iron\_loss}}{\pi(D_o+D_i)L_s+\pi(D_o^2-D_i^2)/2} & \text{for TORUS}\\[2ex] \dfrac{\text{Copper\_loss}+\text{Iron\_loss}}{2\pi(D_o+D_i)L_s+\pi(D_o^2-D_i^2)} & \text{for AFIR}\end{cases} \qquad (27)$$

The summary of the machine comparison is given in Figures 5 through 10.

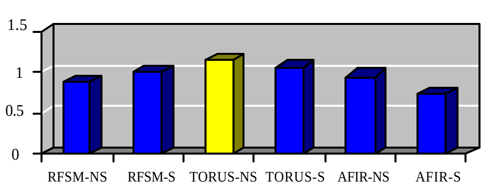


Fig. 5. Power/Torque density comparison [ratio]

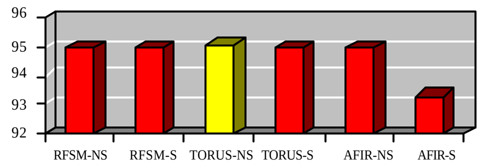


Fig. 6. Efficiency comparison [%]

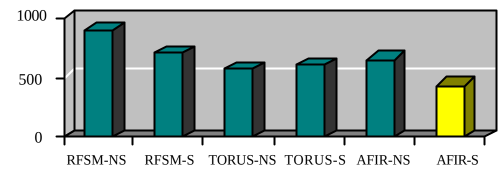


Fig. 7. Weight comparison [lb]

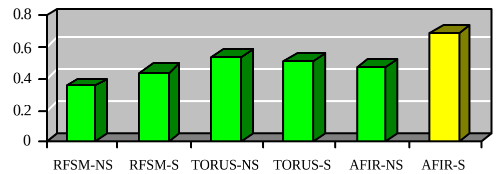


Fig. 8. Utilization factor comparison [kW/Kg]

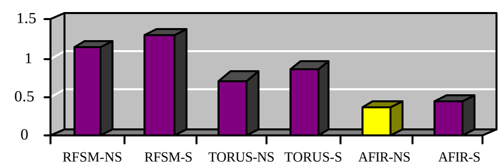


Fig. 9. Heat dissipation comparison [w/cm^2]

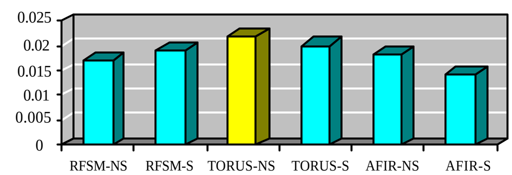


Fig. 10. Torque density comparison [Nm/cm ^{3}]

From this information and data gathered, following conclusions can be obtained:

- TORUS type machines are better than the other machines in terms of power/torque density and efficiency. In addition, TORUS-NS topology has the highest power / torque density and efficiency.
- AFIR type machines are better than the others in terms of weight and utilization. Furthermore, the AFIR-S topology has the lowest weight and the highest utilization factor.
- Generally, the external rotor topologies (TORUS) are better than the internal rotor topologies (AFIR) in terms of power/torque density and efficiency.
- Non-slotted topologies of AFM are always better than slotted topologies of AFM in terms of power/torque density, efficiency and heat dissipation.
- The internal rotor topology of AFM can be easily used with cooling plate for conductor heat transfer.
- The slotless TORUS topology has the highest torque density ratio compared to other topologies.

6. Conclusion

In this paper, general purpose sizing equations are obtained as a function of machine outer diameter and axial length, and developed for both radial flux surface mounted PM machine and axial flux surface mounted slotless TORUS type PM machine. Optimization of the TORUS machine shows that the machines can be designed for maximum power density or efficiency points. Comparison of radial and axial flux surface mounted PM machines for 200 HP 6 pole 1200 rpm PM drive motors in terms of power/torque density, efficiency, heat dissipation, weight and utilization factor were accomplished and the results were provided in the paper. It was observed that slotted and non-slotted TORUS topologies possess higher power or torque density ratio than the other radial and axial flux topologies.

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