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Sizing Equations and Power Density Evaluation of Dual-Rotor, Radial-Flux, Toroidally Wound, Permanent-Magnet Machines

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Abstract — **A novel machine family – the dual-rotor, Radial-Flux, Toroidally wound, Permanent-Magnet (RFTPM) machine – has been proven in a previous paper to be able to improve the machine efficiency and boost the torque density. This paper will develop the sizing equations for the RFTPM machines based on the machine overall sizes, material properties, and electrical and magnetic loadings to provide a quick method to evaluate the RFTPM machines. The accuracy of the developed sizing equations is proven by a prototype machine. A comparison among induction, axial-flux PM, and RFTPM machines are made based on the sizing equations. The RFTPM structure appears to be capable of substantially higher power density than equivalent induction machines, and more potential to achieve higher torque density than axial-flux PM machines.**

Keywords — **RFTPM Machine, Sizing Equations, PM Machine, Power Density**

I. INTRODUCTION

In general, comparison of different machine types is a very formidable task since many variables exist for each machine and it is difficult to select those quantities which should be held constant and those that should be left free to vary. One traditional method of comparison is to use the $D_g^2L_e$ sizing equation, which compares the machine power on the basis of the airgap volume, where D_g is the airgap diameter and L_e is the effective stack length. However, the machine outer diameter D_o is more directly related to the volume and thus to the cost and size of the machine. Meanwhile, given the great number of possible motor choices it has become important to compare power potential of machines with vastly different topologies, having a variety of different waveforms of back EMF and current. A systematic and easy-to-use method based on sizing equations is therefore very desirable to compare the capability of machines with different structures.

The general-purpose sizing and power density equations have recently been developed in terms of the main machine dimensions $D_o^2L_e$ instead of air gap dimension $D_g^2L_e$ at Wisconsin Electric Machine and Power Electronics Consortium (WEMPEC) at the University of Wisconsin-Madison [1-4]. This approach permits a comparison of the capability of different machine topologies based upon the overall occupied volume. Several sample applications of the general-purpose sizing and power density equations have been accomplished. The application to induction machines with small and medium power ratings (up to 100 hp) have been proven to be valid by comparing the actual motor data to the results estimated by the sizing equations.

Meanwhile, a novel machine family – the Dual-Rotor, Radial-Flux, Toroidally Wound, Permanent Magnet (RFTPM) machine – has been proposed and proven to be able to improve the machine efficiency and boost the torque density. It will be very interesting to develop sizing equations for the RFTPM machines based on the machine overall volume, material properties, and electrical and magnetic loadings, so that the RFTPM machine can be easily compared as other machines based on the same method – sizing equations.

In this paper, after the introduction of the general-purpose sizing and power density equations and the RFTPM machines, the sizing and power density equations based the machine outside diameter and length are developed for both the non-slotted and slotted RFTPM machines. Then, the power density of this machine type is compared with induction machines and axial-flux toroidally wound PM (AFTPM) machines.

The following SI units are used as default units for all the variables in this paper except those specially noted.

Length:	Meter	Voltage:	Volt
Power:	Watt	Power density:	W/m^3
Current:	Ampere	Current density:	A/m^2
Flux density:	Tesla	Electrical loading:	A/m
Frequency:	Hertz	Flux linkage:	Web
Time:	Second	Torque:	Nm

II. DUAL-ROTOR, RADIAL-FLUX, TOROIDALLY WOUND, PERMANENT-MAGNET MACHINES

Briefly, the dual-rotor, RFTPM machine is constructed so that two machines nested inside one another. The outer alternator has magnets at the outside surface of the outer airgap with the flux directed inward/outward, and the inner alternator has magnets at the inside surface of the inner airgap with the flux directed outward/inward. The two sets of stator coils are back-to-back toroidally wound, sharing common back iron. In this topology, the magnets drive flux across the two radial air gaps into the stator core; the flux then travels circumferentially along the core, back across the air gaps, and then through the rotor back iron of the rotor. Fig. 1 shows an example of the slotted RFTPM machine.

