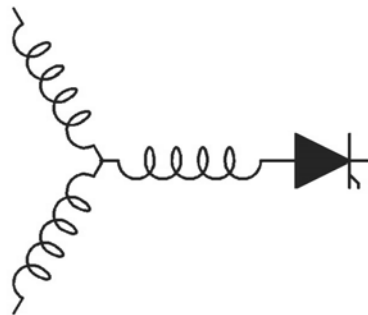


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**A Simplified Model of a Nine Phase Synchronous
Machine Using Vector Space Decomposition**

A. Rockhill and T.A. Lipo

Dept. of Elect. & Comp. Engr.
University of Wisconsin-Madison
1415 Engineering Drive
Madison, WI 53706



**Wisconsin
Electric
Machines &
Power
Electronics
Consortium**

University of Wisconsin-Madison
College of Engineering
Wisconsin Power Electronics Research Center
2559D Engineering Hall
1415 Engineering Drive
Madison WI 53706-1691

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A Simplified Model of a Nine Phase Synchronous Machine Using Vector Space Decomposition

Andrew A. Rockhill and T.A. Lipo
University of Wisconsin
Madison WI USA

Abstract - This paper describes the use of vector space decomposition for development of a simplified model of a nine phase synchronous machine. It is shown that the nine phase machine can be reduced to the usual d - q equivalent circuit model plus seven additional circuits to represent zero sequence components. These additional components are shown to provide additional current paths for both zero sequence components as well as negative sequence components.

I. INTRODUCTION

Wind power generation is tending toward ever larger ratings and has reached power levels exceeding 5 MW [1]. As electrical machine ratings increase beyond this level, it is well known that the wound field synchronous machine has traditionally had performance and cost advantages over any other type of electrical machine in large sizes. Also, as the ratings of the wind turbine continually progress upward, the presence of a gear train to convert mechanical power from the turbine shaft to the electrical machine shaft becomes progressively more expensive and unreliable. Work is in progress by several wind turbine manufacturers to adopt direct conversion schemes in which the gear set is eliminated and machine directly driven at the low speed of the turbine. Such large machines require corresponding large solid state power converters. It has been recognized that improved reliability and lower cost may be realized by adopting a six or nine phase arrangement where smaller, lower cost power converters can be utilized to provide the necessary power conversion for use with the 50 or 60 Hz power grid. However, the modeling of such machines has only recently been undertaken for induction machines [2] and no work appears to have been carried out on similar modeling of a synchronous machine[3].

In this paper a d - q model is developed for a nine phase salient pole synchronous machine. Matrix theory together with the principle of vector space decomposition is used to develop a model in which the complex coupling of the nine phase machine is replaced by a d - q model as well as zero sequence circuits in which coupling between circuits only occurs as speed voltages in the d - q model in the conventional manner.

II. VECTOR SPACE DECOMPOSITION

Figure 1 shows a simplified diagram of the nine phase synchronous machine and associated power converters. In a nine phase system the adjacent phases are shifted by 20 electrical degrees as shown. In general, being nine phases, nine independent currents can flow making up a nine dimensional system. Two of these cur-

rent can be determined by assuming projections of nine components of flux, current and voltage on a stationary set of axes (α - β axes or non-rotating d - q axes) located in such a manner that the α axis lines up with one of the phases, designated by phase a_1 .

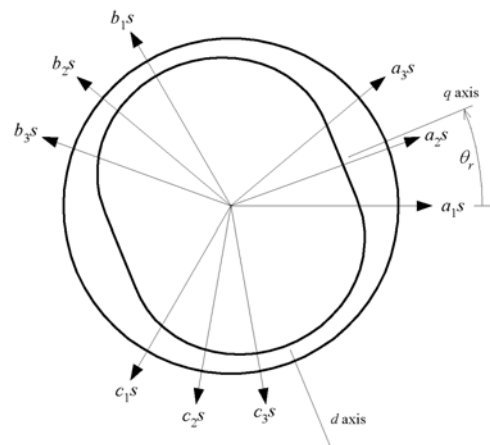


Figure 1 Nine phase salient pole synchronous machine showing orientation of the phase with respect to the d - q axes.

