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General Closed-form Analytical Expressions of Air-gap Inductances for Surface-mounted Permanent Magnet and Induction Machines

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***Abstract*—General closed-form analytical expressions for air-gap inductance of surface-mounted PM and induction machines are derived without the assumption of sinusoidally wound windings. The expressions are suitable for all single- or double-layer, concentrated or distributed windings, and demonstrated to be identical with the winding function approach and share the same advantage that all the harmonics are inherently included. These expressions are straightforward analytical equations, which are easily used by machine designers.**

I. INTRODUCTION

Air-gap inductance, defined as the inductance due to flux crossing the air gap, is one of the most important parameters for Permanent Magnet (PM) and induction machine designs.

The well-known inductance calculation method — the winding function [1] — has the advantage that all the harmonics are inherently included. It is often convenient to calculate the inductance of a special case by using the winding function. However, a tedious solution of an integral is needed in the winding function method, which is not very straightforward and difficult to be used in computer programs. Therefore, a general closed-form analytical expression that should be suitable for computer programs, a necessary tool for modern machine designs, is desirable, but has not been available.

This paper will derive such an equivalent analytical expressions which are suitable for machine designers, who usually use computer programs to design machines and compare the inductances of several different designs.

To derive the analytical expressions for the air gap inductance, the following assumptions are made:

- The surface-mounted magnets and spacers between them have essentially the same relative permeability, and
- The rotor and stator relative permeabilities are high, so that the rotor and stator back iron reluctance are negligible and no saturation exists, and
- The fringing effect at both ends of the windings is negligible.

Based upon the assumptions above and the inductance expression of a single full pitch coil, the coil self inductance, coil-to-coil mutual inductance, air-gap inductance per pole, mutual inductance between two coils in two adjacent poles, and phase air-gap inductance will be derived sequentially. The final expression is an analytical equation in terms of the machine physical sizes and the winding distribution.

Given the difference between PM machines and induction machines, the inductance expressions for PM machines are modified to apply to induction machines. A general closed-form analytical expression of the air-gap inductance for induction machines will be given. In addition, two simplified forms for the full pitch windings and the concentrated full pitch windings will be presented.

The derived expressions and the winding function will be demonstrated by two examples to be identical and share the same advantage that all the harmonics are inherently included. Furthermore, these expressions are the straightforward analytical equations without any integral included, which are desirable and easily used by machine designers.

II. GENERAL CLOSED-FORM EXPRESSIONS OF AIR-GAP INDUCTANCES FOR PM MACHINES

A general closed-form expression of air-gap inductance of surface-mounted PM machines will be derived in this section, then applied to induction machines in the next section.

A. Coil Self Inductance

Based upon the assumptions above, the inductance of a single full pitch coil with N_{coil} turns is found to be [3]

$$L_{gcoil} = \frac{N_{coil}^2 \mu_r \mu_o L \tau_p}{2(H_{PM} + \mu_r g_e)} \qquad (1)$$

where g_e is the effective length of air gap including the slotting effects, H_{pm} the magnet height shown in Figure 1, L the axial length of the stator core or rotors, τ_p the pole pitch, μ_o permeability of free space, and μ_r the magnet relative permeability.

Following the same derivation process, the inductance of the coil with the short pitch of τ_c slots can be expressed as

