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Control of Bi-Directional Z-Source Inverter

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Abstract: A new topology bi-directional Z-source inverter is proposed in this paper. It can overcome the uncontrollable and unstable limitations of the Z-source inverter in the discontinuous conduction mode (DCM). The operating principle, switching signal flow graph model and transfer function for the circuit are developed. In addition, the system stability and dynamic performance are analyzed thoroughly. Due to the presence of two control variables D and M, the control of the system is very nonlinear. A dual closed loop controller is proposed to improve the stability and dynamic performance. A prototype of a 55kVA bi-directional Z-source inverter is designed to verify the principle and demonstrate the performance of the proposed topology.

Keywords: Z-source inverter, DCM mode

I. INTRODUCTION

The recently, the proposed Z-source inverter (ZSI) [1] utilizes a unique LC network to handle shoot through states and boost the output voltage. The Z-source inverter can boost and buck the output voltage with a single stage structure. The shoot through caused by EMI can no longer destroy the inverter, which increases the reliability of the inverter greatly. Because no dead time is required, excellent sinusoidal output waveform is obtainable. The operating principle has been described in [1]. However, all the descriptions and analyses are based on an assumption that the inductance of inductor in the Z-source network is great enough to maintain the inductor current almost constant, and that the Z-source inverter works in the continuous conduction mode (CCM). In some applications, the inductance should be minimized in order to reduce cost, volume, and weight. In the case of Z-source inverter with small inductance or light-load operation, The CCM critical condition \( I_L > \frac{1}{2} I_{in} \) is not satisfied [2]. The Z-source inverter then works in the discontinuous conduction mode (DCM), the dc-link voltage increases indefinitely, and the output voltage will be uncontrollable so that the system is unstable.

When using the Z-source LC network, the circuit characteristic becomes much more complex than traditional voltage source and current source inverters. Due to two control variables D and M, the control of system is a nonlinear and complex. Small signal models have been developed [3] for the Z-source inverter. There are also several controllers designed for the Z-source inverter [4-5]. Both controllers control the capacitor voltage and the output voltage using separate control loops, which theoretically cannot guarantee system stability.

This paper proposes a new topology bi-directional Z-source inverter (Bi-ZSI) to overcome the above limitations. The operational principle, PWM signals and operate modes are analyzed thoroughly in section II. In section III, the switching signal flow graph (SFG) models are developed. Based on the SFG mode, the system transfer functions are deduced and the system stability is analyzed with Bode plots. In order to improve the stability and dynamic performance, the two control variables are decoupled first, then a dual closed loop controller with inner current loop and outer voltage loop are proposed in section IV. Saber simulation of the static and dynamic performance for a Z-source inverter with a closed loop controller are discussed in section V. Finally, a prototype of 55kVA bi-directional Z-source inverter is designed to verify the principle and performance of the proposed topology in section VI.

II. THE BI-DIRECTIONAL Z-SOURCE INVERTER

When the inductance is small or the load power factor is low, the inductor current can have high ripple or even become discontinuous. In this case the Z-source inverter works in a DCM condition. Under this condition the circuit may have five different operating modes when viewed from the Z-source network. While in a CCM condition, it only has two modes. By proper design of the Z-source network and proper control, one can avoid the unwanted operation mode under certain conditions. However, it is generally impossible to completely avoid these modes.

Figure 1 shows a new topology termed the bi-directional Z-source inverter. By using this topology, the inverter is able to completely avoid the unwanted operating modes by turning on the switch \( S_7 \) during all active states and traditional zero states. Furthermore, this topology provides the circuit with a bi-directional power flow function. Figure 2 shows the PWM signals for 3-phase power switches and bi-directional switch \( S_7 \). in Z-source inverter with a simple control strategy.

![Figure 1. Bi-directional Z-source inverter.](image-url)
Figure 2. PWM signals for bi-directional Z-source inverter.

Figure 3 shows the possible operation modes of the bi-directional Z-source inverter. In one cycle, the circuit has possible seven operating modes.