The other axis, orthogonal to the α axis, is located clockwise with respect to this axis and is termed the β axis. When these two axes are rotated they form the rotating d - q axes shown in Fig. 1. The projection of the nine phases on the α - β axes can be expressed as vectors as [2],[4]:

$$v_{\alpha} = [1, \cos(\gamma), \cos(2\gamma), \cos(6\gamma), \cos(7\gamma), \cos(8\gamma) \\ \cos(12\gamma), \cos(13\gamma), \cos(14\gamma)]^t \quad (1)$$

$$v_{\beta} = [0, \sin(\gamma), \sin(2\gamma), \sin(6\gamma), \sin(7\gamma), \sin(8\gamma), , \\ \sin(12\gamma), \sin(13\gamma), \sin(14\gamma)]^t \quad (2)$$

where “ t ” denotes the transpose and $\theta = \pi/9$. To show that these two vectors are orthogonal it can be demonstrated that

$$v_{\alpha}^t \cdot v_{\beta} = 0 \quad (3)$$

Other orthogonal vectors can be computed by increasing the argument of Eqs. 1 and 2 by an integer. For example, increasing the argument by a factor of three,

$$v_{z1} = [1, \cos(3\gamma), \cos(6\gamma), \cos(18\gamma), \cos(21\gamma), \cos(24\gamma), \\ \cos(36\gamma), \cos(39\gamma), \cos(42\gamma)]^t \quad (4)$$

$$v_{z2} = [0, \sin(3\gamma), \sin(6\gamma), \sin(18\gamma), \sin(21\gamma), \sin(24\gamma), \\ \sin(36\gamma), \sin(39\gamma), \sin(42\gamma)]^t \quad (5)$$

However, since $\cos(18\gamma) = \cos\left(18\frac{\pi}{9}\right) = \cos(2\pi) = 1$ and so forth, Eqs. 4 and 5 can also be written as

$$\mathbf{v}_{z1} = [1, \cos(3\gamma), \cos(6\gamma), 1, \cos(3\gamma), \cos(6\gamma), 1, \cos(3\gamma), \cos(6\gamma)]^t \quad (6)$$

$$\mathbf{v}_{z2} = [0, \sin(3\gamma), \sin(6\gamma), 0, \sin(3\gamma), \sin(6\gamma), 0, \sin(3\gamma), \sin(6\gamma)]^t \quad (7)$$

Increasing the argument of Eqs. 1 and 2 by 5, after simplification

$$\mathbf{v}_{z3} = [1, \cos(5\gamma), \cos(10\gamma), \cos(12\gamma), \cos(17\gamma), \cos(4\gamma), \cos(6\gamma), \cos(11\gamma), \cos(16\gamma)]^t \quad (8)$$

$$\mathbf{v}_{z4} = [0, \sin(5\gamma), \sin(10\gamma), \sin(12\gamma), \sin(17\gamma), \sin(4\gamma), \sin(6\gamma), \sin(11\gamma), \sin(16\gamma)]^t \quad (9)$$

If the argument of Eqs. 1 and 2 are increases by 7,

$$\mathbf{v}_{z5} = [1, \cos(7\gamma), \cos(14\gamma), \cos(6\gamma), \cos(13\gamma), \cos(2\gamma), \cos(12\gamma), \cos(1\gamma), \cos(8\gamma)]^t \quad (10)$$

$$\mathbf{v}_{z6} = [0, \sin(7\gamma), \sin(14\gamma), \sin(6\gamma), \sin(13\gamma), \sin(2\gamma),$$