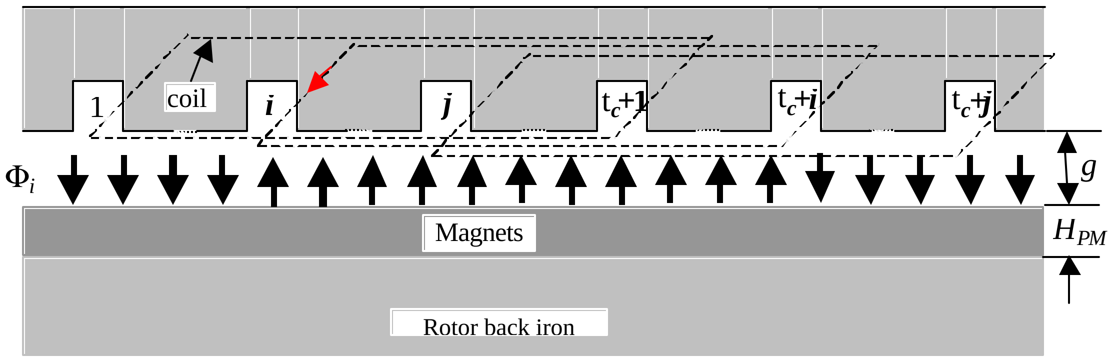

Fig. 1. Mutual coupling between two coils

$$L_{gc} = \frac{N_{coil}^2 \mu_r \mu_o L K_\tau \tau_p}{2(H_{PM} + \mu_r g_e)} \quad (2)$$

where the coil pitch factor K_τ is defined as

$$K_\tau = y(2 - y) \quad (3)$$

The coil pitch ratio y is defined as the ratio of the coil pitch to the pole pitch and is given by

$$y = \frac{\tau_c}{mq} \quad (4)$$

where τ_c is the coil pitch measured in number of slots, m the machine phase number, and q the slot number per phase per pole. For full pitch coils, K_τ is equal to 1.

The pole pitch measured in number of slots, τ_{ps}, defined in the (5) will be used in the following sections.

$$\tau_{ps} = mq \quad (5)$$

B. *Coil-to-coil mutual inductance*

Mutual inductance is defined in terms of the flux linked by one coil due to the current in another. Air gap mutual inductances between coils are a function of the relative placement of the coils and therefore are a function of the slot number q per phase per pole. As shown in Figure 1, consider any two coils, coil i and coil j ($i < j \le q < \tau_c$), with the coil pitch τ_c, Φ_i is the air gap flux created by current flowing in coil i. This flux couples to coil j in such a way that $\frac{\tau_c - (j-i)}{\tau_c}$ of the flux is coupled in one direction and $\frac{j-i}{2\tau_{ps} - \tau_c}$ of the flux is coupled in the opposite direction. Thus, the net flux from coil i linking coil j is $\frac{\tau_c - (j-i)}{\tau_c} - \frac{j-i}{2\tau_{ps} - \tau_c}$. Since the self inductance of coil i is linearly related to the flux created by coil i, the ratio of the air gap mutual and self inductance is $\frac{\tau_c - (j-i)}{\tau_c} - \frac{j-i}{2\tau_{ps} - \tau_c}$. i.e.,

$$M_{gij} = \left[\frac{\tau_c - (j-i)}{\tau_c} - \frac{j-i}{2\tau_{ps} - \tau_c}\right] L_{gi} = \left(1 - \frac{j-i}{\tau_c} - \frac{j-i}{2\tau_{ps} - \tau_c}\right) L_{gc} \quad (6)$$

Due to the symmetry, all the coil self inductances are equal to each other, and to L_{gc}.

C. *Phase inductance per pole.*

For each pole, there are q coils in series, and mutual inductances exist between any two coils. Then the total phase inductance per pole must be

$$\begin{aligned} L_{pole} = qL_{gc} &+ 2M_{g12} + 2M_{g13} + 2M_{g14} + \cdots + 2M_{g1q} \\ &+ 2M_{g23} + 2M_{g24} + \cdots + 2M_{g2q} \\ &\qquad \ddots \qquad \vdots \\ &+ 2M_{g(q-1)q} \end{aligned} \quad (7)$$