The toroidal machine works like two conventional machines in series. One is inside, the other is outside. They have the same armature current. Their back EMFs are in series. The outer and inner portion of the rotor are connected together by one end disc, which can work as a cooling fan. The stator is fixed at the other end to a frame.

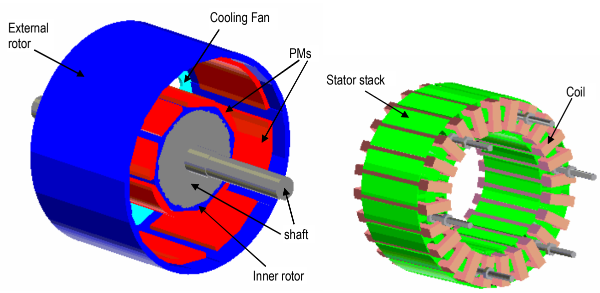


Figure 1. Dual-rotor, toroidally wound, slotted, RFTPM machine

It has been demonstrated that the dual-rotor RFTPM machine has the following features [6-8]:

- Greatly shortened end windings
- High efficiency
- High torque density
- Low-cost techniques available to reduce cogging torque

III. General Purpose Sizing Equations and Power Density

The general-purpose sizing equations have been developed in [1-5] and take the form of

$$P_R = \frac{1}{1+K_\phi}\frac{m}{m_1}\frac{\pi}{2}K_e K_i K_{pw}\eta B_{gmax} A \frac{f}{p}\lambda_0^2 D_o^2 L_e \qquad (1)$$

and

$$P_R = \frac{1}{1+K_\phi}\frac{m}{m_1}\frac{\pi}{2}K_e K_i K_{pw} K_L \eta B_{gmax} A \frac{f}{p}\lambda_0^3 D_o^3 \qquad (2)$$

The definitions of the variables in both equations are listed below:

P_R rated output power

K_ϕ $=A_r/A_s$, ratio of electric loading on rotor and stator. In a machine topology without a rotor winding, $K_\phi=0$.

m number of phases of the machine

m_1 number of phases of each stator (if there is more than one stator, each stator has the same m_1).

K_e EMF factor that incorporates the winding distribution factor K_w and the ratio between the area spanned by the poles and the total airgap area.

K_i current waveform factor in order to indicate the effect of the current waveform, $K_i = \frac{I_{phmax}}{I_{rms}} = \left[\frac{1}{T}\int_0^T\left(\frac{i(t)}{I_{phmax}}\right)^2 dt\right]^{-1/2}$ where the current $i(t)$ and I_{phmax} are the phase current and the peak phase current, respectively, and I_{rms} is the rms current.

K_{pw} electrical power waveform factor, $K_{pw} = \frac{1}{T}\int_0^T \frac{e(t)i(t)}{E_{pk}I_{phmax}}dt = \frac{1}{T}\int_0^T f_e(t)f_i(t)\,dt$ where $f_e(t)=e(t)/E_{pk}$ and $f_i(t)=i(t)/I_{phmax}$ are the expressions for the normalized EMF and current waveforms. $e(t)$ and E_{pk} are the phase air gap EMF and its peak value. T is the period of one EMF cycle.

K_L $=\frac{L_e}{D_g}$, defined as the aspect ratio coefficient

λ_o $=\frac{D_g}{D_o}$, the diameter ratio

η machine efficiency,

B_{gmax} flux density in the air gap, maximum value

A total electric loading, including stator and rotor loading

N_t number of turns per phase,

f power supply frequency

p number of machine pole pairs

Finally, the machine power density for the total volume can be defined as

$$\xi = \frac{P_R}{\frac{\pi}{4}D_o^2 L_t} \qquad (3)$$

where L_t is the total length of the machine including the stack length and the protrusion of the end winding from the iron stack in the axial direction.

By examining the back EMF and current waveform for a particular machine type, the factors K_i and K_{pw} can be determined. K_i and K_{pw} for several typical waveforms have been well summarized in [1].

IV. Sizing Equations of RFTPM Machines

In the last section, the general purpose sizing equation was introduced. For dual-rotor, radial-flux machines, the sizing equation is still applicable but may take different forms. As a further study, a detailed approach will be presented in this section for the application of the general-purpose sizing and power density equations to dual-rotor radial-flux machines, focusing specifically on the RFTPM machine.