$$\sin(12\gamma), \sin(1\gamma), \sin(8\gamma)]^t \quad (11)$$

Finally, if the argument increases by 9,

$$\mathbf{v}_{z7} = [1, -1, 1, -1, 1, -1, 1, -1, 1]^t \quad (12)$$

$$\mathbf{v}_{z8} = [0, 0, 0, 0, 0, 0, 0, 0, 0]^t \quad (13)$$

Altogether 10 orthogonal vectors have been produced which can be used to transform the differential equations of the nine phase machine to orthogonal components. Either of the two vectors obtained by increasing the argument by 9 can be used and \mathbf{v}_{z7} has been selected since \mathbf{v}_{z8} is a trivial case. The 9 vectors can now be assembled to form a transformation matrix \mathbf{T} as shown in Eq. 14. It can be shown that increasing the arguments of Eq. 1 and 2 by factors greater than 9 will reproduce the vectors already obtained. The scaling of all coefficients by $\sqrt{2}/3$ and the 9th column also by $1/(\sqrt{2})$ is needed to make the transformation orthogonal, that is $\mathbf{T}\mathbf{T}^t = \mathbf{I}$.

$$\mathbf{T} = \frac{\sqrt{2}}{3} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ \cos(\gamma) & \sin(\gamma) & \cos(3\gamma) & \sin(3\gamma) & \cos(5\gamma) & \sin(5\gamma) & \cos(7\gamma) & \sin(7\gamma) & -1/\sqrt{2} \\ \cos(2\gamma) & \sin(2\gamma) & \cos(6\gamma) & \sin(6\gamma) & \cos(10\gamma) & \sin(10\gamma) & \cos(14\gamma) & \sin(14\gamma) & 1/\sqrt{2} \\ \cos(6\gamma) & \sin(6\gamma) & 1 & 0 & \cos(12\gamma) & \sin(12\gamma) & \cos(6\gamma) & \sin(6\gamma) & 1/\sqrt{2} \\ \cos(7\gamma) & \sin(7\gamma) & \cos(3\gamma) & \sin(3\gamma) & \cos(17\gamma) & \sin(17\gamma) & \cos(13\gamma) & \sin(13\gamma) & -1/\sqrt{2} \\ \cos(8\gamma) & \sin(8\gamma) & \cos(6\gamma) & \sin(6\gamma) & \cos(4\gamma) & \sin(4\gamma) & \cos(2\gamma) & \sin(2\gamma) & 1/\sqrt{2} \\ \cos(12\gamma) & \sin(12\gamma) & 1 & 0 & \cos(6\gamma) & \sin(6\gamma) & \cos(12\gamma) & \sin(12\gamma) & 1/\sqrt{2} \\ \cos(13\gamma) & \sin(13\gamma) & \cos(3\gamma) & \sin(3\gamma) & \cos(11\gamma) & \sin(11\gamma) & \cos(\gamma) & \sin(\gamma) & -1/\sqrt{2} \\ \cos(14\gamma) & \sin(14\gamma) & \cos(6\gamma) & \sin(6\gamma) & \cos(16\gamma) & \sin(16\gamma) & \cos(8\gamma) & \sin(8\gamma) & 1/\sqrt{2} \end{bmatrix} \quad (14)$$

III. NINE PHASE SYNCHRONOUS MACHINE EQUATIONS IN PHASE VARIABLE FORM

The depiction of the machine in terms of phase variables involve complete inducting coupling between the nine phase units. Regardless of the number of phases, when the resistance is the same in each phase, the differential equations of the machine may be written as

$$\mathbf{v}_s = \mathbf{R}_s \mathbf{i}_s + \frac{d}{dt} \lambda_s \quad (15)$$

and

$$\mathbf{v}_r = \mathbf{R}_r \mathbf{i}_r + \frac{d}{dt} \lambda_r \quad (16)$$

where \mathbf{R}_s is a scalar,

$$\mathbf{v}_s = [v_a, v_b, v_c, v_d, v_e, v_f, v_g, v_h, v_k]^t \quad (17)$$

and \mathbf{i}_s and λ_s are similarly defined.

For the rotor is the diagonal matrix

$$\mathbf{R}_r = \begin{bmatrix} R_{fr} & 0 & 0 \\ 0 & R_{dr} & 0 \\ 0 & 0 & R_{qr} \end{bmatrix} \quad (18)$$

and

$$\mathbf{v}_r = [v_{fr}, 0, 0]^t \quad (19)$$

where \mathbf{i}_r and λ_r are defined below.