Substituting (6) yields

$$\begin{aligned} L_{pole} = {} & qL_{gc} + q(q-1)L_{gc} \\ & - 2L_{gc}\Big[+ \frac{1}{\tau_c} + \frac{2}{\tau_c} + \cdots + \frac{q-1}{\tau_c} \\ & \quad + \frac{1}{\tau_c} + \frac{2}{\tau_c} + \cdots + \frac{q-2}{\tau_c} \\ & \quad \vdots \qquad \iddots \\ & \quad + \frac{1}{\tau_c} \Big] \\ & - 2L_{gc}\Big[+ \frac{1}{2\tau_{ps} - \tau_c} + \frac{2}{2\tau_{ps} - \tau_c} + \cdots + \frac{q-1}{2\tau_{ps} - \tau_c} \\ & \quad + \frac{1}{2\tau_{ps} - \tau_c} + \frac{2}{2\tau_{ps} - \tau_c} + \cdots + \frac{q-2}{2\tau_{ps} - \tau_c} \\ & \quad \vdots \qquad \iddots \\ & \quad + \frac{1}{2\tau_{ps} - \tau_c} \Big] \end{aligned} \quad (8)$$

Noting that

$$(q-1) + 2(q-2) + 3(q-3) + \cdots + (q-2)2 + (q-1) = \frac{q(q^2-1)}{6} \quad (9)$$

and after some manipulation, (8) reduces to

$$L_{pole} = \left[q^2 - \frac{q(q^2-1)}{3\tau_c} - \frac{q(q^2-1)}{3(2\tau_{ps} - \tau_c)}\right] L_{gc} \quad (10)$$

Combining (2) and (10) yields

$$L_{pole} = \left[\frac{q^2}{2} - \frac{q(q^2-1)}{6\tau_c} - \frac{q(q^2-1)}{6(2\tau_{ps} - \tau_c)}\right] \times \frac{N_{coil}^2 \mu_r \mu_o L K_\tau \tau_p}{H_{PM} + \mu_r g_e} \quad (11)$$

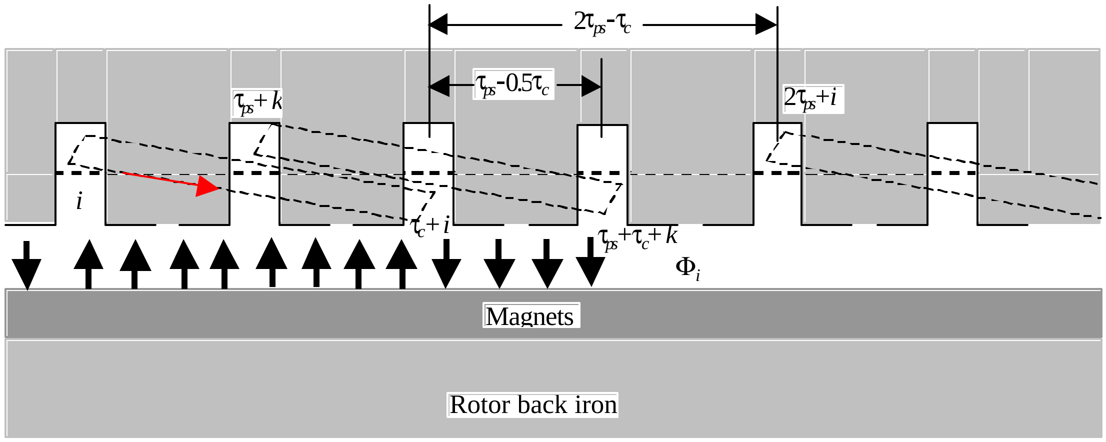

Fig. 2. Mutual coupling between two coils in two adjacent poles (case A)