Referring to the structure of the RFTPM machine shown in Fig. 2 and assuming the same air gap flux densities for inside and outside, the back EMF for the machines is given by

$$e(t) = \frac{d\Lambda_g}{dt} = 2\pi K_w N_t B_{gmax}\frac{f}{p}L_e\frac{1+\lambda}{2}D_o f_e(t) \qquad (4)$$

where Λ_g is the air-gap flux linkage per phase, N_t is the number of turns per phase. The ratio λ is defined as

$$\lambda = \frac{D_i}{D_o} \qquad (5)$$

where D_i is the stator inner diameter and D_o the stator outer diameter. From (4) it is apparent that

$$E_{pk} = 2\pi K_w N_t B_{gmax}\frac{f}{p}L_e\frac{1+\lambda}{2}D_o \qquad (6)$$

From section II, the factor K_i is

$$K_i = \frac{I_{phmax}}{I_{rms}} = \left[\frac{1}{T}\int_0^T\left(\frac{i(t)}{I_{phmax}}\right)^2 dt\right]^{-1/2} \qquad (7)$$

where I_{rms} is the rms phase current which is related to the stator electric loading A_s. For the RFTPM machine the electric loading A_s includes both the inner and outer parts. The inner electric loading A_{s1} has the form of

$$A_{s1} = 2m_1\frac{N_t}{2}\frac{I_{rms}}{\pi D_i} \qquad (8)$$

while the outer part A_{s2} is

$$A_{s2} = 2m_1 \frac{N_t}{2} \frac{I_{rms}}{\pi D_o} \tag{9}$$

Taking both parts into consideration, the overall stator electric loading A_s should be

$$A_s = A_{s1} + A_{s2} = 2m_1 N_t \frac{I_{rms}}{\pi D_g} \tag{10}$$

where the equivalent diameter of air gap coming from the two air gaps in “parallel” is

$$D_g = \frac{2D_oD_i}{D_o + D_i} = \frac{2\lambda}{1+\lambda}D_o \quad or \quad \frac{2}{D_g} = \frac{1}{D_i} + \frac{1}{D_o} \tag{11}$$

Since the total electric loading A includes both the stator electric loading A_s and rotor electric loading A_r.

$$A_s = A - A_r = \frac{A}{1+K_\phi} \tag{12}$$

From (7), (10) and (12), an expression for the peak current is

$$I_{phmax} = \frac{1}{1+K_\phi} K_i A \pi \frac{D_g}{2m_1N_t} \tag{13}$$

In general, if stator leakage inductance and resistance can be neglected, the output power for any electrical machine can be expressed as

$$P_R = \eta \frac{m}{T} \int_0^T e(t)\, i(t)dt = \eta\, m K_{pw} E_{pk} I_{phmax} \tag{14}$$

Substituting (6), (11) and (13) into (14), the general purpose $D_o^2L_e$ sizing equation for the RFTPM machines takes the form of

$$P_R = \frac{1}{1+K_\phi} \frac{m}{m_1} \pi^2 K_w K_i K_{pw} \eta B_{gmax} A \frac{f}{p} \lambda D_o^2 L_e \tag{15}$$

To realize the required D_o^3 sizing equation, it is useful to define the aspect ratio K_L considering the special rotor-stator-rotor structure of the RFTPM machines,

$$K_L = \frac{L_e}{D_o} \tag{16}$$

The general purpose D_o^3 sizing equation for the RFTPM machines ultimately takes the unique form of

$$P_R = \frac{1}{1+K_\phi} \frac{m}{m_1} \pi^2 K_w K_i K_{pw} K_L \eta B_{gmax} A \frac{f}{p} \lambda D_o^3 \tag{17}$$

The overall power density can be defined as

$$\xi = \frac{P_R}{\frac{\pi}{4} D_t^2 L_t} \tag{18}$$

where D_t is the total outer diameter of the machine including the stator diameter and the thickness of both the outer rotor and magnets. L_t is the total machine length including the stack length and the protrusion of the end winding from the iron stack.