The stator flux linkage can be expressed as

$$\lambda_s = L_{ls} \mathbf{i}_s + \mathbf{L}_{ms} \mathbf{i}_s + \mathbf{L}_{sr} \mathbf{i}_r \quad (20)$$

where, L_{ls} is a scalar and

$$\mathbf{i}_r = [i_{fr}, i_{dr}, i_{qr}]^t \quad (21)$$

The \mathbf{L}_{ms} and \mathbf{L}_{sr} are 9x9 and 3x9 matrices which represent the coupling of the nine phases with itself and with the rotor respec-

tively. The matrix L_{ms} can, in turn, be broken up into the following

$$L_{ms} = L_{os}S - L_{2s}R(\theta_r) \quad (22)$$

Here S is a 9x9 matrix which represents the mutual coupling of a round rotor machine. The inductance $R(\theta_r)$ corresponds to the additional coupling which exists for a salient pole machine. The first row of the matrix representing a symmetric machine is

$$S(1, :) = \left[1, \cos\left(\frac{\pi}{9}\right), \cos\left(\frac{2\pi}{9}\right), \cos\left(\frac{6\pi}{9}\right), \cos\left(\frac{7\pi}{9}\right), \cos\left(\frac{8\pi}{9}\right), \right. \\ \left. \cos\left(\frac{12\pi}{9}\right), \cos\left(\frac{13\pi}{9}\right), \cos\left(\frac{14\pi}{9}\right) \right] \quad (23)$$

Since the matrix S is symmetric, the remaining rows are readily determined. For example, the second row is

$$S(2, :) = \left[\cos\left(\frac{14\pi}{9}\right), \cos\left(\frac{\pi}{9}\right), \cos\left(\frac{5\pi}{9}\right), \cos\left(\frac{6\pi}{9}\right), \cos\left(\frac{7\pi}{9}\right), \right. \\ \left. \cos\left(\frac{11\pi}{9}\right), \cos\left(\frac{12\pi}{9}\right), \cos\left(\frac{13\pi}{9}\right) \right] \quad (24)$$

and so forth.

The first row of matrix $R(\theta_r)$ representing the additional effect due to saliency is

$$R(1, :) = \left[\cos 2\theta_r, \cos 2\left(\theta_r - \frac{\pi}{9}\right), \cos 2\left(\theta_r - \frac{2\pi}{9}\right), \cos 2\left(\theta_r - \frac{6\pi}{9}\right), \right. \\ \left. \cos 2\left(\theta_r - \frac{7\pi}{9}\right), \cos 2\left(\theta_r - \frac{8\pi}{9}\right), \cos 2\left(\theta_r - \frac{12\pi}{9}\right), \right. \\ \left. \cos 2\left(\theta_r - \frac{13\pi}{9}\right), \cos 2\left(\theta_r - \frac{14\pi}{9}\right) \right] \quad (25)$$

Since $R(\theta_r)$ is also symmetric, the remainder of the rows are also readily determined.

The inductance coefficients of S and $R(\theta_r)$ are

$$L_{os} = \mu_o r l N_s^2 \left(\frac{\pi}{8}\right) \left(\frac{1}{g_{max}} + \frac{1}{g_{min}}\right) \quad (26)$$

$$L_{2s} = \mu_o r l N_s^2 \left(\frac{\pi}{8}\right) \left(\frac{1}{g_{max}} - \frac{1}{g_{min}}\right) \quad (27)$$

where r is the radius of the machine at the stator inner surface, l is the effective length of the machine, N_s is the number of effective stator turns, g_{max} and g_{min} are the maximum and minimum gap. If the machine has a round rotor then

$$L_{os} = \mu_o \frac{r l}{g} N_s^2 \frac{\pi}{4} \quad (28)$$

$$L_{2s} = 0 \quad (29)$$

The equations describing the flux linkages which link the stator due to rotor currents can be expressed in the form

$$\lambda_{sr} = L_{sfr}i_{fr} + L_{sdr}i_{dr} + L_{sqr}i_{qr} \quad (30)$$

in which the inductances are the vectors

$$L_{sfr} = L_{sf} \left[\cos(\theta_r), \cos\left(\theta_r - \frac{\pi}{9}\right), \cos\left(\theta_r - \frac{2\pi}{9}\right), \dots, \cos\left(\theta_r - \frac{4\pi}{9}\right) \right]^t \quad (31)$$