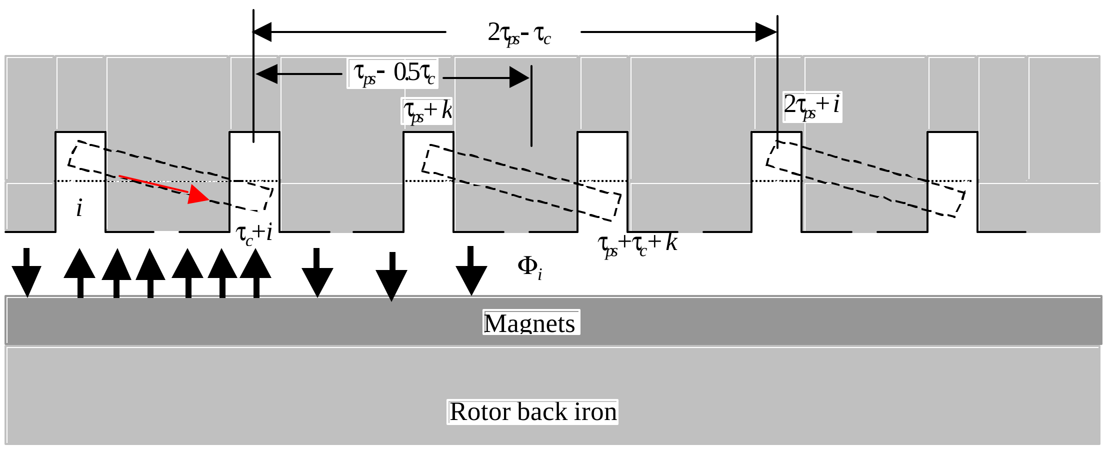

Fig. 3. Mutual coupling between two coils in two adjacent poles (case B)

This expression denotes the inductance of q coils distributed in q slots, each with N_{coil} turns and a pitch of τ_c slots.

D. Mutual inductance between two coils in two adjacent poles.

For double layer windings, the mutual inductance between two coils, one of which is on one pole and the other on an adjacent pole, should be calculated before the phase inductance can be found. There is no mutual inductance between any coils on different poles for sing-layer case since these kind coils have at least a 360° degree phase shift.

Considering the flux Φ_i produced by coil i ($1 \le i \le q$) consisting of the top layer conductors in slot i and the bottom layer conductors in slot τ_c+i, as shown in Figure 2, one half travels through its one side on which coil $\tau_{ps}+k$ ($1 \le k \le q$) is, and one half through the other side. Coil i and coil $\tau_{ps}+k$, which is consisted of the top layer conductors in slot $\tau_{ps}+k$ and the bottom conductors in slot $\tau_{ps}+\tau_c+k$, are in the two adjacent poles.

For the case shown in Figure 2, τ_c+i is larger than $\tau_{ps}+k$. The flux Φ_i couples to coil k in such a way that $\frac{(\tau_c+i)-(\tau_{ps}+k)}{\tau_c}$ of the flux is coupled in one direction and half the flux is coupled in the opposite direction. Thus the net flux linking coil k is

$$\Phi_{ik} = \frac{1}{2}\Phi_i - \frac{(\tau_c+i)-(\tau_{ps}+k)}{\tau_c}\Phi_i = [\frac{1}{2} - \frac{(\tau_c+i)-(\tau_{ps}+k)}{\tau_c}]\Phi_i \qquad (12)$$

Since the self inductance of coil i is linearly related to the flux created by coil i, the mutual inductance between two coils in two adjacent poles is related to the coil self inductance by

$$M_{ppik} = [\frac{1}{2} - \frac{(\tau_c + i) - (\tau_{ps} + k)}{\tau_c}] L_{gc} \quad \text{for } 0.5\tau_c \geq (\tau_c + i) - (\tau_{ps} + k) > 0 \tag{13}$$

Given the relationship of

$$i - k \leq q - 1 < \frac{mq}{2} = \frac{\tau_{ps}}{2} = \tau_{ps} - \frac{\tau_{ps}}{2} \leq \tau_{ps} - \frac{\tau_c}{2} \quad \text{for } m > 1 \tag{14}$$

the condition of $(\tau_c + i) - (\tau_{ps} + k) \leq 0.5\tau_c$, which is equivalent to $i - k \leq \tau_{ps} - 0.5\tau_c$, is always met. Since the fluxes created by coil i and coil $2\tau_{ps} + i$, which is in the third pole or coil i itself for the 2 pole machine, share the area between $\tau_c + i$ and $2\tau_{ps} + i$, one additional condition is needed to make (13) valid. It is