In practice, the optimal value of K_L is different depending upon the optimization goal. It is a major design parameter that has significant effect on the characteristic of the machine. Even when the optimization criterion is the same the optimal value of K_L also differs for different rated power, pole pairs, power supply frequency, etc. Further, if different materials or different structures are involved, the optimal K_L will have the different value.

A. Sizing and Power Density Equations of Non-slotted RFTPM Machines

For the non-slotted RFTPM machines shown in Fig. 2A, the ratio λ can be derived using the relationship between D_i and D_o.

$$D_i = D_o - 2d_{ys} \tag{19}$$

where d_{ys} is the thickness of the stator core.

If B_{cs} is defined as the flux density in the stator core, then d_{ys} can be found as

$$d_{ys} = \frac{\pi\alpha_p}{4pK_{fe}} (D_i + D_o) \frac{B_{gmax}}{B_{cs}} \tag{20}$$

Combining above two equations, it can be determined that

$$d_{ys} = D_i \frac{\pi\alpha_p B_{gmax}}{2pK_{fe}B_{cs} - \pi\alpha_p B_{gmax}} \tag{21}$$

Substituting (19) and (21) into (5), the ratio λ turns out as

$$\lambda = \frac{2pK_{fe}B_{cs} - \pi\alpha_p B_{gmax}}{2pK_{fe}B_{cs} + \pi\alpha_p B_{gmax}} = 1 - \frac{2\pi\alpha_p B_{gmax}}{2pK_{fe}B_{cs} + \pi\alpha_p B_{gmax}} \tag{22}$$

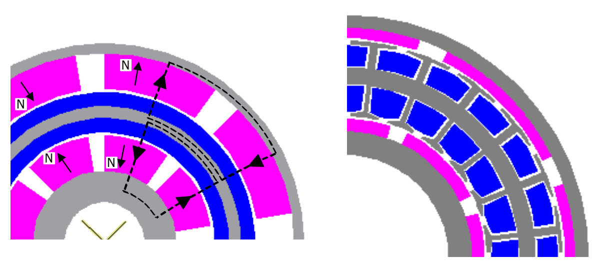
It is interesting to note that the ratio λ is not a function of D_i and D_o. Table I shows λ values for the different pole pairs and magnet types at B_{cs} =1.7 Tesla.

The machine total outer diameter D_t is given by

$$D_t = D_o + 2H_{cu2} + 2g + 2H_{pm2} + 2d_{yr2} \tag{23}$$

where

- H_{cu2} out side part of the winding thickness in the radial direction.
- g air gap length.
- H_{pm2} thickness of the outer permanent magnets.
- d_{yr2} outer rotor core thickness.



A: non-slotted structure B: slotted structure

Figure 2. Cross-section view of RFTPM machines

Table I λ VALUES FOR THE DIFFERENT POLE PAIRS & MAGNET TYPES

Pole pairs p	1	2	3	4	5
B_{gmax} =0.24 T (Ferrite magnets)	0.669	0.820	0.876	0.906	0.924
B_{gmax} =0.6 T (Rare earth magnets)	0.337	0.603	0.716	0.779	0.816

at B_{cs} =1.7 Tesla, α_p=0.85, K_{fe} =0.95

The amplitudes of both the inner and outer winding thickness depend on the stator equivalent electric loading A_s, the current density J_c, and the copper filling factor K_{cu}. They can be derived as follows.

Substituting (5), (8) and (9) into (10), A_{s2} and A_{s1} can be expressed in term of A_s as

$$A_{s2} = \frac{\lambda}{1+\lambda} A_s \quad (24)$$

$$A_{s1} = \frac{1}{1+\lambda} A_s \quad (25)$$

In general, the relationship between the thickness of the non-slotted winding and electric loading A_s, the current density J_c, and the copper filling factor K_{cu} is

$$H_{cu} = \frac{A_s}{J_c K_{cu}} \quad (26)$$

Thus, it is clear that H_{cu2} and H_{cu1} take the form of

$$H_{cu2} = \frac{\lambda}{1+\lambda}\frac{A_s}{J_c K_{cu}} \quad (27)$$

$$H_{cu1} = \frac{1}{1+\lambda}\frac{A_s}{J_c K_{cu}} \quad (28)$$

The PM thickness H_{PM2} can be calculated as [5]

$$H_{PM2} = \frac{\mu_r B_{gmax}}{B_r - B_m}(g + H_{cu2}) \quad (29)$$

where μ_r is the recoil relative permeability of the permanent magnet, B_r is the residual flux density and depends on PM material, and B_m is the attainable flux density on the surface of the permanent magnets.