$$L_{sdr} = L_{sd} \left[\cos(\theta_r), \cos\left(\theta_r - \frac{\pi}{9}\right), \cos\left(\theta_r - \frac{2\pi}{9}\right), \dots, \cos\left(\theta_r - \frac{4\pi}{9}\right) \right]^t \quad (32)$$

$$L_{sqr} = L_{sq} \left[\sin(\theta_r), \sin\left(\theta_r - \frac{\pi}{9}\right), \sin\left(\theta_r - \frac{2\pi}{9}\right), \dots, \sin\left(\theta_r - \frac{4\pi}{9}\right) \right]^t \quad (33)$$

wherein

$$L_{sf} = \mu_o \frac{r l}{g_{min}} N_s N_f \left(\frac{\pi}{4}\right) \quad (34)$$

$$L_{sd} = \mu_o \frac{r l}{g_{min}} N_s N_{dr} \left(\frac{\pi}{4}\right) \quad (35)$$

$$L_{sq} = \mu_o \frac{r l}{g_{max}} N_s N_{qr} \left(\frac{\pi}{4}\right) \quad (36)$$

The rotor flux linkages can be expanded to the form

$$\lambda_r = (L_{lr} + L_{mr})i_r + L_{rs}i_s \quad (37)$$

in which

$$L_{lr} + L_{mr} = \begin{bmatrix} L_{lfr} + L_{mfr} & 0 & 0 \\ 0 & L_{ldr} + L_{mdr} & 0 \\ 0 & 0 & L_{lqr} + L_{mqr} \end{bmatrix} \quad (38)$$

The rotor self inductances are identical to those obtained for three phase machines, namely,

$$L_{rs} = \begin{bmatrix} L_{sfr}^t \\ L_{sdr}^t \\ L_{sqr}^t \end{bmatrix} \quad (39)$$

The values of the inductances associated with the rotor leakage and magnetizing components are clearly the same as for a conventional three phase machine [4].

IV. DECOUPLED SYNCHRONOUS MACHINE EQUATIONS

The complexity of the nine phase machine is clear motivation to produce a set of equivalent equations where the coupling between components is a minimum. This result can be obtained by successively operating on portions of the overall system equation. For example, if the stator differential equation, Eq. (15) is multiplied by T then,

$$T v_s = R_s T i_s + \frac{d}{dt} T \lambda_s \quad (40)$$

Defining $T\mathbf{v}_s = \mathbf{v}_{st}$, $T\mathbf{i}_s = \mathbf{i}_{st}$, and $T\lambda_s = \lambda_{st}$, where the additional subscript 't' denotes the "transformed" version of the quantity. Eq.(40) becomes

$$\mathbf{v}_{st} = R_s \mathbf{i}_{st} + \frac{d}{dt} \lambda_{st} \quad (41)$$

While the change of variables is straightforward for the voltages and currents, the change is, of course, not so simple for the flux linkages. Upon multiplying Eq. (20) by T and changing the flux linkage and current variables to the new components results in

$$\lambda_{st} = L_{ls} \mathbf{i}_{st} + \mathbf{TL}_{ms} T^{-1} \mathbf{i}_{st} + \mathbf{TL}_{sr} T^{-1} \mathbf{i}_{rt} \quad (42)$$

After performing the indicated operations on the second and third term, Eq.(42) eventually reduces to

$$\lambda_{st} = L_{ls} \mathbf{i}_{st} + \mathbf{L}_{mst} \mathbf{i}_{st} + \mathbf{L}_{srt} \mathbf{i}_{rt} \quad (43)$$

where,

$$\mathbf{L}_{mst} = \left(\frac{9}{2}\right) L_{0s} \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \quad (44)$$

$$\mathbf{L}_{srt} = \left(\frac{9}{2}\right) L_{2s} \begin{bmatrix} \sin\theta_r & \cos\theta_r & 0 & \dots & 0 \\ \cos\theta_r & -\sin\theta_r & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (45)$$