$$\tau_{ps} + \tau_c + k - (\tau_c + i) \geq \tau_{ps} - \frac{\tau_c}{2} \tag{15}$$

which is equivalent to

$$\frac{\tau_c}{2} \geq k - i \tag{16}$$

For machines, τ_c is usually greater than $\frac{2}{3}\tau_{ps}$ and hence greater than $\frac{2mq}{3}$, while the maximum value of $k - i$ is q-1. That implies condition (16) is always valid for multiphase machines (m>2) since $\frac{mq}{3} > q$-1. These discussions keep (13) valid for multiphase machines (m >2).

For the case shown in Figure 3, $\tau_c + i$ is smaller than or equal to $\tau_{ps} + k$, so that only $\frac{1}{2} - \frac{[(\tau_{ps} + k) - (\tau_c + i)]}{2\tau_{ps} - \tau_c}$ of the flux couples to coil k. Thus the net flux linking coil k is

$$\Phi_{ik} = [\frac{1}{2} - \frac{(\tau_{ps} + k) - (\tau_c + i)}{2\tau_{ps} - \tau_c}] \Phi_i \tag{17}$$

Similarly, the mutual inductance between two coils in two adjacent poles in this case is related to the coil self inductance by

$$M_{ppik} = [\frac{1}{2} - \frac{(\tau_{ps} + k) - (\tau_c + i)}{2\tau_{ps} - \tau_c}] L_{gc} \quad \text{for } 0 \leq [(\tau_{ps} + k) - (\tau_c + i)] \leq (\tau_{ps} - 0.5\tau_c) \tag{18}$$

The condition of $[(\tau_{ps} + k) - (\tau_c + i)] \leq (\tau_{ps} - 0.5\tau_c)$ is always valid for $m > 2$ and equivalent to Condition (16). Equation (13) and (18) can be combined to form

$$M_{ppik} = [\frac{1}{2} - \frac{1}{\tau_q} |(\tau_{ps} + k) - (\tau_c + i)|] L_{gc} \quad \text{for } 1 \leq i \leq q \text{ and } 1 \leq k \leq q \tag{19}$$

where

$$\tau_q = \begin{cases} \tau_c & \text{for } \tau_c + i > \tau_{ps} + k \\ 2\tau_{ps} - \tau_c & \text{for } \tau_c + i \leq \tau_{ps} + k \end{cases} \tag{20}$$

E. Pole-to-pole phase mutual inductance for double layer windings

The total pole-to-pole phase mutual inductance is the sum of the coil mutual inductances between two adjacent poles:

$$M_{pp} = \sum_{i=1}^{q} \sum_{k=1}^{q} M_{ppik} \tag{21}$$

Substituting (19) into (21), it yields

$$M_{pp} = [\frac{q^2}{2} - \sum_{i=1}^{q} \sum_{k=1}^{q} \frac{|(\tau_{ps} + k) - (\tau_c + i)|}{\tau_q}] L_{gc} \quad \text{for all } 1 \leq i \leq q,\ 1 \leq k \leq q \tag{22}$$

Equation (22) is in a concise format and very easily used in any computer program. In practice, q is about 2~5 for small or middle power level machines, τ_{ps} is equal to mq, and τ_c is not greater than τ_p. Therefore, this equation is not hard to be evaluated by hand.