Figure 4a and Figure 4b show the inductor current and period of each operation mode for the ZSI in the DCM condition as well as the Bi-ZSI. The ZSI in CCM operates with mode 1,2. Also the ZSI in DCM can operate from mode 1 to mode 5, while BI-ZSI can operate from mode 1 to mode 7. With different control methods, different circuit parameters, and load, the inverter can operate differently with different combinations and sequences of the above 7 modes.

From Figure 4b, considering the voltage relationship, only two modes of the bi-directional Z-source inverter is left in Figure 5. This result simplifies the modeling and analysis of the inverter system. The defining equations are:

\[ V_{c1} = V_{c2} = V_c = \frac{1 - D_s}{1 - 2D_s} V_0 \]  
\[ \dot{V}_c = \frac{M}{2(1 - 2D_s)} V_0 \]  

III. THE MODEL AND ANALYSIS OF BI-DIRECTIONAL Z-SOURCE INVERTER

To model the Z-source inverter, assuming that the Z-source network is symmetrical, \( L_1 = L_2 = L \), \( C_1 = C_2 = C \), so that the currents through the two inductors and the voltages across the capacitors are the same. The switching signal flow graph (SFG) is a graphic modeling method for nonlinear power converter systems. The SFG model of Bi-Directional Z-Source Inverter can be developed in Figure 6.
From the SFG model, one can deduce the transfer function of the system. The system transfer functions over the shoot through duty ratio $D_s$ are as follows:

$$\hat{V}_i = \frac{(2V_s-V_o-RI_1(D_1-D_s))+(I_1-2I_s)Ls-(2V_s-V_o)LCs\delta}{LC\delta+(R_1+R_s)Cs+(D_1-D_s)^2} \quad (3)$$

$$\hat{V}_c = \frac{(2V_s-V_o)(D_4-D_s)+(I_1-2I_s)Ls}{LC\delta^2+(R_1+R_s)Cs+(D_4-D_s)^2} \quad (4)$$

$$\hat{I}_s = \frac{(2V_s-V_o-R_sI_s)Cs}{LC\delta^2+(R_1+R_s)Cs+(D_4-D_s)^2} \quad (5)$$

From the transfer function, the root locus plot and stability analysis were obtained from a Matlab simulation. To confirm the small signal model, a Bode diagram can be drawn and compared with simulation results, shown in Figure 7.

![Bode plots](image)

From (3), it is clear that there is a right half plane (RHP) zero in the transfer function DC link voltage over the shoot through duty cycle ratio $\hat{V}_i / \hat{d}_s$, and that this transfer function is semi-proper. This condition means that the system can be unstable in certain conditions. From Figure 7 (a), the system has negative gain margin, which corresponds to an unstable system. On the other hand from (4) the transfer function capacitor voltage over the shoot through duty ratio $\hat{V}_c / \hat{d}_s$ is proper. However, it still has a RHP zero when $I_i < 2I_L$, which may lead the transfer function to be unstable. This result means that the system may be unstable under the DCM condition. From Figure 7 (b), the system has negative amplitude margin and phase margin, which again means that the system is unstable. From (5), the transfer functions of the inductor current over the shoot through duty ratio $\hat{I}_L / \hat{d}_s$ has no RHP zero, so it is possible to design a linear PID controller to stabilize it. From Figure 7 (c), the system have positive phase margin, which indicates that the system is stable.

**IV. CONTROLLER DESIGN**

In order to design a controller, one has to define the equilibrium points that are related to the targeted output. For a Z-source inverter, the targeted output values of system are $(V_{CSS}, V_{XSS})$. In a three phase Z-source inverter, there exist two control variables, $D$ (shoot through duty ratio) and $M$ (modulation index). Both two variables control the voltage of capacitor $V_{CSS}$ and output phase voltage $V_{XSS}$ respectively. These two control variables $D$ and $M$ are not independent. Thus it is difficult to design the controller for a Z-source inverter. One needs both controllers to control the capacitor voltage and the output voltage using separate control loops, which theoretically cannot guarantee the system stability.