The outer rotor core thickness d_{yr2} is expressed as

$$d_{yr2} = \frac{\pi K_\alpha D_o}{4p}\frac{B_m}{B_{cr}} \quad (30)$$

where K_α is the ratio of the portion occupied by magnets to the whole rotor circumference, and B_{cr} is the flux density in the inner or outer rotor core, which is typically 1.6 –1.8 Tesla.

Combining (23), (27), (29), and (30), the total machine diameter D_t is

$$D_t = D_o\left(1+\frac{\pi K_\alpha}{2p}\frac{B_m}{B_{cr}}\right) + \left(1+\frac{\mu_r B_{gmax}}{B_r - B_m}\right)\left(2g+\frac{2\lambda}{1+\lambda}\frac{A_s}{J_c K_{cu}}\right) \quad (31)$$

The protrusion of the end winding from the iron stack L_{cue} has the same value as H_{cu1} :

$$L_{cue} = \frac{1}{1+\lambda}\frac{A_s}{J_c K_{cu}} \quad (32)$$

The total machine length L_e can then be expressed as

$$L_t = L + 2L_{cue} + L_{re} \quad (33)$$

where L is the axial length of the stator core and L_{re} is the connection portion between two rotors, which is necessary from mechanical point of view.

Due to the structure of an RFTPM machine, the flux in the stator and rotor have different characteristics. In the stator core an AC flux exists, while, on the other hand, in the rotor core a nearly constant flux exists. The relationship between flux density B_{cs} and converter/power frequency f can be estimated as[1]

$$B_{cs} = \begin{cases} 5.4 f^{-0.32} & f > 40\ Hz \\ 1.7 \text{ to } 1.8 & f \leq 40 \text{ Hz} \end{cases} \quad (34)$$

The air gap flux density B_{gmax} can also be expressed as

$$B_{gmax} = \frac{4}{\pi} K_{focus} K_{FL} \sin\frac{K_\alpha \pi}{2} B_m \quad (35)$$

where K_{FL} is the flux leakage factor of the PM machines obtained usually through a finite element study or through design experience, and K_{focus} is the flux focusing factor which is related to the structure of the permanent magnet machine. Generally

$$K_{focus} = \frac{A_{PM}}{A_P} \quad (36)$$

where A_{PM} is the surface area of permanent magnets and A_P is the area of the gap surface physically crossed by the flux,. For a surface mounted machine like the RFTPMs shown in Fig. 2A and 2B, A_{pm} is equal to A_p.

For the RFTPM machine, there is no rotor winding, hence $K_\phi = 0$. Considering the typical trapezoidal waveforms in PM machines, $K_i K_{pw}$ is given as 0.881[2].

From (15), (35) and (36), the following RFTPM machine sizing equations are obtained

$$P_{R(RFTPM)} = 8.6951 \eta K_w B_{gmax} A \frac{f}{p} \lambda D_o^2 L_e \quad (37)$$

$$P_{R(RFTPM)} = 11.07 \eta K_w K_{FL} \sin\frac{K_\alpha \pi}{2} B_m A \frac{f}{p} \lambda D_o^2 L_e \quad (38)$$

and the power density of the RFTPM machine is

$$\xi = \frac{P_{R(RFTPM)}}{\frac{\pi}{4} D_t^2 L_t} \quad (39)$$

Equation (22), (31), (33), (38), and (39) well define the RFTPM machine power density. In these equations, the only variable to be determined is efficiency η. It closely depends upon the materials used, electric loading A, magnetic loading B_{gmax}, current density J_c, and the selection of the number of pole pairs, frequency, machine structure and geometric sizes. Usually, the efficiency of PM machines may vary from 80% to 95% for small and medium power ratings.