F. Phase inductance

It is interesting to note that, the short pitch windings are not suitable for the single-layer windings, and that full pitch windings ($\tau_c = \tau_{ps}$ and hence K_τ = 1) must be used. The number of the coil turns N_{coil} must be equal to the conductor number per slot N_c. For a single-layer machine with P poles and C circuits in parallel, given P/2C circuits in series and using (11), the total inductance of each circuit is

$$L_{ckt} = \frac{P}{2C} [\frac{q^2}{2} - \frac{q(q^2 - 1)}{6\tau_c} - \frac{q(q^2 - 1)}{6(2\tau_{ps} - \tau_c)}] \frac{N_c^2 \mu_r \mu_o L \tau_p}{H_{PM} + \mu_r g_e} \tag{23}$$

Again given C circuits in parallel, the overall machine phase inductance with the Single Layer (SL) windings is found as

$$L_{ph} = \frac{\mu_r \mu_o L P N_c^2}{2C^2} \frac{\tau_p}{H_{PM} + \mu_r g_e} (\frac{q^2}{2} - \frac{q(q^2 - 1)}{6\tau_c} - \frac{q(q^2 - 1)}{6(2\tau_{ps} - \tau_c)}) \quad \text{for SL} \tag{24}$$

For the double-layer winding case, the number of the coil turns N_{coil} is equal to one half of the conductor number per slot N_c, the pole inductance per phase takes the form of

$$L_{pole} = \frac{1}{4} (\frac{q^2}{2} - \frac{q(q^2 - 1)}{6\tau_c} - \frac{q(q^2 - 1)}{6(2\tau_{ps} - \tau_c)}) \frac{N_c^2 \mu_r \mu_o L K_t \tau_p}{H_{PM} + \mu_r g_e} \tag{25}$$

In addition, the mutual inductance between any two adjacent poles exists and takes the form of (22). Given P pole inductances, P mutuals between any two adjacent poles, and C circuits in parallel, the overall inductance of each circuit for the double layer case is

$$L_{ckt} = \frac{P}{C} (L_{pole} + 2M_{pp}) \tag{26}$$

It can be extended to

$$L_{ckt} = \frac{P}{4C} [q^2 - \frac{q(q^2 - 1)}{6\tau_c} - \frac{q(q^2 - 1)}{6(2\tau_{ps} - \tau_c)} - \sum_{i=1}^{q} \sum_{k=1}^{q} \frac{|(\tau_{ps} + k) - (\tau_c + i)|}{\tau_q}] \frac{N_c^2 \mu_r \mu_o L K_\tau \tau_p}{H_{PM} + \mu_r g_e} \tag{27}$$

Given C circuits in parallel, the PM machine phase inductance with the Double Layer (DL) windings is found as

$$L_{ph} = \frac{\mu_r \mu_o K_\tau L P N_c^2}{2C^2} [\frac{q^2}{2} - \frac{q(q^2 - 1)}{12\tau_c} - \frac{q(q^2 - 1)}{12(2t_{ps} - \tau_c)} - \frac{1}{2} \sum_{i=1}^{q} \sum_{k=1}^{q} \frac{|(\tau_{ps} + k) - (\tau_c + i)|}{\tau_q}] \times \frac{\tau_p}{H_{PM} + \mu_r g_e} \quad \text{for DL} \tag{28}$$

For the full-pitch windings ($\tau_c=\tau_{ps}=\tau_q$), (28) can be reduced to

$$L_{ph} = \frac{\mu_r\mu_o L P N_c^2}{2C^2}\left[\frac{q^2}{2} - \frac{q(q^2-1)}{6\tau_c} - \frac{1}{2\tau_c}\sum_{i=1}^{q}\sum_{k=1}^{q}|k-i|\right] \times \frac{\tau_p}{H_{PM}+\mu_r g_e} \tag{29}$$

Noting that

$$\frac{q(q^2-1)}{6} = \frac{1}{2}\sum_{i=1}^{q}\sum_{k=1}^{q}|k-i| \tag{30}$$

(29) is reduced to the identity of (24). This result means that the phase inductance of the single-layer full pitch windings is exactly same as that of the double-layer full pitch windings if all the other conditions are kept same. This is easily understood since the flux distributions are same for the both cases. This implies that (24) can be applied to the double-layer winding if the effect of the short pitch is neglected when calculating the pole-to-pole mutual inductance.