Figure 8 shows the DC equivalent circuit of a three phase Z-source inverter when operating at shoot through duty ratio of $D$ and modulation index of $M$. In Figure 8, there are two switches, S1 is switching with a duty ratio of $D$, the duty ratio of S2 is $M$, and $D+M \leq 1$.

![DC equivalent circuit](image)

From Figure 8, the following equation can be reached:

$$V_{CSS} = \frac{(1-D_s)}{1-2D_s}V_{in} \quad (6)$$

$$V_{ZSS} = M \frac{1}{1-2D_s}V_{in} \quad (7)$$

$$M \leq 1-D_s \quad (8)$$

$$V_{ZSS} \leq \frac{1-D_s}{1-2D_s}V_{in} = V_{CSS} \quad (9)$$
It is useful to define $V_{ZSS} = kV_{CSS}$, where $k_{ratio} \leq 1$.

In this one can use only one parameter to parameterize the controller for a given ratio $k$.

In last section, we know the transfer functions of the inductor current over the shoot through duty ratio $\hat{I}_L / \hat{d}_S$ is stable, and the transfer function capacitor voltage overshoot through duty ratio $\hat{V}_C / \hat{d}_S$ is proper but still has a RHP zero. Hence, one can design an inner current loop to obtain good dynamic performance. To stabilize the output voltage, the simplest way is to design an outer voltage loop, and a well-designed linear PID controller can adjust the stable performance of the outer voltage loop. Figure 9(a) shows the dual loops controller with inner current loop and outer voltage loop designed for a Z-source inverter. The simplified control schematic is shown in Figure 9(b).

The inner current loop Bode plots with and without a current compensator is shown in Fig. 10(a). The system has a 89-degree phase margin, which indicates that the system is stable. The cross-off frequency is 11 kHz, and system has a quick dynamic response.

The outer voltage loop Bode plots with and without voltage compensator is shown in Figure. 10(b). The system has a 80-degree phase margin and a 16.3 db gain margin, which means system very stable. The cross-off frequency is 1kHz.

**V. SIMULATION RESULTS**

The circuit simulations of static and dynamic performance have been accomplished using the Saber software. The circuit parameters of Bi-Z source inverter are as follows and are the same as the real hardware prototype: $P=55kW$, $L_1 = L_2 = 50\mu H$, $C_1 = C_2 = 380\mu F$. Power switches $S_1 - S_6$ are designed for the desired crossover frequency and phase margin. The current compensator and voltage compensator are,

$$G_v (s) = 50 * \frac{s / 40 + 1}{s(s/10000 + 1)}$$

$$G_v (s) = 3.75 * \frac{s / 3140 + 1}{s(s / 70000 + 1)}$$

The inner current loop Bode plots with and without a current compensator is shown in Fig. 10(a). The system has a 89-degree phase margin, which indicates that the system is stable. The cross-off frequency is 11 kHz, and system has a quick dynamic response.

The outer voltage loop Bode plots with and without voltage compensator is shown in Figure. 10(b). The system has a 80-degree phase margin and a 16.3 db gain margin, which means system very stable. The cross-off frequency is 1kHz.
IPM PM600CLA060 for the inverter bridge, and IPM PM800HSA060 for the bi-directional switch $S_7$.

Figure. 11 (a) simulation shows that Z-source DC link voltage $V_{pn}$ and output voltage are uncontrollable in DCM condition, while bi-directional Z-source inverter can keep the $V_{pn}$ and output voltage constant, shown in Figure 11 (b).

![Figure 11 Simulation of ZSI in DCM and Bi-ZSI](image)

Figure 11. Simulation of ZSI in DCM and Bi-ZSI.