B. Sizing and Power Density Equations of Slotted RFTPM Machines

The general sizing equations for the non-slotted RFTPM machines have been derived in the last section. For the slotted RFTPM machines shown in Fig. 2B, the expressions of the diameter ratio λ and the total length L_t should be updated due to the presence of slots. The new relationship among d_{ys} ,D_i , and D_o is

$$D_i = D_o - 2d_{ys} - 2d_{ss} \quad (40)$$

where d_{ss} is the sum of the depths of inner and outer stator slots, which is composed of the inner slot depth d_{ss1} and the outer slot depth d_{ss2} indicated in (41) and (42).

$$d_{ss1} = \frac{A_{s1}}{K_{ds} K_{slot} J_c K_{cu}} \quad (41)$$

$$d_{ss2} = \frac{A_{s2}}{K_{ds} K_{slot} J_c K_{cu}} \quad (42)$$

where K_{slot} is the ratio of the slot width to the total width occupied by a pair of slot and tooth. In two equations above, it is assumed that the factor K_{ds} and K_{slot} are same for both inner and outer slots. Given $A_s = A_{s1} + A_{s2}$, d_{ss} can therefore expressed as

$$d_{ss} = d_{ss1} + d_{ss2} = \frac{A_s}{K_{ds}K_{slot}J_c K_{cu}} \tag{43}$$

Substituting (20), which is still correct for the slotted case, into (40), and after some manipulation, it yields

$$\frac{D_i}{D_o} = 1 - \frac{2\pi\alpha_p B_{gmax}}{2pK_{fe}B_{cs} + \pi\alpha_p B_{gmax}} - \frac{4pK_{fe}B_{cs}d_{ss}}{2pK_{fe}B_{cs} + \pi\alpha_p B_{gmax}} \frac{1}{D_o} \tag{44}$$

Substituting (43), the ratio λ is finally found as

$$\lambda = \frac{D_i}{D_o} = 1 - \frac{2\pi\alpha_p B_{gmax}}{2pK_{fe}B_{cs} + \pi\alpha_p B_{gmax}} - \frac{4pK_{fe}B_{cs}A_s}{K_{ds}K_{slot}K_{cu}J_c(2pK_{fe}B_{cs} + \pi\alpha_p B_{gmax})} \frac{1}{D_o} \tag{45}$$

Note that the first two terms are same as the non-slotted case in (22), the last one is caused by the slots and is inversely proportional to the stator out diameter D_o.

Since the winding thickness is already included in D_o for this case, the machine total outer diameter D_t should be updated to

$$D_t = D_o + 2g + 2H_{PM2} + 2d_{yr2} \tag{46}$$

Also, the magnet thickness, H_{PM2}, will be different due to the change in the effective air gap.

$$H_{PM2} = \frac{\mu_r K_{c2}\, g B_{gmax}}{B_r - B_m} \tag{47}$$

where K_{c2} is the Carter coefficient for stator outside slotting. Substituting (47) and (30), which is kept unchanged for the slotted case, into (46), it yields

$$D_t = D_o\left(1 + \frac{\pi K_\alpha}{2p}\frac{B_m}{B_{cr}}\right) + 2g\left(1 + \frac{\mu_r K_{c2} B_{gmax}}{B_r - B_m}\right) \tag{48}$$

The total machine length L_t can be expressed as

$$L_t = L + 2L_{cue} + L_{re} \tag{49}$$

where L_{cue} is the protrusion of the end winding from the iron stack and should be updated to.

$$L_{cue} = \frac{1}{1+\lambda}\frac{A_s}{K_{slot}J_c K_{cu}} \tag{50}$$

Equation (34) through (36) are still correct. Although the expressions of the overall sizing equation and power density indicated in (51), (52), and (53) have the same forms as those in the last section, the calculation formulas of λ, D_t, and L_t are modified.