For concentrated full-pitch windings ($q=1$, $\tau_c=\tau_{ps}$), substituting $q=1$ and $\tau_c=\tau_{ps}=mq$ into (24), this expression reduces to

$$L_{ph} = \frac{\mu_r\mu_o LPN_c^2}{4C^2}\frac{\tau_p}{H_{PM}+\mu_r g_e} \quad \text{for concentrated full pitch windings} \tag{31}$$

It is worth noting that the air gap inductance is relatively small because of the low relative permeability and large length of the PM with respect to the air gap.

III. Application to Induction Machines and Comparison with the Winding Function

Given the difference between PM machines and induction machines, (24), (28) and (31) can be modified to apply to induction machines as

$$L_{ph} = \frac{\mu_o\tau_p LPN_c^2}{2C^2 g_e}\left[\frac{q^2}{2} - \frac{q(q^2-1)}{3\tau_c}\right] \quad \text{for SL IM} \tag{32}$$

$$L_{ph} = \frac{\mu_o\tau_p K_\tau LPN_c^2}{2C^2 g_e}\left[\frac{q^2}{2} - \frac{q(q^2-1)}{12\tau_c} - \frac{q(q^2-1)}{12(2\tau_{ps}-\tau_c)} - \frac{1}{2}\sum_{i=1}^{q}\sum_{k=1}^{q}\frac{|(\tau_{ps}+k)-(\tau_c+i)|}{\tau_q}\right] \text{for DL IM} \tag{33}$$

$$L_{ph} = \frac{\mu_o\tau_p LPN_c^2}{4C^2 g_e} \quad \text{for concentrated full pitch IM} \tag{34}$$

Equation (32) through (34) could be rewritten into the forms

$$L_{ph} = \frac{\mu_o\pi LR_{is}}{g_e}\frac{N_c^2}{C^2}\left[\frac{q^2}{2} - \frac{q(q^2-1)}{3\tau_c}\right] \quad \text{for SL IM} \tag{35}$$

$$L_{ph} = \frac{\mu_o\pi K_\tau LR_{is}}{g_e}\frac{N_c^2}{C^2}\left[\frac{q^2}{2} - \frac{q(q^2-1)}{12\tau_c} - \frac{q(q^2-1)}{12(2\tau_{ps}-\tau_c)} - \frac{1}{2}\sum_{i=1}^{q}\sum_{k=1}^{q}\frac{|(\tau_{ps}+k)-(\tau_c+i)|}{\tau_q}\right] \text{for DL IM} \tag{36}$$

$$L_{ph} = \frac{\mu_o\pi LR_{is}}{g_e}\frac{N_c^2}{2C^2} \quad \text{for concentrated full pitch IM} \tag{37}$$

Equation (37) and (35) are both the special cases of (36) for the full pitch windings and the concentrated full pitch windings, respectively. The three equations above and the well-known inductance calculation method — the winding function — share the same advantage that all the harmonics are already included. However, an integral is needed in the winding function method, which is not very straightforward and difficult to be used in computer programs. On the other hand, (36) is a closed-form analytical equation and suitable for computer programs, a necessary tool to design machines nowadays. In other words, (36) could be taken as the integral result of the winding function approach for the general case, in which the windings with the coil pitch of τ_c slots are uniformly distributed in q slots for each phase each pole. There are a total m phases, P poles, and C parallel circuits. To calculate the inductance of a special case, the winding function might be more convenient while (36) is suitable for machine designers, who usually use computer programs to design machines and compare the inductances of several different designs.

In order to show that the equations derived here are equivalent to the winding function approach, consider an example used in course ECE 711 at the University of Wisconsin-Madison: a total of N conductors are uniformly distributed along the stator inner surface of a 3 phase 2 pole machine with the air gap g_e. Assume N is large enough that the steps in the winding function caused by individual conductors can be approximated as a smooth curve. The winding function approach gives the phase inductance is

$$L_{ph} = \frac{7}{648}\frac{\mu_o\pi LR_{is}N^2}{g_e}\ \text{H} \tag{38}$$

In this example, q is equal to infinity so that $q^2-1=q^2$, $m=3$, $\tau_c=mq=3q$, and $C=1$. Substituting these conditions into (35), it is reduced to

$$L_{ph} = \frac{\mu_o\pi LR_{is}}{g_e}\frac{7q^2N_c^2}{18}\ \text{H} \tag{39}$$

Given $N=N_c qmP=6N_c q$, the two results above are found to be identical.