Figure 12 show the static and dynamic performance of the Bi-Directional Z-Source Inverter with a dual closed loop controller. Figure. 12 (a) show the capacitor voltage follows the reference signal $V_{cref}$ very well, and the output voltage and current is kept constant. The dynamic overshoot on the capacitor Voltage and PN voltage $V_{pn}$ across the device are less than 5%. Figure. 12 (b) shows the waveforms of Bi-ZSI when input voltage $V_{in}$ varies. The voltage on capacitor keep track to the reference signal quiet well. With the forward control, the overshot on the capacitor Voltage and PN voltage $V_{pn}$ are eliminated. For the sake of safety and reliability, it is very important to limit the voltage/current stress on the power switches and capacitor during the dynamic period. The peak value of voltage $V_{pn}$ decreases when the input voltage increases, which means the shoot through duty cycle decrease, and vice-versa. In this case, with the $V_{in}$ variation, the Bi-ZSI inverter can operate switching between boost mode and normal mode automatically.

![Figure 12 simulations of Bi-ZSI with close loop controller](image)

Figure 12 simulations of Bi-ZSI with close loop controller
Upper: Va- output phase voltage (blue), Ia- output current (red);
Middle: Vc-capacitor voltage (blue), Vcref- voltage reference (red);
Vin- input voltage (green).
Lower: Vpn- voltage on switches (blue), Vin- input voltage (red).

**VI. PROTOTYPE & EXPERIMENTAL RESULTS**

A 55kW Bi-Directional Z-Source inverter prototype for electrical vehicles has been developed and is shown in Figure. 13.

![Figure 13. 55kW Bi-Directional Z-Source inverter prototype.](image)

Figure 13. 55kW Bi-Directional Z-Source inverter prototype.

Figure 14 shows the static performance of the Bi-Directional Z-Source Inverter. Figure 14 (a) shows the capacitor voltage follows the reference signal $V_{cref}$ very well and the static error is less than 1%. Figure 14 (b) shows the capacitor voltage and PN voltage $V_{pn}$ across the device at the same time. The inductor current $I_L$ is continuous and bi-directional, which confirms that the bi-directional Z source inverter can eliminate the DCM condition and provide the circuit with a bi-directional power flow function.

Figures 15 and 16 show the dynamic performance of the Bi-Directional Z-Source Inverter. Figure 15(a) shows starting waveforms of the Bi-ZSI when $V_{in}$ is equal to 200V. The capacitor voltage keeps track to the reference signal $V_{cref}$, the voltage overshot is less than 5%, and dynamic response time less than 1ms. Figure 15 (b) shows start and stop waveforms in one trace, which confirms that the dynamic response of the Bi-ZSI system is rapid.

Figure 16(a) shows the waveforms of the Bi-ZSI when $V_{in}$ varies from 140V~210V. The voltage on the capacitor has a little variation along the reference signal $V_{cref}$, and the voltage error is less than 5%.
Figure 16(b) shows the voltage $V_{pn}$ and inductor current $I_L$. When the input voltage increases, in order to keep $V_{cap}$ constant, the shoot through period become small gradually, the Bi-ZSI operate mode change from boost mode to normal mode automatically, the peak value $V_{pn}$ and $I_L$ decrease gradually, and vice versa.

Figure 14. Static performances of the Bi-ZSI.

Figure 15. Dynamic start/stop performance the Bi-ZSI.

Figure 16. Dynamic performance of the Bi-ZSI with Vin variation.

**VII. CONCLUSION**

In this paper, a new topology bi-directional Z-source inverter is proposed. The topology can operate in a DCM condition with a small inductor as well as overcome the uncontrollable and unstable limitations of the Z-source inverter and improve system stability. Furthermore, the new topology provides the circuit with bi-directional power flow capability. The operating principle and modes of bi-directional Z-source inverter have been analyzed. Based on the SFG method, a small signal model and transfer functions were developed. Meanwhile, system stability analysis is provided with Bode plot. For closed loop control, the two control variables are decoupled. The dual closed loop controller with an inner current loop and outer voltage loop are effective to improve the stability and dynamic performance. Experimental results verify the principle of the proposed bi-directional Z-source inverter. Static & dynamic performance and stability of the Bi-ZSI suggest that the topology to be excellent for electrical vehicle applications, as well as renewable energy applications.

**REFERENCES**