$$P_{R(RFTPM)} = 8.6951\eta K_w B_{gmax} A \frac{f}{p} \lambda D_o^2 L_e \tag{51}$$

$$P_{R(RFTPM)} = 11.07\eta K_w K_{FL} \sin\frac{K_\alpha \pi}{2} B_m A \frac{f}{p} \lambda D_o^2 L_e \tag{52}$$

$$\xi = \frac{P_{R(RFTPM)}}{\frac{\pi}{4} D_t^2 L_t} \tag{53}$$

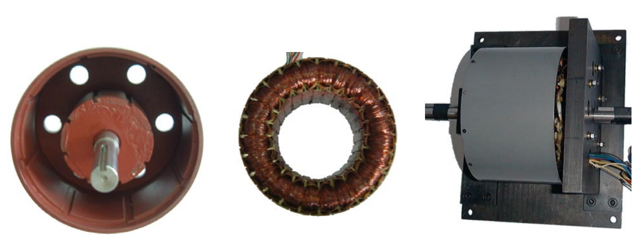
V. Sizing Equation Evaluation and Power Density Comparison

A three-phase 3 HP RFTPM prototype machine shown in Fig. 3 was designed based on the detailed design equations in [6], built, and tested. The main machine performance parameters as well as the machine sizes and power density are listed in Table II. The same prototype machine parameters are used in the sizing equations derived above to evaluate their accuracy. The resultant machine overall length and diameter are 10.24 cm and 17.47 cm, respectively. They are very close to the prototype data of 11.1cm length and 17.7cm diameter. The power density from the sizing equation is 0.91 W/cm^3, which is only 11% higher than the prototype data of 0.82 W/cm^3. For a sizing equation with simplifications, 11% error is reasonable, which proves the sizing equation's accuracy.

It is now possible to compare the power densities of dual-rotor RFTPM machines and other machines based on the sizing equations. The power density comparison between IM, AFTPM and RFTPM machines is shown in Fig. 4, where the data of the RFTPM machines are calculated based on the sizing and power density equations derived before, while those of AFTPM and induction machines were summarized by Dr. Luo [1].

Every point on the curves represents a different machine design optimized for that particular rated mechanical speed. The curves show that the non-slotted rare earth RFTPM with 8 poles and 4 poles achieve the higher power density by a factor of 1.3~1.4 than the AFTPMs with 8 poles and 4 poles, respectively. Even the ferrite magnets are used in the slotted topologies, they still reveal as high power density capability as the AFTPM machines with rare earth magnets.

It should be mentioned that the sizing equations are derived to provide a simple comparison method of the capability of different machine topologies. They are not suitable and should not be used for the machine detailed design due to the simplification, which usually introduces some error. For example, the error caused by the simplification makes the power density lines, Curve 3 and 6, Curve 2 and 4 in Fig. 4, crossed, respectively.



Rotor Stator Overview

Figure 3. RFTPM prototype machine

Table II RFTPM Prototype Parameters

Torque	12.87 Nm	Speed	1800 r/min
Efficiency	0.87	Power factor	0.78
DC bus voltage	380 V	Phase current	7.15 A, rms
Inner elec. loading	245 A/cm	Current density	452 A/cm^2
Outer elec. loading	186 A/cm	Air gap flux	0.217 T
Overall diameter	17.7 cm	Overall length	11.1 cm
Volume	2730 cm^3	Power density	0.82 W/cm^3

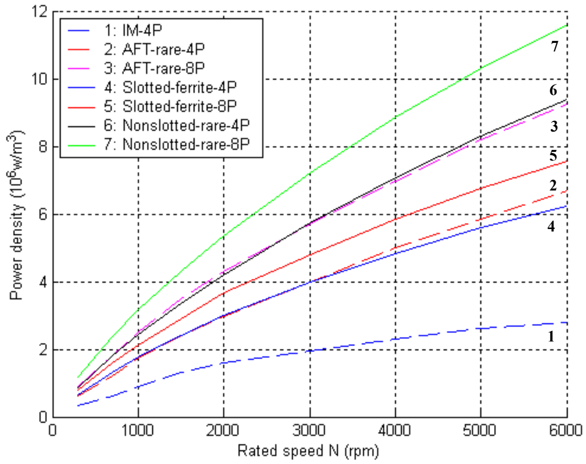


Figure 4. Power density comparison among RFTPM, AFTPM and induction machines

For IM and AFTPMs, A=60 kA/m, J_c=6.2 ×10^6 A/m^2, P_R=75 kW
For RFTPM, A=50/45 kA/m, J_c=6.2 ×10^6 A/m^2, P_R=65 kW(Rare) / 75 kW(Ferrite)