Consider another more general example: a double layer winding with $q=3$, $m=3$, $\tau_{ps}=mq=9$, $\tau_c=7$ (short pitch), $P=8$, and $C=1$. Equation (36) can be easily evaluated to be

$$L_{ph} = 3.0556\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e} \tag{40}$$

To find the phase inductance by the winding function, the winding function $N(\theta)$ should be drawn first. It is shown in Figure 4

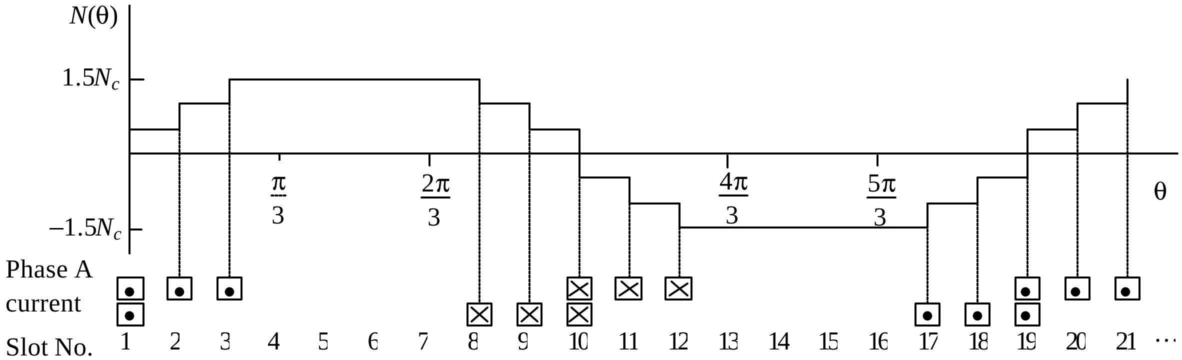

Fig. 4. Winding function for the double layer windings with q=3, m=3, $\tau_{ps}=mq$=9, τ_c=7, P=8, C=1

The phase A inductance as per the winding function theory will be given by

$$L_{ph} = \frac{\mu_o LR_{is}}{g_e C^2}\int_0^{2\pi} N^2(\theta)\,d\theta = 3.0556\times\frac{\mu_o \pi LR_{is}N_c^2}{g_e} \quad (41)$$

which is the same as that given by (36).

Table I summarized the results calculated using the winding function method and (36) for 4 different cases. The identity between the two approaches is demonstrated.

TABLE I INDUCTANCES CALCULATED FROM THE WINDING FUNCTION AND (36)
@ q=3, m=3, τ_{ps}=9, P=8

	The winding function results	Equation (36) results
C=1, τ_c=7	$3.0556\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e}$	$3.0556\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e}$
C=1, τ_c=8	$3.4444\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e}$	$3.4444\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e}$
C=2, τ_c=7	$0.7639\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e}$	$0.7639\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e}$
C=2, τ_c=8	$0.8611\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e}$	$0.8611\times\frac{\mu_o\pi LR_{is}N_c^2}{g_e}$

IV. CONCLUSIONS

General closed-form analytical expressions of air-gap inductance for 3- or more-phase surface-mounted permanent magnet machines have been derived and applied to induction machines. The assumption of sinusoidally wound phase windings is unnecessary. The derived expressions are suitable for all single- or double-layer, concentrated or distributed windings. The expressions and the winding function method have been demonstrated to be identical and share the same advantage that all the harmonics are inherently included, while these expressions are straightforward analytical equations and can be easily used in computer programs.

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