Equations 38, 41, 44 and 45 suggest the block diagram representing the transformed nine phase synchronous machine as shown in Figure 2. Park's model for the synchronous machine is well

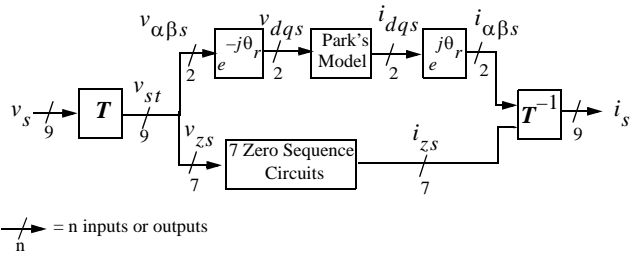


Figure 2 Block diagram of nine phase synchronous machine model.

$e^{-j\theta_r}$ represents the α - β to d - q rotor frame transformation.

known and presented in many texts, e.g. [5]. The seven zero sequence circuits are simple R - L circuits comprised of the stator resistance and the stator leakage inductance respectively.

V. EFFECT OF UNBALANCED OPERATION

Since the equivalent circuit contains seven zero sequence circuits it is interesting to determine how these seven circuits respond to various types of unbalances including the presence of the conventional zero component. The identity of which of the seven circuits obtained by the transformation T correspond to a particular three phase group can be determined by applying a zero sequence component to one of the three phase groups, for example $a1, b1, c1$ as designated in Fig.1 while leaving the remaining three phase groups balanced. Figure 3 shows the nine decoupled voltage components obtained by passing the nine applied voltages in phase variable form through the transformation

$$\mathbf{v}_{st} = T\mathbf{v}_s \quad (46)$$

where

$$\mathbf{v}_s = V_s \left[\cos\theta, \cos\left(\theta - \frac{\pi}{9}\right), \cos\left(\theta - \frac{2\pi}{9}\right), \cos\left(\theta - \frac{2\pi}{3}\right), \cos\left(\theta - \frac{7\pi}{9}\right), \cos\left(\theta - \frac{8\pi}{9}\right), \cos\left(\theta - \frac{4\pi}{3}\right), \cos\left(\theta - \frac{13\pi}{9}\right), \cos\left(\frac{14\pi}{9}\right) \right]^t + V_{s3} [\cos(3\theta), \cos(3\theta), \cos(3\theta), 0, 0, 0, 0, 0, 0]^t \quad (47)$$

The results are shown in Fig. 3.

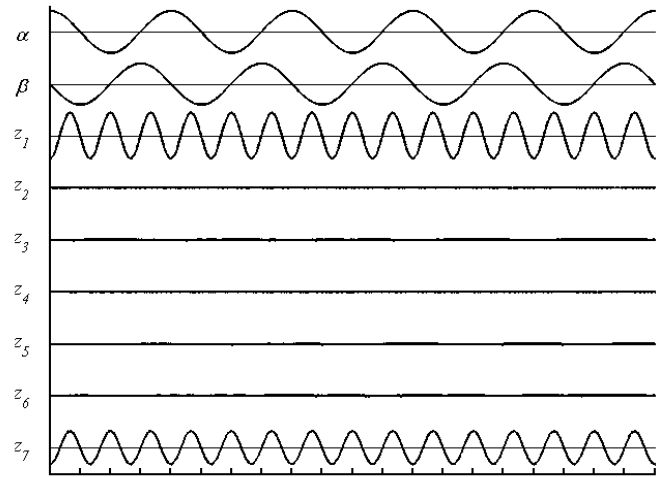


Figure 3 Voltages of the transformed system of equations when the stator is excited with a balanced fundamental component set plus a third harmonic set in phase group #1. Zero components scaled 10 times larger than the α, β components.

It is interesting to note zero sequence voltages are produced not only in two of the zero components circuits rather than one as might be expected. In some applications the neutrals of the three groups 1,2 and 3 are tied together thereby forming a nine phase star. In this case it can be shown that circulating zero sequence components will then flow in all nine phases of such a machine if

any of the phase groups contain a zero sequence component even if the neutral is not tied to ground.