VI. Conclusion

After the review of the general purpose sizing and power density equations and novel RFTPM machine structures, the sizing equations for the radial-flux, toroidally wound, permanent magnet machines have been derived. Both slotted and non-slotted machine topologies are included. These equations are in term of the machine overall sizes, material properties, electrical loading, magnetic loading, and some factors and constants. They are easy to be employed.

This new approach permits a comparison of the novel RFTPM machines with other machine topologies based upon the overall occupied volume, not the airgap volume. It has been proven by a prototype machine that the sizing equation, together with experience parameters, can provide quite accurate estimation for the machine sizes and performances.

A fair comparison among induction, AFTPM, and RFTPM machines was carried out based upon the sizing and power density equations. The RFTPM structure appears to be capable of substantially higher power density than equivalent induction machines, and more potential to achieve higher torque density than AFTPM machines.

The sizing equations were derived to provide a simple and quick comparison method among different machine topologies. They are simplified and lack of design details. To design a RFTPM machine, more detailed calculations may be needed.

Reference

[1] J. Luo, *Axial Flux Circumferential Current Permanent Magnet Electric Machines*, Ph.D. Thesis, University of Wisconsin-Madison, Dec. 1999.

[2] S. Huang, J. Luo, F.Leonardi, and T. A. Lipo, “A general Approach to Sizing and Power Density Equations for Comparison of Electrical Machines”, *IEEE Transactions on Industry Applications*, Vol. 34, No.1, Jan.Feb 1998, pp.92-97.

[3] S. Huang J. Luo, F. Leonardi, and T. A. Lipo, “Comparison of Power Density for Axial Flux Machines Based on General Purpose Sizing Equations”, *IEEE Transactions on Energy Conversion*. Vol. 14 No.2, 1999, pp.185-192.

[4] S. Huang, J. Luo, and T. A. Lipo, “Evaluation of the Transverse Flux Circumferential Current Machine by the Use of Sizing Equations”, *IEEE International Electric Machines and Drives Conference*, IEMDC. Milwaukee, WI, USA, 1997, pp.15.1-15.3.

[5] V. B. Honsinger, “Sizing Equations for Electrical Machinery”, *IEEE Transactions on Energy Conversion*, Vol. EC-2, No. 1, March 1987, pp. 116-121.

[6] R. Qu, *Design and Analysis of Dual-Rotor, Radial-Flux, Toroidally Wound, Surface-Mounted-PM Machines*, Ph.D. Thesis, University of Wisconsin-Madison, Aug, 2002.

[7] R. Qu and T. A. Lipo, “Dual-rotor, radial-flux, toroidally wound, permanent-magnet machines”, *IEEE Transactions on Industry Applications*, Vol. 39, No.6, Nov/Dec 2003, pp. 1655-1673.

[8] R. Qu, and T. A. Lipo, “Design and parameter effect analysis of dual-rotor, radial-flux, toroidally wound, permanent-magnet machines”, *IEEE Transactions on Industry Applications*, Vol. 40, No.3, May/June 2004, pp.771-779

[9] R. Qu, and T. A. Lipo, “Analysis and modeling of airgap & zigzag leakage fluxes in a surface-mounted-PM machine”, *IEEE Transactions on Industry Applications*, Vol. 40, No.1, Jan/Feb 2004, pp. 121-127.

[10] D. C. Hanselman, *Brushless Permanent-Magnet Motor Design*, New York, McGraw-Hill, 1994.

[11] J. R. Hendershot Jr and T. J. E. Miller, *Design of Brushless Permanent-Magnet Motor*, Magna Physics Publishing and Clarendon Press, Oxford, 1994.

[12] T. A. Lipo, *Introduction to AC Machine Design*, Vol. 2, Wisconsin Power Electronics Research Center, University of Wisconsin-Madison. 1998.