The obvious approach to eliminating these components is to let the three groups remain isolated with three independent neutrals. If the neutrals are not grounded, the zero sequence currents are zero even though the zero sequence voltages continue to exist as voltages of the neutral points with respect to the system ground.

Although the detrimental effects of the zero sequence component can be resolved, four more uncoupled components having the same parameters as the three zero sequence circuits are present. Figure 4 shows the transformed set of voltages which occur when the amplitude of voltage set of the second group is 90% that of the other two sets. It can be noted that four of the zero sequence components are affected

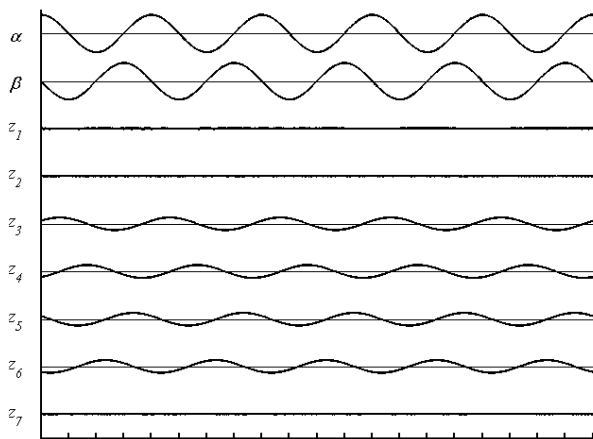


Figure 4 Voltages of the transformed system of equations when all three voltage sets are balanced but the amplitude of set #2 is 90% that of sets #1 and #3. Zero components scaled 10 times larger than the α, β components.

The effect of unbalances which create negative sequence components can also be examined by means of Eq. (46). In Figure 5, a negative voltage set was created by letting the amplitude of the voltage of phase c2 to be 50% that of the other two phases, a2 and b2. The voltages of sets #1 and #3 are assumed to be balanced rated voltages.

VI. CONCLUSION

This paper has developed the equivalent of Park's Equations of a three phase salient pole synchronous machine for the case of a nine phase stator winding. The resulting analysis results in a transformed set of variables in which two of the variables correspond to the conventional α, β quantities, three of the new variable correspond to traditional zero sequence components and 4 of the variables correspond to quantities in circuits which again contain only stator resistance and stator leakage inductance but are excited whenever the phases become an unbalanced voltage set. This representation should help shed some light on the difficulties associ-

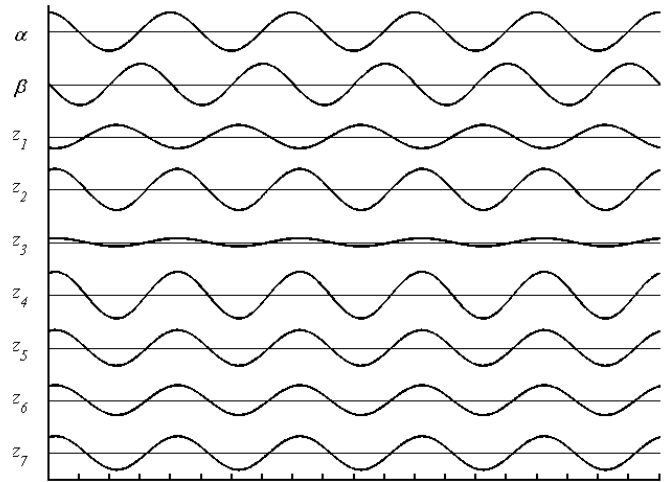


Figure 5 Voltages of the transformed system of equations when amplitude of the phase voltage c2 of stator set #2 is 50% that of phase a2 and b2. The voltages of sets #1 and #3 are assumed to be balanced. Zero components scaled 10 times larger than the α, β components

ated with the application of such machines to applications such as wind turbines where the stator current is controlled by solid state power converters. It is interesting to observe that with this relatively common and simple unbalance condition all 7 zero sequence components are affected.

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